Corrections

PHYSICS. For the article “Branch-cut singularities in thermodynamics of Fermi liquid systems,” by Arkady Shekhter and Alexander M. Finkel’stein, which appeared in issue 43, October 24, 2006, of Proc Natl Acad Sci USA (103:15765–15769; first published October 12, 2006; 10.1073/pnas.0607200103), the authors note that, due to a printer’s error, Fig. 2 appeared incorrectly. The corrected figure and its legend appear below.

Fig. 2. The diagrams at the top present the ring diagrams in the electron-hole channel. The diagram on the left shows the two-section term that is controlled by the backward scattering; the momenta in the four Green’s functions are along the same direction: 1, 2 = +k_F and 3, 4 = −k_F. The Green’s functions are numbered to keep track of them after the rearrangement in different channels. The diagram on the right at the bottom shows how the two-section term can be read in the 2k_F channel. Here, the shaded areas represent the interaction amplitudes in the 2k_F channel. The lines inside the shaded areas are drawn to clarify the spin structure and to indicate the source of the relevant renormalizations. The diagrams on the left at the bottom show the result of twisting of the two-section term into the Cooper channel. In the series of Cooper ladder diagrams obtained in this way, only two sections (marked by numbered Green’s functions) are responsible for the linear in T term in the spin susceptibility. The role of all other sections is to renormalize logarithmically the e−e interaction amplitudes.

www.pnas.org/cgi/doi/10.1073/pnas.0609409103
PLANT BIOLOGY. For the article “Cryptochrome-1-dependent execution of programmed cell death induced by singlet oxygen in Arabidopsis thaliana,” by Antoine Danon, Núria Sánchez Coll, and Klaus Apel, which appeared in issue 45, November 7, 2006, of Proc Natl Acad Sci USA (103:17036–17041; first published October 30, 2006; 10.1073/pnas.0608139103), due to a printer’s error Fig. 1 was printed in black and white. The correct figure and its legend appear below.

Fig. 1. Blue-light dependency of the cell death response induced in seedlings of the flu mutant. Seedlings of flu were kept under continuous light for 5 days, followed by 15 h of darkness and exposure to white light for 1 h at 100 μmol·m⁻²·s⁻¹ to allow singlet oxygen production. The seedlings were subsequently placed for 23 h in white light at 100 μmol·m⁻²·s⁻¹ (A), in darkness (B), or in red (C) or blue (D) light, both at 10 μmol·m⁻²·s⁻¹. (E and F) Control seedlings that were kept for 24 h under red (E) or blue (F) light at 10 μmol·m⁻²·s⁻¹ without the 1-h white-light treatment.

www.pnas.org/cgi/doi/10.1073/pnas.0609704103

BIOPHYSICS. For the article “General structural motifs of amyloid protofilaments,” by Neil Ferguson, Johanna Becker, Henning Tidow, Sandra Tremmel, Timothy D. Sharpe, Gerd Krause, Jeremy Flinders, Miriana Petrovich, John Berriman, Hartmut Oschkinat, and Alan R. Fersht, which appeared in issue 44, October 31, 2006, of Proc Natl Acad Sci USA (103:16248–16253; first published October 23, 2006; 10.1073/pnas.0607815103), the authors note that the atomic coordinates discussed in this paper have been deposited in the Protein Data Bank, www.pdb.org (PDB ID code 2NNT).
Branch-cut singularities in thermodynamics of Fermi liquid systems

Arkady Shekhter* and Alexander M. Finkel’stein

Communicated by Elihu Abrahams, Rutgers, The State University of New Jersey, Piscataway, NJ, August 21, 2006 (received for review June 10, 2006)

The recently measured spin susceptibility of the two-dimensional electron gas exhibits a strong dependence on temperature, which is incompatible with the standard Fermi liquid phenomenology. In this article, we show that the observed temperature behavior is inherent to ballistic two-dimensional electrons. Besides the single-particle and collective excitations, the thermodynamics of Fermi liquid systems includes effects of the branch-cut singularities originating from the edges of the continuum of pairs of quasiparticles. As a result of the rescattering induced by interactions, the branch-cut singularities generate nonanalyticities in the thermodynamic potential that reveal themselves in anomalous temperature dependences. Calculation of the spin susceptibility in such a situation requires a nonperturbative treatment of the interactions. As in high-energy physics, a mixture of the collective excitations and pairs of quasiparticles can effectively be described by a pole in the complex momentum plane. This analysis provides a natural explanation for the observed temperature dependence of the spin susceptibility, both in sign and in magnitude.

T he temperature dependences of the thermodynamic quantities in the Fermi liquid have been originally attributed to the smearing of the quasiparticle distribution near the Fermi surface (1). This smearing yields a relatively weak, quadratic in temperature effect. A contribution of collective excitations, which in dimensions larger than one has a small phase space, has been ignored. There is a lacuna in this picture. Both the single-particle and collective excitations are described by poles in the corresponding correlation functions. However, besides the poles, there are branch-cut singularities originating from the edges of the continuum of pairs of quasiparticles. Such branch-cut singularities have not been given adequate attention in the theory of Fermi liquid systems. In the Fermi liquid theory, a rescattering of pairs of quasiparticles is considered for the description of the collective excitations that exist under certain conditions. This is not all that the rescattering of pairs does. Regardless of the existence (or absence) of the collective modes, the excitations near the edges of the continuum cannot be treated as independent as a consequence of the rescattering. The thermodynamics of Fermi liquid systems is not exhausted by the contributions of the single-particle and collective excitations. In interacting systems, as a result of the multiple rescattering, the branch-cut singularities generate anomalous temperature dependences in the thermodynamic potential.

Motivated by recent measurements in the silicon metal-oxide-semiconductor field-effect transistors (Si-MOSFETs) (2), we study here the temperature dependence of the spin susceptibility, \( \chi(T) \), in the 2D electron gas in the ballistic regime. Experimental results indicate that in the metallic range of densities and for temperatures exceeding the elastic scattering rate, \( T > 1/\tau_{el} \), the electrons in Si-MOSFET behave as an isotropic Fermi liquid with moderately strong interactions. In particular, the Shubnikov–de Haas oscillations both without and with an in-plane magnetic field indicate clearly the existence of a Fermi surface (3–5). The only observation (2) incompatible with the simple Fermi liquid phenomenology is a surprisingly strong temperature dependence of \( \chi(T) \). This behavior occurs in a wide range of densities that rules out proximity to a \( T = 0 \) quantum critical point as an explanation of the observed temperature effect. In this article, we show that such a temperature behavior of the spin susceptibility is inherent to 2D ballistic electrons. We explain the experiment by means of anomalous linear in \( T \) terms (6) generated by the electron–electron (e–e) interactions in \( \chi(T) \). In recent articles, linear in \( T \) terms have been studied intensely within perturbation theory (7–11). However, these works predict the susceptibility increasing with temperature, whereas the trend observed in the experiment is opposite. Taken seriously, this discrepancy indicates that we encounter a nonperturbative phenomenon. Here we show that a consistent treatment of the effect of rescattering of pairs of quasiparticles in different channels provides an explanation of the observed temperature dependence of \( \chi \).

How Anomalous Temperature Terms Are Generated in Spin Susceptibility

Technically, the multiple rescattering of pairs of quasiparticles is represented by ladder diagrams in which each section describes a propagation of a pair of quasiparticles between the rescattering events (see Fig. 1). The collective excitations reveal themselves as pole singularities in the ladder diagrams. When the pole enters into the continuum of two-particle excitations, collective excitation decays. Each of the intermediate sections in the ladder diagrams carries two branch-point singularities that reflect the edges of the continuum of pairs of quasiparticles. Therefore, the correlation function describing a free propagation of a pair of quasiparticles has a branch cut. The analysis of the effects of the branch-cut singularities on temperature dependences in the thermodynamic potential is the object of this article. In the thermodynamic potential, the contribution of the processes of multiple rescattering is given by the so-called ring diagrams, i.e., a series of closed ladder diagrams. For the ladder diagrams, the constraints imposed by the conservation of the momentum and energy are most effective because they are applied to a minimal number of quasiparticles. In this way, the dominant terms are generated in the thermodynamic potential. Otherwise, summations over a large number of intermediate states smear out the singularities generated by the rescattering processes.

We have to consider series of the ring diagrams in three different channels, i.e., in the particle-hole (p–h), the particle-particle (Coo–per), and the \( 2\sqrt{p} \)-scattering channels. The first two channels are standard for Fermi liquid theory. The third one is known mostly in connection with the Kohn anomaly in the polarization operator (12). We start by analyzing the anomalous temperature terms in the

Author contributions: A.S. and A.M.F. performed research; A.S. and A.M.F. analyzed data; and A.S. and A.M.F. wrote the paper.

The authors declare no conflict of interest.

Abbreviations: e–e interaction, electron–electron interaction; p–h, particle-hole; Cooper, particle-particle.

*To whom correspondence should be addressed. E-mail: arkadi shekhter@weizmann.ac.il.

© 2006 by The National Academy of Sciences of the USA

www.pnas.org/cgi/doi/10.1073/pnas.0607200103
p–h channel. Within Fermi liquid theory, the e–e interaction amplitude depends on the angle between the incoming and outgoing directions of a scattered particle θ and θ' and commonly is described in terms of the angular harmonics. To understand how the anomalous temperature terms are generated in the spin susceptibility, let us assume for a moment that the zero harmonic, $\Gamma_0$, dominates the interaction amplitude: $\gamma = \Gamma_0$. In the case of a single harmonic, propagation of a p–h pair is described by the angular averaged dynamic correlation function,\(^1\) $S_0 = \langle S(\theta) \rangle = \om/\sqrt{(\om + \Delta)^2 - (qvp)^2}$, where $\Delta = g\mu_B(1 + \Gamma_0)H$ is the spin split energy induced by an external magnetic field $H$, and $(1 + \Gamma_0)$ describes the Fermi liquid renormalization of the $g$ factor. The function $S_0$ is imaginary when (for a given momentum) the frequency lies within the continuum of the p–h excitations. The edges of the p–h continuum reveal themselves in $S_0$ as branch-point singularities. Because the position of the branch cut $|\om + \Delta| < qvp$ depends on the magnetic field, the magnetization of the electron gas becomes sensitive to the analytical properties of the two-particle correlation function near the edge of the continuum. The series of ladder diagrams describing the rescattering of p–h pairs generates the following term in the magnetization (for derivation see Eqs. 12 and 13 in Supporting Text, which is published as supporting information on the PNAS web site):

$$\delta M = \int_{-\pi}^{\pi} \frac{d\om}{2\pi} \coth \left( \frac{\beta \om}{2} \right) \chi_{q, \om}^\gamma \left( \frac{\gamma \om + \sqrt{\om^2 - (qvp)^2}}{\gamma \om + \sqrt{\om^2 - (qvp)^2}} \right)^2$$  \hspace{1cm} [1]

where $\om = \om + \Delta$: we temporarily put $g\mu_B(1 + \Gamma_0)/2$ equal to 1. Besides the branch-cut singularities originating from the p–h continuum, the expression in Eq. 1 exhibits a pole generated by $\gamma \om + \sqrt{\om^2 - (qvp)^2}$, which determines the spectrum of the collective excitations, i.e., the spin-wave excitations (14). Note that the expansion, either in $\gamma$ or in $\Delta$, destroys the subtle structure of the denominator, changing its analytical properties. Obviously, we encounter a nonperturbative phenomenon.

