\[ \Delta = \left| \frac{\partial f_i(\alpha, \beta)}{\partial y_j} \right| \pm 0, \quad \alpha = (a_1, a_2, \ldots), \beta = (b_1, b_2, \ldots). \quad (9) \]

Suppose that there exists a number \( B \) such that

\[ \sum_{k=1}^{\infty} \left| D_{ki} \right| \leq B r_i \quad i = 1, 2, \ldots, \]

where \( D_{ki} \) is the cofactor of the element \( \partial f_i / \partial y_i \) in (9). Let it be assumed that for every \( e > 0 \) there can be found a number \( d_0 > 0 \) such that, for \( \left| x_j - a_j \right| \leq d_0 \) \( (j = 1, 2, \ldots) \) it follows that \( \left| f_i(\xi, \beta) \right| \leq e \) \( (i = 1, 2, \ldots) \). Then there exist positive constants \( (c, d) \), \( 0 < c \leq d_0 \) such that to every \( \xi \) satisfying \( \left| y_j - b_j \right| \leq d r_j \) there corresponds one and only one solution of (2) in the region defined by \( \left| y_j - b_j \right| \leq d r_j \). Furthermore, the solution \( \eta \) \((x_1, x_2, \ldots) \) so determined is \( C_1 \) for all \( \xi \) satisfying \( \left| x_j - a_j \right| \leq c r_j \).

The problem of obtaining continuation properties for the solution \( \eta \) \((\xi) \) is not developed in the present paper because the form of the hypotheses used here does not readily lend itself to this generalization.

---

When it was established that larvae could be produced from unfertilized eggs by chemical methods,\(^1\) a number of secondary problems arose. Foremost among these was the question whether or not the organisms thus produced were capable of developing into normal adults. This was at first considered improbable, especially by those who accepted Oscar Hertwig’s statement that fertilization consisted in the fusion of the egg and sperm nucleus. Since no such fusion takes place in artificial parthenogenesis, it became obvious that either Hertwig’s definition of fertilization was wrong or that artificial parthenogenesis was merely a pathological phenomenon not capable of leading to the formation of a normal organism.

---

\(^1\) Cf. an equivalent theorem proved in Bolza’s Vorlesungen über Variationsrechnung, p. 423, part (b).
\(^2\) Öfversigt af Kongliga Vetenskaps Akademien Förhandlingar, 56, 395-411 (1899).
\(^3\) These Proceedings, 1, 350 (1915).
\(^4\) Atti dei IV Congresso Internazionale dei Mathematici, 2, 98 (Roma, 6-11 Aprile, 1908).
\(^5\) Introduction to a Form of General Analysis, New Haven Mathematical Colloquium.
\(^7\) Ibid., 36, 95 (1908).
\(^8\) Cf. Kowalewski, Einführung in die Determinanten-Theorie.
My first experiments on artificial parthenogenesis were carried out on marine invertebrates, sea urchins, starfish, annelids, and molluscs, and since it is difficult to raise the normal larvae of such animals to the adult stage it seemed hopeless to attempt the task in the case of parthenogenetic specimens. Delage had the courage to undertake it on sea urchins and succeeded in raising two parthenogenetic larvae to the adult stage, one of which was far enough advanced to permit the recognition of its sex, which was male.

It seemed more hopeful to make the attempt in vertebrates. Guyer had found that by injecting lymph into the unfertilized egg of a frog development could be induced and Bataillon found later that the mere pricking of the unfertilized egg of the frog with a needle suffices for the purpose; although he believes that with the pricking a blood cell must be introduced in order to induce a normal development of the egg. According to my experience the essential feature in fertilization is an alteration of the surface layer of the egg which in my experiments was brought about by chemical agencies; while in the experiments of Guyer and Bataillon on the egg of the frog it was accomplished mechanically. It may be stated incidentally that this mechanical method has failed in every other form thus far tried in our laboratory.

