If $\theta_1$ is a linear function of $x$, $y$, $z$, it is possible to choose the quadric (5) so that by means of (7) and (14) $\theta_1/\sqrt{\sigma}$ is expressible as a homogeneous linear function of $X$, $Y$, $Z$, $W$, involving $W$ at least. Hence:

When the function $\theta_1$ determining a transformation $K$ of a surface $S$ is a linear function of the cartesian coordinates of $S$, the corresponding tangent planes to $S$ and its transform meet in line of a fixed plane.

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1. Transformations of conjugate systems with equal point invariants, *Trans. Amer. Math. Soc.*, 15, 397–430 (1914). This will be referred to as memoir $M_1$.

2. Conjugate systems with equal tangential invariants and the transformation of Moutard, *Palermo, Rend. Circ. Mat.*, 39, (1915). This will be referred to as memoir $M_2$.


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**ON THE POLE EFFECT IN THE IRON ARC**

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In a communication to the Academy which appeared in the *Proceedings* for March 1915, we reported some results of our investigations on the pole effect in the iron arc under normal conditions. We have found between two and three hundred lines whose wave-lengths at the negative pole are distinctly longer than when the light is taken from a cross section at the center of the arc.

Aside from the theoretical interest in such changes in wave-length, reference may be made to the following points:

1. A number of these lines are included among the international standards of the second order. Their wave-lengths depend upon interferometer measurements made by three independent observers, the means of which have been adopted as standards by the International Union for Coöperation in Solar Research.

2. There is a region of the iron spectrum extending from $\lambda$ 5500 to $\lambda$ 6000 in which no other class of lines is available for standards.

3. In various laboratories there are in progress redeterminations, based upon the iron standards, of the wave-lengths in international units of the lines of many elements. In these redeterminations the instrument most frequently used is the concave grating in the usual Rowland mounting. In ordinary practice, the slit of the spectrograph is parallel to the axis of the arc and includes the major portion of its length. We have found that the pole effect appears at a considerable distance from the negative pole and that for high precision
the light should not be taken from a point less than 2 mm. from the pole of the arc. The astigmatism of the concave grating in the usual mounting introduces more or less pole effect and to that degree vitiates results involving the lines under consideration. The practice of reversing the current in the arc in order to overcome the tendency to produce wedge-shaped lines, when the slit and the axis of the arc are parallel, obscures but does not eliminate the pole effect. Since the redeterminations aim at a precision of 0.002 to 0.003 angstrom, it is important to take the pole effect into consideration.

4. Lines of the type considered are not limited to iron but occur in the spectra of other elements, the detailed investigation of which is necessary before safe deductions can be made from their use in astrophysical investigations.

Bearing upon the explanation of the pole effect, our recent investigations yield the following results. The absence of a general increase in pressure at the negative pole, evidenced by the unchanged wavelength of whole classes of lines known to be affected by pressure, led us to suggest an increased density of the radiating vapor as a possible cause, but our measurements of furnace spectra taken with a 10 fold increase in the quantity of iron vaporized show no change of wavelength for these lines. Furthermore, the lines of manganese appearing as an impurity in the iron poles show displacements of the same order as neighboring iron lines; it is difficult to imagine a high density, even at the pole, for manganese vapor appearing only as a trace.

The effect of temperature was examined by comparing the wavelengths of these lines at temperatures as widely different as is practicable in the furnace (2100°–2600°C.). No evidence was found of any dependence of wavelength upon temperature.

As the result of a study of the behavior of these lines in an arc in vacuo, it was found that the pole effect in general does not occur at pressures below 10 cm. of mercury. The arc which we employ in vacuo is of the same type and length and carries the same current as that used at normal pressure, but in appearance it is strikingly different. Under normal pressure the vapor mainly concerned in the production of these lines issues from a point source on the electrode, is highly luminous and is confined to a comparatively limited volume; in vacuo the luminosity covers the entire surface of the pole more uniformly and is much less intense, while definite structure is for the most part lacking. The disappearance of the pole effect indicates that the electrical conditions play a subordinate rôle, if any, in producing the displacements, but a more definitive investigation is about to be undertaken.
In view of the absence of a general increase in pressure at the negative pole, it is probable that some agency other than pressure is involved, but there is the possibility of a local increase in pressure affecting only the innermost portion of the vapor in the vicinity of the pole. To obtain light upon this point, a determination of the pressure shift per atmosphere has been made for a large number of the lines under consideration. Their wave-lengths in vacuo and at normal pressure have been compared, the light in each case being taken from the central section of the arc. These conditions are well suited to a measurement of the pressure shift for lines of this type, as they are of good quality in both spectra. In a paper by St. John and Ware they said:

Neither the small pressure-changes of about one-fifth of an atmosphere taken advantage of in this investigation, nor the high pressures used by Gale and Adams are well adapted to the study of lines of this type, and it is purposed to examine in vacuo and under normal pressure the behavior of an extended list of lines belonging to groups d and e.

If the pole effect is due to pressure alone, the connection between it and wave-length should be similar to that relating pressure shift and wave-length. A brief summary of our results is given in the accompanying table.

<table>
<thead>
<tr>
<th>Group</th>
<th>No. of Lines</th>
<th>Mean λ</th>
<th>Pressure Shift per Atm.</th>
<th>Pole Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>25</td>
<td>4085.14</td>
<td>+0.0048A</td>
<td>+0.0096A</td>
</tr>
<tr>
<td>e5</td>
<td>16</td>
<td>4766.41</td>
<td>+0.0093</td>
<td>+0.0119</td>
</tr>
<tr>
<td>d</td>
<td>12</td>
<td>5528.44</td>
<td>+0.0089</td>
<td>+0.0206</td>
</tr>
<tr>
<td>d</td>
<td>5</td>
<td>6350.74</td>
<td>+0.0160</td>
<td>+0.0185</td>
</tr>
</tbody>
</table>

The three sections of group d show pressure displacements which vary as the cube of the wave-length, a result in harmony with the observations of Gale and Adams. On the other hand there is no relation apparent between pole effect and wave-length. If we attempt to determine the difference in pressure between the center of the arc and a point in the core near the negative electrode by comparing the last two columns in the table, we find 2.0; 1.3; 2.3; and 1.1 atmospheres, numbers whose discrepancies exceed the errors of observation. Such a treatment of the data assumes that the pressure in the core of the arc at the negative pole is the same for all wave-lengths. The increase of pressure, if it exists, must be produced by rapid vaporization and the almost explosive expulsion of the emission centers, which are not necessarily the same for all wave-lengths. Investigations are now to be undertaken in order to obtain some measure of the velocities of the emitting particles.
Summary. 1. Emphasis has been placed upon the necessity of considering the pole effect in the redetermination of wave-lengths in international units.

2. The wave-lengths of these sensitive lines are not affected by a wide variation of density of the radiating vapor.

3. Their wave-lengths are independent of changes in temperature over the range of our observations.

4. For the lines considered the pole effect does not occur in vacuo, and in so far appears independent of electrical conditions.

5. The observed pole effect does not vary with wave-length in the same way as pressure shift, and cannot be explained as a pure pressure effect unless the pressure changes in certain definite ways with the wave-length.

INHERITANCE IN THE ASEXUAL REPRODUCTION OF HYDRA VIRIDIS

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Do heritable variations commonly occur among the offspring of a single individual multiplying asexually? May selection among such offspring produce strains differing in hereditary characters? The investigation here resumed is designed to contribute data toward the answer to these much debated questions.

A number of specimens of Hydra viridis, taken at random from wild populations, gave rise by asexual reproduction to clones differing from one another in their average number of tentacles and in other characters. As a test of whether such differences are the result of internal factors or of environmental differences two clones were bred in large numbers for a period of five months, during which time the members of the two clones were kept under environmental conditions as nearly as possible the same. Each polyp was kept in a separate culture dish and food was distributed uniformly to all. The number of tentacles was recorded at the time when the polyps began their independent life after separation from the parents (the initial number of tentacles), and records were kept, also, of changes in the number of tentacles of parents.

The two clones kept under parallel conditions gave the following results. The average number of tentacles of 1353 members of one