THE SPACE MOTIONS OF STARS IN THE ORION AND SCORPIO-CENTAURUS CLUSTERS

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1. Introduction.—Since the appearance of Kapteyn's exhaustive investigations of the extended moving clusters in the regions of Orion and Scorpio-Centaurus, the motions of the B-type stars have been considered primarily with regard to second order effects such as galactic rotation and the K-term. However, the recent publication of catalogues of radial velocities and spectroscopic parallaxes of B stars suggests the possibility of gaining additional information concerning the general state of the motions within these two clusters. In Harvard Bulletin 886 were published some preliminary results which seemed to call for a more extensive study. Accordingly, the present paper summarizes an investigation of the space motions of 166 stars of classes B0–B9 in the region of Scorpio-Centaurus, and 57 in that of Orion. The limits of these areas in galactic coordinates are those fixed by Kapteyn: for the former, \( l = 216-360^\circ, b = \pm 30^\circ \); and for the latter, \( l = 150-216^\circ, b = \pm 30^\circ \).

2. Computation of the Velocity Components.—Lick Observatory radial velocities,\(^a\) Boss proper motions\(^b\) and spectroscopic parallaxes by C. J. Anger\(^c\) and W. B. Rimmer\(^d\) have been utilized for the computation of the space velocities of the stars mentioned above. The corrections of Boss and Jenkins\(^e\) and of Oort\(^f\) have been applied to the proper motions, which are therefore as free as possible from systematic errors. The spectroscopic parallaxes from the two sources have been reduced to a mean zero point, those contained in Dr. Anger's second list being first reduced to the scale and zero point of her first paper. From this material, the galactic components of the space velocities have been computed in the usual manner. The \(+X\) axis is here defined as lying toward \( l = 0^\circ, b = 0^\circ \); the \(+Y\) axis toward \( l = 90^\circ, b = 0^\circ \); and the \(+Z\) axis toward \( b = 90^\circ \).

A check on the results was provided by the tables of A. Köhlschutter.\(^g\) The components have been corrected for a solar motion of 20 km./sec. toward R. A. 18\(^h\) 20\(^m\), Dec. +30\(^o\). Unless otherwise stated, all velocities...
referred to in this paper are so corrected. In addition, it is necessary to consider the effects of galactic rotation, and $K$-term. The former may amount to 2 or 3 km./sec., but its influence, if any, will depend directly on distance from the galactic center, and should therefore be easy to detect. The effect of $K$-term, on the other hand, cannot be predicted. Corrections assuming $K = +4$ km./sec. were therefore computed, and could be omitted or applied at will. It may be supposed that the true velocity distribution will lie between the distributions found for the corrected and uncorrected velocities.

3. Errors of the Observed Quantities.—In a recent paper Plaskett and Pearce find that the Lick radial velocities differ by only $-0.4$ km./sec. from their mean system. The systematic errors must therefore be small. They also find $\pm 2.03$ km./sec. as the mean probable error of one velocity. This and other published results indicate that the probable error may be assumed to be $\pm 2.0$ km./sec.

The systematic errors of $\mu_x$ and $\mu_y$ may be comparatively large. Since they must be of the same order of magnitude as the uncertainties in the corrections which were applied, it is not unlikely that they are as great as 0.004, about 10% of the proper motion in either coordinate. The mean probable error is found from Boss's catalogue to be close to $\pm 0.006$ in both coordinates.

It is difficult to make a preliminary estimate of the probable size of the systematic errors in the spectroscopic absolute magnitudes. It is therefore permissible to present here some results which were derived later in the investigation. If the systematic errors in the proper motions are as large as indicated above, the stream parallaxes on which the adopted zero point is partly based may be in error by an amount equivalent to 0.2 magnitude in the resulting absolute magnitudes. A comparison of the absolute magnitudes of Dr. Anger and Rasmuson, the latter derived from stream parallaxes, shows systematic differences of 0.2 magnitude, depending on distance from the convergent of the cluster. Finally, the stream velocity of the Scorpio-Centaurus cluster as derived from the cross motions alone is but 84 per cent of that found from the radial velocities. The discrepancy may be partly accounted for by supposing that the assumed absolute magnitudes are, in the mean, too faint. These considerations, however, do not unite in giving evidence as to the direction in which the zero point is in error, but serve to indicate that this error may be as large as 0.2 or 0.3 magnitude. The probable error of an absolute magnitude is certainly no greater than $\pm 0.5$ magnitude.