I have now seven parthenogenetic frogs, (Rana pipiens) over a year old, produced by pricking the unfertilized egg. The growth of these animals was normal and the variations in growth observed were due to differences in taking food. This species of frogs apparently requires two years to become fully mature, and some of these seven parthenogenetic frogs have now grown to more than half their normal size. They are normal in every respect as regards appearance and behavior. One of the frogs became infected and was killed at the age of ten months; figure 1 gives a photograph in natural size. The legs and the abdomen are slightly distended as a consequence of the infection. The egg had been pricked April 17, 1915, the metamorphosis of the tadpole to the frog stage took place August 29. The frog was killed the 26th of February, 1916. Some of the surviving frogs are larger than the one photographed.
These experiments prove that the methods of artificial parthenogenesis can give rise to normal animals, even in forms so high in the scale as the frog; and that these animals are able to live and grow normally.

The second problem connected with the raising of these frogs was to ascertain their sex. Loeb and Bancroft had investigated this problem three years ago on a frog and a tadpole four and a half months old. The investigation met with some difficulty owing to the fact that in young tadpoles and frogs both sexes have eggs in their sex glands. These eggs gradually disintegrate in the testicles, but at the age of four months eggs are still found in the gonads of those which will develop into males. This was the case in the gonads examined by Loeb.
and Bancroft, but the eggs were so few in number that it was safe to assume that the two organisms examined would have developed into normal males had they lived; and the other structure of the gonads was such as to support this conclusion. Still it seemed desirable to make sure of this conclusion by examining older specimens of parthenogenetic frogs if they could be obtained. This opportunity offered itself in the case of the ten-months old parthenogenetic frog. Its gonads were hardened in Zenker's fluid and sectioned. It was found that the gonads (figs. 2 and 3) were normal testicles containing normal spermatozoa in large numbers, which leaves no doubt as to the sex of this parthenogenetic frog.

According to our present knowledge of the determination of sex it would appear from this that in the frog the male is heterozygous for sex; i.e., that the eggs are all alike and that there are two kinds of sperma-
GENETICS: J. A. HARRIS

Genetics: one with and one without a sex chromosome; and that if a spermatozoon of the former type enters an egg a female is produced. Since in artificial parthenogenesis no sex chromosome is introduced the parthenogenetic frogs should be males.

It will be of further interest to find out whether the spermatozoa of such frogs when used for the fertilization of eggs will give rise to normal offspring and to both sexes and it is my intention to carry the experiments if possible to a decision of this question. It is further of interest to study the number and nature of chromosomes in the spermatozoa of the parthenogenetic frogs.

[Since the proof of this paper was read, another of the parthenogenetic frogs died, at the age of thirteen months. It was also a male, possessing well developed testicles of more than 1 mm. in diameter, and the typical pads on the thumb of the forelegs characteristic of the male.]

1 Loeb, J., Amer. J. Physiol., 3, 135 (1899); 3, 434 (1900).
2 Loeb, Artificial Parthenogenesis and Fertilization, Chicago, 1913.
4 Loeb, J., and Bancroft, F. W., J. Exp. Zool., 14, 275 (1913); 15, 379 (1913).
5 Dr. Uhlenhuth was kind enough to do this for me.

DE VRIESIAN MUTATION IN THE GARDEN BEAN, PHASEOLUS VULGARIS

By J. Arthur Harris

STATION FOR EXPERIMENTAL EVOLUTION, COLD SPRING HARBOR, N. Y.

Received by the Academy, April 25, 1916

The extensive experience of experimental breeders with Phaseolus vulgaris during the past fifteen years has yielded few cases of unquestionable de Vriesian mutation. Such seems the most logical explanation of the origin of a race now under cultivation at the Station for Experimental Evolution.

Among the plants which survived from a lot of 4286 morphological aberrant plants secured in a study of 238,015 seedlings in 1912 were 9 which were distinguished from the remainder by producing only highly abnormal offspring in 1913. Practically without exception the 8000 first, second and third generation offspring produced in 1913, 1914 and 1915 were of a similar morphologically aberrant type. The same is true of a smaller culture of fourth generation plants grown in the greenhouse in 1915.

In this race the whole morphological organization of the seedling has apparently been changed. The new race is also characterized by a high degree of variability.