In the case of a weak magnetic field, $\Delta < T$, the collective excitations and the continuum of p–h excitations are not well separated. Therefore, calculations of the thermodynamic quantities, e.g., magnetization, should be performed with care as the contributions from the collective and single-particle excitations are not independent. Performing the $q$ integration by contours in the complex $q$ plane (one should keep in mind that the analytical properties in the $\om$ plane differ from that in the $q$ plane), we find that this mixture of excitations is captured effectively by a pole in the complex momentum plane. This finding is reminiscent of the Regge pole description of the scattering processes in high-energy physics (15). For $\Delta \neq 0$, the $q$ integral is nonvanishing only when the pole in the complex $q$ plane (a footprint of the spin-wave excitations) moves into the imaginary interval.\(^2\) This move occurs within an interval $-\Delta < q < -\Delta/(1 + \gamma)$. At small $\gamma$, this interval has a width $\gamma \Delta$ that, by the way, explains why in $\delta M/\delta \Delta$ we cannot set $\Delta$ to zero. Only after the $q$ integration, we get for $\delta M$ an expression that (at nonzero $T$) is regular both in $\gamma$ and $\Delta$:

$$\delta M = -\frac{\nu}{2e_F} \int_{-\Delta}^{\Delta}(\om + \Delta) \coth \left( \frac{\beta \om}{2} \right) d\om,$$  \hspace{1cm} [2]

where $\nu$ is the density of states (per spin) at the Fermi surface. Expanding in $\Delta$, we obtain a linear in $T$ correction in the spin susceptibility:

$$\delta \chi_{\nu, p} = -2\nu \frac{T}{e_F} \left( \ln \frac{1}{1 + \gamma} + \frac{\gamma}{1 + \gamma} \right).$$  \hspace{1cm} [3]

A comment is in order here. At first glance, a linear in $T$ term in $\chi(T)$ cannot be reconciled with the third law of thermodynamics, $S_{T=0} = 0$, in view of the Maxwell relation ($\partial M/\partial T)_H = (\partial S/\partial H)_T$. This observation is the core of the statement on the textbook level that the paramagnetic behavior with $\chi^{-1} = T + const$ cannot exist at sufficiently low temperature (see, e.g., ref. 16). The well known vanishing of the coefficient of thermal expansion at the absolute zero has the same origin. In this kind of argumentation, it is assumed indirectly that the thermodynamic potential has a regular expansion in both of its arguments around $H, T = 0$.\(^3\) In fact, Eq. 2 demonstrates that the magnetization $\delta M = TH\chi(H/T)$ has a strong dependence on the order of limits $H, T \rightarrow 0$. We see from Eq. 2 that $\delta M \propto HT$ when $T > H$, but for $T < H$, the temperature dependence disappears and $\delta M \propto HT$. The solution to the conflict with the third law of thermodynamics is that the magnetic field range over which $\delta M \propto HT$ shrinks to zero as $T \rightarrow 0$, and $\delta M$ acquires a nonlinear in $H$ behavior outside this range. At $T \rightarrow 0$, which unavoidably brings us into the region $T < H$, the only indisputable condition imposed by the third law is limited to vanishing of $(\partial M/\partial T)_H$. Evidently, Eq. 2 complies with this requirement at $T/H \rightarrow 0$. Therefore, the existence of a linear in $T$ correction in the spin susceptibility is legitimate and may persist down to $T \rightarrow 0$, provided that $T > H$.

**Why Spin Susceptibility Decreases with Temperature**

The spin susceptibility as given by Eq. 3 contradicts the experiment. According to Eq. 3, the spin susceptibility should increase with $T$, whereas in the experiment it decreases. Below we offer a resolution to this puzzle. We would like first to indicate a subtlety in the thermodynamic potential term with two rescattering sections [i.e., in the term proportional to $G^2$ (see Fig. 2; obviously in the ring diagrams the number of sections is equal to the number of the interaction amplitudes)]. We show below that the term $\sim G^2$ in the spin susceptibility is heavily dominated by the scattering sharply peaked near the backward direction, $\theta - \theta' = 0$ (throughout the article the term $``\text{backward scatter-}$

\(^1\) We work with the dimensionless static amplitudes known in Fermi liquid theory (13) as $G$, whereas the propagation of a p–h pair is described by the dynamic correlation function $S(\theta) = \om/\sqrt{(\om + \Delta)^2 - (qvp)^2}$, see Eq. 7 in Supporting Text. Repulsion corresponds to $G > 0$, and Pomernack's instability is at $G = \infty$.