4. Treatment of the Material.—It was desired to present the general characteristics of the velocity distributions in graphical form. The direction of motion and projected velocities in the $XY$, $XZ$ and $YZ$ galactic planes have been computed for each star. The direction in the $XY$ plane
corresponds to the galactic longitude of the direction of motion of the star, while the directions in the two remaining planes are defined by \( \theta_{XZ} = \tan^{-1} \frac{V_Z}{V_X} \), and \( \theta_{YZ} = \tan^{-1} \frac{V_Z}{V_Y} \). Then for each plane, the sum of the projected velocities falling within successive ten degree intervals of the corresponding angle \( \theta \) was computed, the results smoothed by taking running means of three ten degree intervals, and the smoothed means plotted against the corresponding values of \( \theta \). This procedure, it will be seen, is similar to that employed by Kapteyn\textsuperscript{11} in his first discussion of star streaming as exhibited by proper motions.

5. The Theoretical Velocity Distribution.—Before discussing the observed velocity distribution, it is necessary to consider in detail the effect of accidental errors. In the study of proper motions in a limited region of the sky, it may be expected that the accidental errors of \( \mu_x \) and \( \mu_y \) will be nearly the same. In dealing with space motions, the corresponding assumption cannot be made concerning the errors of the three space velocity components. The components are computed from the radial velocities and cross motions, and in a specific case, one space velocity component may be influenced primarily by the errors of the radial velocities, another by those of the proper motions and parallaxes. Since the radial velocities are observed in a manner fundamentally different from the cross motions, it cannot be assumed \textit{a priori} that the errordispersions along the \( X, Y \) and \( Z \) axes will be the same. It is apparent that the general problem is analytically similar to that of the Schwarzschild Ellipsoidal Hypothesis, which also supposes unequal velocity dispersions along the axes, and a superposed group motion relative to the sun. Considering only two dimensions, three cases merit consideration. The first is that in which the error dispersion is the same in both coordinates, the second that in which the dispersion perpendicular to the direction of group motion is the larger, and in the third the axis of greatest dispersion is oriented arbitrarily with respect to the direction of motion of the cluster. The necessary formulæ may be adapted from those given by Eddington\textsuperscript{12}. For the first case, let \( N \) be the total number of stars, \( V \) the group velocity, \( r \) the observed velocity of any star and \( h \) related to \( \eta \), the observed mean peculiar motion in either coordinate by \( h = 0.5641/\eta \). Then the number of stars moving with velocities between \( r \) and \( r + dr \) in directions between \( \theta \) and \( \theta + d\theta \) is given by

\[
\frac{Nh^2}{\pi} e^{-h^2(r^2 + V^2 - 2rV \cos \theta)} r dr d\theta
\]

whence it follows that the sum of the velocities of all stars with velocities equal to or less than \( R \), moving between \( \theta \) and \( \theta + d\theta \), is
\[
\frac{Nh^2}{\pi} d\theta \int_0^R e^{-kt(r^2 + V^2 - 2rV \cos \theta)} r^2 dr.
\]

It is necessary to set a finite upper limit \( R \), since the computed curve is for comparison with those found by observation, and in practice, the inclusion of stars of extremely high velocity produces irregularities out of proportion to the number of stars responsible. The function was integrated numerically with \( V = 5 \) km./sec., and a value of \( h \) chosen on the assumption that the mean of the errors in either coordinate combined with a true mean peculiar motion of \( \pm 2.0 \) km./sec. is \( \pm 3.8 \) km./sec. The integration was performed with two values of the upper limit \( R \). The resulting curves have a single symmetrical maximum, and are both strongly concentrated in the direction of motion of the cluster; the higher the upper limit, the more concentrated the curve. The development of the second case is similar to the preceding one. The number of stars with velocities between \( r \) and \( r + dr \) traveling between \( \theta \) and \( \theta + d\theta \) is given by

\[
\frac{Nh}{\pi} e^{-r(t + h \cos \theta + h \sin \theta)} + 2rV \cos \theta - kV^2 r^2 dr d\theta
\]

which leads at once to the following expression for the sum of all velocities less than or equal to \( R \) lying between \( \theta \) and \( \theta + d\theta \):

\[
\frac{Nh}{\pi} e^{-kV^2} d\theta \int_0^R e^{-r(t + h \cos \theta + h \sin \theta)} + 2rV \cos \theta - kV^2 r^2 dr.
\]

Here \( V \), \( r \) and \( R \) are defined as before, and \( k \) and \( h \) are related as described above to the observed mean peculiar motions along the \( X \) and \( Y \) axes. Since \( V \) is along the \( X \) axis, \( k \) will be larger than \( h \). For the integration, \( V \) was again taken as \( 5 \) km./sec., and the observed mean peculiar motions along the \( X \) and \( Y \) axes were supposed to be \( \pm 2.5 \) and \( \pm 5.0 \) km./sec., respectively. The resulting distribution makes apparent the need for care in the interpretation of the observations. The function has a pronounced minimum in the direction of group motion, with maxima distant about \( 40^\circ \) on both sides. This is in general agreement with the appearance of the observed distribution discussed in Harvard Bulletin 886, and demonstrates that a non-spherical error dispersion is capable of simulating the existence of two intermingled clusters or streams. The appearance of the velocity distribution in the third case may be predicted qualitatively from its analogue in the Ellipsoidal Hypothesis. If the axis of greatest dispersion is inclined at some angle not \( 90^\circ \) to the direction of motion of the cluster, there will be two maxima of unequal size, the more pronounced being the nearer to the true direction of motion of the cluster.

6. Comparison with Observation.—The distribution of the velocities of
the fifty-seven stars of the Orion region is illustrated in figures 1a and 1b. The heavily drawn curves represent velocities corrected for K-term, the lighter curves are for velocities not so corrected. In addition, in figure 1b, centered on an arbitrary zero point, is included the theoretical distribution of the fifty-seven stars, computed on the assumption of a spherical error dispersion. Inspection indicates that while the inclusion of the K-term
correction effects numerous minor changes in the appearance of the curves, the general contours are unaltered. It will also be noted that the distributions are somewhat asymmetric, probably due to the fact that the error dispersion is not truly spherical. A comparison with the theoretical distribution shows that although the forms are not strictly comparable, the observed dispersion is no greater than would be anticipated if practically all the stars with velocities less than 21 km./sec. belonged to a single cluster. The number of velocities greater than 30 km./sec. is six, rather more than would be expected, but in all cases except one it may be shown that the high velocity is probably due to a large error in the parallax, or that the star is a very distant one lying far beyond the cluster. It may therefore be concluded that the great bulk of the class $B$ stars falling within the Orion region form a single homogeneous cluster.

In figures 1c to 1f are presented the principal characteristics of the velocity distribution of the stars in the Scorpio-Centaurus region. In figure 1c the dotted outline is a reproduction of the velocity distribution of the original ninety-five stars treated in Harvard Bulletin 886. It will be seen that its general appearance is well reproduced by the more complete material. Considering first the $XY$ velocities corrected for $K$-term, the outstanding feature is the double maximum representing the motions of the greater part of the stars. In figure 1d this curve is repeated, and, superposed on it, the theoretical distribution computed as in the second case of Section 5, on the supposition that the error dispersion is not spherical. The constants $V, h$ and $k$ in the function were chosen purely according to their most probable values. The only deliberate fitting consisted in assuming that there are only 105 stars in the cluster. The true number is larger, but in view of the number of disposable constants in such a function, it would have been entirely feasible to fit a curve with the right number of stars to the observed distribution. The purpose of the computation is served equally well, however, by this demonstration that the distribution of purely accidental errors can so well imitate the appearance of two intermingled clusters or streams. The distributions in the $XZ$ and $YZ$ planes are not reproduced here, since they contribute little information concerning the state of motion within this region. They show, however, a strongly systematic tendency toward positive $Z$ components. Since this is shared by the Orion stars, it may be the result of systematic errors, or an error in the assumed value of the $Z$ component of the solar motion. In addition to the principal maxima, there appear two secondary peaks which will be considered in a later section. However, it is clear that by far the greater part of the stars in this region form a single cluster, and that the appearance of a double stream is entirely due to an ellipsoidal error dispersion.