\(^2\) We are not aware of a similar discussion of the thermal expansion coefficient (as well as elastic constants) at low temperatures. In the context of the spin susceptibility, the question has been raised by Misawa (17) who guessed (incorrectly) a nonanalytic form of the thermodynamic potential.

\(^3\) An alternative calculation without referring to the complex $q$ plane is presented in Supporting Text.
Fig. 2.  The diagrams at the top present the ring diagrams in the electron-hole channel. The diagram on the left shows the two-section term that is controlled by the backward scattering; the momenta in the four Green’s functions are along the same direction: 1, 2, 3, 4. The Green’s functions are numbered to keep track of them after the rearrangement in different channels. The diagram on the right at the bottom shows how the two-section term can be read in the 2kF channel. Here the shaded areas represent the interaction amplitudes in the 2kF channel. The lines inside the shaded areas are drawn to clarify the spin structure and to indicate the source of the relevant renormalizations. The diagrams on the left at the bottom show the result of twisting of the two-section term into the Cooper channel. In the series of the Cooper ladder diagrams obtained in this way, only two sections (marked by numbered Green’s functions) are responsible for the linear in T term in the spin susceptibility. The role of all other sections is to renormalize logarithmically the e–e interaction amplitudes.

δχ(2) = ν \frac{T}{ε_F} \sum_{mn} (-1)^{n+m} Γ_m Γ_n.

Because Σn(−1)nΓn is equal to the backward-scattering amplitude Π(π), this contribution reduces to δχ(2) = (T/ε_F)Π2(π). We have checked that exactly the same result can be obtained by the calculation of two rescattering sections in the Cooper channel or in the 2kF channel. In the calculation of the Cooper channel, we use the angular harmonics of the particle-particle correlation functions. Once again, despite the nontrivial dependence of these correlation functions on their harmonics indices, the result reduces to the backward-scattering amplitude (19). Moreover, this calculation yields the same coefficient as in Eq. 4. In the case of the 2kF channel, the presence of Π′(π) in Eq. 4 is evident, but one has to check the coefficient. On the level of two rescattering sections, the contributions generated in three channels coincide (i.e., in δχ(2) all three channels overlap), as we described above.

We now analyze the problem of the renormalizations of the linear in T terms. It is easy to check that, unlike the case of 1D electrons (18), the higher-order terms in the 2kF-scattering channel are not important in 2D. Therefore, we will not discuss this channel further and will concentrate on the interplay between the other two channels. Up to this point, the interaction amplitudes have played a rather passive role in our calculations. The peak near the backward-scattering direction has been generated by the dynamic correlation functions describing the propagation of pairs of particles in each of the channels. The interaction amplitudes simply have supplied a featureless coefficient Π(π) in the two-section term. To understand the true role

In fact, an arbitrary number of rescattering sections appears in the Cooper channel after such twisting, but only two of them are used here for the extraction of the anomalous in temperature terms. The role of all other sections is to renormalize logarithmically the e–e interaction amplitudes in the Cooper channel. In the text, we refer the term “section” in the Cooper channel only to those of them that generate linear in T terms; this allows us to speak simultaneously about two sections and the renormalized e–e amplitudes without confusion.

A calculation with the use of angular harmonics has been performed in ref. 11 for the anomalous terms in the specific heat; it also leads to the backward-scattering amplitude Π(0).
of the e–e interaction in the anomalous temperature corrections, we have to abandon the central assumption of the microscopic Fermi liquid theory that different sections in the ladder diagrams are independent. Indeed, when the rescattering is dominated by the backward scattering, a strong dependence of the interaction amplitude on its arguments in the p–h channel emerges from the logarithms in the Cooper channel [this is a weak version of the parquet known for 1D electrons (18)]. In view of this circumstance, in the case of two rescattering sections we have to take into consideration the dependence of the scattering amplitude \( \Gamma(p, -p + q + k, -p + k, p + q) \) on the arguments \( q \) and \( k \). We resolve the problem of the logarithms by moving the term with two rescattering sections to the Cooper channel where the logarithmic renormalizations originate. This move is possible because the terms with two sections in different channels coincide. Note also that as a result of moving the two-section term to the Cooper channel, we avoid the double counting of the two-section term. In this way, we resolve the overlap of the two-section term. In this way, we resolve the overlap of the two-section term. In this way, we resolve the overlap of the two-section term. In this way, we resolve the overlap of the two-section term.

In contrast to the previous calculations, this result provides the contribution to the spin susceptibility from the Cooper channel. The rescattering in the Cooper channel leads to the logarithmic renormalizations of the interaction amplitudes \( \Gamma_0^n(T) = \Gamma_0^n/(1 + \Gamma_0^n \ln \varepsilon_F/T) \), where \( \Gamma_0^n \) are harmonics of the amplitude in the Cooper channel. At sufficiently small temperatures, the repulsive amplitudes, \( \Gamma_0^n > 0 \), vanish as \( \Gamma_0^n(T) \sim 1/(\ln \varepsilon_F/T) \). [We do not consider here the developing of the instability for the attractive amplitudes (19) as it is most likely blocked by the disorder in the system studied in ref. 2.] Therefore, the linear in \( T \) terms generated in the Cooper channel are suppressed at low temperatures. Coming back to the discussion preceding Eq. 5, we now see that the logarithmic renormalization of the amplitudes in the term with two rescattering sections in the p–h channel results in full elimination of this term at low enough temperatures. Therefore, for the repulsive e–e interaction when only the zero harmonic is kept, the temperature dependence of the spin susceptibility is given by

\[
\delta \chi = -2\nu \frac{T}{\varepsilon_F} \left( \gamma^2/2 + \ln \frac{1}{1 + \gamma} + \frac{\gamma}{1 + \gamma} \right).
\]

In contrast to the previous calculations, this result provides the sign of the temperature dependence of the spin susceptibility that coincides with that observed experimentally (2). The expression in Eq. 6 has been obtained by summation of the ladder diagrams in two channels and taking into consideration the overlap of the two-section term. In this way, we resolve the puzzle of the sign of the temperature trend in \( \chi(T) \).

We next note that the intervention of the Cooper renormalizations in the p–h channel is effective only for the term with two rescattering sections. We have checked that the situation with a dominant role of the backward scattering is not general and it does not occur for terms with more than two rescattering sections. A calculation of the term with three interaction amplitudes, \( \delta \chi(3) = (T/\varepsilon_F)^2 \int \alpha(\theta_1, \theta_2, \theta_3) \Gamma(\theta_1 - \theta_2) \Gamma(\theta_2 - \theta_3) \Gamma(\theta_3 - \theta_1) \frac{d\theta_1 d\theta_2 d\theta_3}{d^3 \theta_1 d^3 \theta_2 d^3 \theta_3} \), performed with the use of the methods sketched above, shows that there is only a weak (logarithmic) singularity near the backward scattering. This singularity is far weaker than the sharp \( \delta \) function peak near the backward direction, \( \theta - \theta' = \pi \), in the case of two rescattering sections. It is therefore safe to conclude that, unlike the case of the two-section term, the logarithmic renormalizations are ineffective for three and more sections in the p–h channel.

We now consider the contribution to the spin susceptibility from the Cooper channel. The rescattering in the Cooper channel

\[
\delta \chi = \delta \chi_{p-h} - \delta \chi_{(2)}.
\]

Finally, let us discuss the result of our analysis in connection with the measurement of the spin susceptibility in Si-MOSFET (2). In Fig. 3 the data for a quantity \( n/\chi \) are presented, where \( n \) is the density of the 2D electron gas. We focus here on the curves corresponding to the ballistic range of the densities. These curves exhibit a noticeable rise with temperature at \( T \sim 1.5 \) K, which is too strong for the conventional Fermi liquid theory; the conventional Fermi liquid temperature dependence will be invisible on scales used in the plot of Fig. 3. The data correspond to the range of densities and temperatures where the transport is ballistic. The rising curves in this plot indicate that the spin susceptibility decreases with temperature. We assume here that this temperature dependence is attributable to the term \( \delta \chi \) given by Eq. 6, which we multiply by the factor 4 to account for two valleys. At lower temperatures, the discussed effect of the anomalous temperature corrections is cut off by disorder. One can expand \( n/\chi \) with respect to the temperature corrections: \( \delta(n/\chi) = T f(\gamma) \), where \( f(\gamma) = \gamma^2/2 - \ln(1 + \gamma) + \gamma/(1 + \gamma) \). The modification of the spin susceptibility by the Stoner factor \((1 + G_0)\) drops out from \( \delta(n/\chi) \). It is cancelled by the two factors \((1 + G_0)\) in \( \delta \chi \) ignored so far because in the definition of \( \Delta \) the combination \( g_{\mu B}(1 + G_0)/2 \) has been put equal to 1. The main advantage of the combination \( \delta(n/\chi) \) is that its temperature dependence is determined by the dimensionless interaction amplitudes only. In the discussed range of densities, parameter \( r_i \) is \( \sim 3 + 4 \) \((r_i \) is the ratio of the energy of the e–e interaction to the kinetic energy). Therefore, one may expect the dimensionless interaction amplitudes to have a magnitude \( \sim 1 \). Perhaps even a few leading harmonics may be involved. For \( n \neq 0 \) harmonics enter in pairs, \( \Gamma_{r_n} = \Gamma_{-r_n} \), and consequently \( f(\gamma) \) should be slightly modified because of mixing between \( \Gamma_{r_n} \) and \( \Gamma_{-r_n} \); see Supporting Text for details. When the amplitude \( \gamma \sim 1 \), the function \( f(\gamma) \) is of order unity (e.g., for \( \gamma = 1.5 \) it is equal to 0.7). The slope of the curves \( \delta(n/\chi) \) presented in Fig. 3 is also \( \sim 1 \), i.e., of the same order of magnitude. Together these facts support our conclusion that at low temperature, the sign and the magnitude of the temperature dependence of the spin susceptibility can be explained by the theory of the anomalous corrections presented in this article. At temperatures comparable with the Fermi energy, the logarithmic suppression of the interaction amplitudes in the \( \delta \chi_{(2)} \) term should become ineffective. If so,
when $1/\ln(e_F/T) \geq \gamma$, the temperature dependence will change sign, leading to a nonmonotonic spin susceptibility. Unfortunately, the temperature range of the existing measurement, $T/e_F \leq 0.1$, does not allow us to verify this consequence of our theory.

To conclude, the thermodynamics of Fermi liquid systems is not exhausted by the contributions of the single-particle and collective excitations. These two types of excitations are described by the poles in the corresponding correlation functions. However, the theory of Fermi liquid systems is not complete without consideration of the branch-cut singularities. In interacting systems, as a result of the rescattering of quasiparticles, the branch-cut singularities generate nonanalyticities in the thermodynamic potential that reveal themselves in anomalous temperature dependences. The observed temperature dependence in the spin susceptibility of the 2D electron gas can be explained in this way. The mechanism determining the sign of the anomalous terms in the spin susceptibility discussed here may have implications for the physics near the quantum critical point at the ferromagnetic instability.

We thank M. Reznikov and C. Varma for valuable discussions. A.M.F. is supported by the Minerva Foundation.