7. The Convergents.—The convergent of the Orion cluster has been
computed by taking the mean values of the $X$, $Y$ and $Z$ velocity components, respectively. Since all stars in the Scorpio-Centaurus region certainly do not belong to a single cluster, it seems that better results may be obtained simply from an inspection of the positions and forms of the principal maxima of the distribution curves. The elements given below are the means of the determinations with and without the $K$-term correction, and have been reduced to the sun for comparison with the values derived by Kapteyn. As inspection of the curves does not give the velocity, a value of 5 km./sec. was assumed for the Scorpio-Centaurus cluster for making the reduction to the sun.


<table>
<thead>
<tr>
<th>ORION CLUSTER</th>
<th>SCORPIO-CENTAURUS CLUSTER</th>
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<tbody>
<tr>
<td>KAPTEYN SPACE MOTIONS</td>
<td>KAPTEYN SPACE MOTIONS</td>
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<tr>
<td>$5^h 44^m$</td>
<td>$-11^\circ$</td>
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Part of the discrepancy between the velocities in the Orion cluster can be traced to the difference between the $K$-term correction applied by Kapteyn, and that assumed here.

8. **Secondary Maxima.**—Two secondary maxima appear in the heavily drawn curve of figure 1c. In the uncorrected distribution, both have disappeared, but there is still evidence of excess motion in the same directions. The more prominent peak at longitude 95° is of particular interest, since Kapteyn's elements for the Orion cluster, after being freed of solar motion, give a longitude of 108°, and Rasmuson finds 80°. Although the convergent indicated by the curves of figure 1a is nearer 130°, it must be remembered that the cluster velocities are on the order of 5 km./sec., and that the direction of the velocity vector is therefore at the mercy of systematic errors. Dr. Anger's space diagrams of the Scorpio-Centaurus and Orion clusters show that their outlying members appear to intermingle, and suggest that the Orion cluster is not highly concentrated, whence it would not be surprising to find a few of its scattered members in the region of space occupied by the Scorpio-Centaurus group. However, the physical reality of the secondary maximum must first be tested. It was noted in Section 5, that the concentration of a velocity distribution with a single maximum should be increased with increase of the upper limit of velocities included. This is, of course, only true if there be a physical moving group. If the maximum be purely accidental, it should be obliterated by the inclusion of stars of higher and higher velocity. Figures 1e and 1f illustrate the result of such an examination. The maximum at longitude 95° increases markedly in concentration, a result which should not appear were the original peak purely fortuitous. The mean velocity of the stars contributing to the peak is found to be in good
agreement with the mean velocity of the stars in the Orion region. Thus, on the grounds of the close proximity of the Orion and Scorpio-Centaurus clusters, and the agreement in velocity and direction of the motions of the stars in this subgroup and in the Orion cluster, it may be concluded that far flung members of the latter group have been detected in this manner.

No identification can be offered of the secondary maximum which appears at longitude 195°, although there is evidence that it is not accidental in character.

9. The True Velocity Dispersion.—Kapteyn considered that the most important results of his discussion of these two clusters were the small values (±1.67 and ±1.00 km./sec.) of the true mean peculiar motions. It is of interest to discover the extent to which the new material checks these figures. The true dispersion is best determined separately from the radial velocities and from the $\tau$ components reduced by means of the parallaxes to km./sec. It was found that after correcting for the probable errors of the radial velocities and proper motions given in Section 3 (the
parallax errors play no part here), the true mean peculiar motion in one coordinate as given by the radial velocities was from 1.5 to 5 times that derived from \( \tau \) components, no matter what method of selection was utilized to eliminate stars not members of the clusters. The only solution lies in assuming smaller errors in the proper motions, or larger ones in the radial velocities. Fortunately, an independent determination of the accidental errors of the proper motions is possible. The stars of either cluster may be divided into two or more groups according to distance from the sun, and for each group, the mean distance computed, and the observed mean peculiar motion determined from the \( \tau \) components. Since the error in km./sec. produced by a given error in the proper motions will be directly proportional to the distance of the star, while the true mean peculiar motion may be supposed constant over the cluster, one has two or more equations of the form

\[
\bar{u}_0^2 = \left[ \frac{4.74\tau \rho_{\mu}}{0.8454} \right]^2 + \bar{r}_u^2
\]

where \( \bar{u}_0 \) is the observed mean peculiar motion in one coordinate, \( \bar{r}_u \) the true mean peculiar motion, \( \bar{r} \) the mean distance and \( \rho_{\mu} \) the probable error of a proper motion component. These equations allow of a solution for the two unknowns, \( \bar{r}_u \) and \( \rho_{\mu} \). Dividing the Orion cluster into two groups, and the Scorpio-Centaurus into three, the true mean motions were found to be \( \pm 1.6 \) and \( \pm 0.8 \) km./sec., respectively, in close agreement with the determinations of Kapteyn, and the mean of the two solutions for \( \rho_{\mu} \) was found to be \( \pm 0.006 \), the value given in Section 3. This must indicate the existence of inherent accidental errors in the measured radial velocities which are not brought out by intercomparison of several measurements or catalogues. The true mean peculiar motions within the clusters are certainly less than \( \pm 2.0 \) km./sec.

10. Star Streaming.—There is no evidence of the two star streams in these space motions. However, Wilson and Raymond\(^{19} \) in a paper on "The Space Motions of 4233 Stars" discuss streaming from the viewpoint of each spectral class, and for 435 stars of class \( B \) determine the axes of the velocity ellipsoid. The division of the stars into the two streams is illustrated by a diagram in which the numbers of stars moving within successive thirty degree intervals of galactic longitude are plotted against longitude. It now seems probable that the two resulting maxima at longitudes 120° and 330° may be caused by stars of the Orion and Scorpio-Centaurus clusters, rather than by star streaming in the ordinary sense. Accordingly, the corresponding counts were made for the stars of the present paper, and the results embodied in figure 2. Here the upper curve is a reproduction of the diagram of Wilson and Raymond, and the
lower curves are derived from the present material. The general agreement between the two sets of curves is so close that there is good reason to suppose that the apparent existence of the two streams is in fact the outcome of the presence of the two large moving clusters.

The writer wishes to express to Dr. Bart J. Bok his appreciation of the continued advice and encouragement which have made this paper possible.

2 Lick Observatory Publ., 16 (1928); and L. O. B., 429 (1929).
3 Boss, Preliminary General Catalogue (1910).
4 Anger, H. C., 383 (1931), and H. C. 372 (1932).
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THE PENETRATION OF IRON METEORITES INTO THE GROUND

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Treatises on exterior ballistics show that objects moving under gravity in a resisting medium of uniform density ultimately descend with a velocity such that the resistance to motion balances the weight: an idea said to be traceable to Huygens. Schiaparelli extended the idea to meteors, and showed that, neglecting the attraction of the earth, the ultimate velocity approached is little affected by the initial or extra-atmospheric velocity, and little affected by the law of variation of air density with height [provided only that the mass of the air immediately disturbed is large in comparison to the mass of the object]. For large meteors the ultimate velocity does depend on the cosmic velocity. His analysis also neglected the effect of ablation by melting in the violent blast of hot air impinging on the front of the meteor.

With meteorites of ordinary size the striking velocity at the ground, being independent of the extra-terrestrial velocity, should depend only on the mass and dimensions of the object and on the density of the air at the ground. For use of this principle it is necessary to take recourse to a formula for spheres: