HOW TO USE THE SPECIFIC FORMULA OF HEREDITY

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The use of the Specific Formula of Heredity will be demonstrated by a specific example which applies the recently discovered formula to the probability-prediction of adult stature of an individual in that section of the British people sampled by Sir Francis Galton in his classical stature-study, 1884-1889.

\[ K = f(M, R) \]

Given \( M \); select \( R \) arbitrarily; find \( K \).

Example.

Given:

Father's stature \( 70.5 \) \]

Mother's stature \( 65.8 \) \]

The foundation of \( M \) or the prediction-basis.

\[
\text{Mid-parental stature} = \frac{(\text{Father's adult height in inches} + \text{Mother's adult height in inches} \times 1.08)}{2} = 70.78''
\]

Find: Probability \( (K) \) that an adult son will develop a stature of, say, \( 73.0 \)''.

\( R \) is any arbitrarily selected adult stature—the thing-predicted in inches \( \pm 0.5'' \).

\( M = \text{Mid-parental stature minus 68.54} \). \( M = 2.24 \) \( M^2 = 4.916 \)

Note that \( M \) is the prediction-basis throughout and that every factor in the resultant formula is a function of \( M \).

1) Find \( FC \) by substituting \( M \) and \( M^2 \) in:

\[
FC = f(M) = [0.5377M + 0.1252M^2] \left( \frac{M + (M-0)}{2M} \right) - 0.6548 \ldots FC = 1.07315
\]

2) Find \( K_f \) by substituting \( M \) and \( M^2 \) in:

\[
K_f = f(M) = 0.0035M + 0.0014M^2 + 0.1785 \ldots K_f = 0.1932
\]

3) Find \( \sigma_1 \) by substituting \( M \) and \( M^2 \) in:

\[
\sigma_1 = f(M) = 0.0314M + 0.0052M^2 + 2.0712 \ldots \sigma_1 = 2.16711
\]
(4) Find $\sigma_s$ by substituting $M$ and $M^2$ in:

$$\sigma_s = \frac{1}{2.5066K_fC}$$

$$\sigma_s = 2.06494$$

Now substituting the above found numerical values of these several functions of $M$ in the formula:

$$K = K_fC \in \sqrt{-\frac{1}{2} \left( \frac{(FC - R)^2}{\sigma_s} + \frac{(\sigma_s - \sigma_0) (FC - R)}{(FC - R)} \right)^2}$$

it is found that $K = 0.1072$

Based upon the generalized evidence of the 1028 actual cases, this particular probability-value is the "soundest mathematical judgment" which at present we know how to render for the given problem.

**Note on Graphical Computation.**—The foregoing method of substitution is, of course, the proper procedure for finding the correct values required by the particular Manerkonic formula, but, in case the particular Manerkonic model is in hand, the graphical computation of $K = f(M, R)$ is sometimes useful also as a check or an approximation. This graphical value is found as follows: Given any selected $M$ and $R$, locate the Manerkonic-surface-point common to the given $M$ and $R$; then read the coordinate value of $K$ on the "post" which carries the $K$-scale. (See illustration: Manerkonic model.)

**The Sex Factor in Stature.**—Galton made his stature-study on the male basis. In his analysis he followed the rule: Female adult stature times 1.08 gives the adult male stature-equivalent, so far as the hereditary growth-factor is concerned. Consequently if it were required by the present specific Manerkonic probability-prediction-formula to find the probability ($K$) that a random-selected daughter (not a son) derived from the given particular mid-parental stature-value would develop an adult stature of $73 \pm 0.5$ inches, it would be equivalent to requiring the computation of the probability that a son would develop the adult stature $(73 \cdot 1.08) \pm 0.5$ inches; or if it were required to find the probability that a daughter would develop an adult stature equivalent so far as heredity is concerned to that of a son with an adult stature of $73 \pm 0.5$ inches, this would be the same biologically as requiring the computation of the probability that the daughter would develop an adult stature of $(73 \div 1.08) \equiv 0.5$ inches.

**The Nature of Data Needed.**—For the Construction of a Specific Probability-prediction-formula—a Manerkon, $K = f(M, R)$.

The General Formula of Heredity\textsuperscript{1} supplies the general pattern. By following this pattern with adequate specific first-hand data the Specific Formula, or Manerkon, is readily computed. This will be specific and directly usable as a probability-prediction-formula in reference to the
particular quantitatively measured subject-trait or quality within the particular population sampled, so far as such subject-trait \( R \) depends upon the particular selected \( M \) or prediction-basis.

**Number of Data Needed.**—In the construction of a specific Manerkon, \( K = f(M, R) \), the mathematical formula and model of the Specific Formula of Heredity which, based upon \( (M) \) the stature of the father and the stature of the mother, predicts by \( (K) \) a definite probability that the adult stature of the offspring, corrected for sex, will fall within any selected stature \( (R) \) ± 0.5 inch. It is a specific application of the General or Pattern Formula of Heredity to a given case.

This particular Specific Formula presents the soundest generalized judgment at present available, based upon the evidence of Galton's 1028 first-hand stature measurements of adult children and of the father and mother of each adult child, within a definite section of the British people, 1884–1889.

that is, a probability-prediction formula \( K = f(M, R) \), fewer than one thousand cases of measured values of \( M \) the prediction-basis and \( R \) the thing-predicted will not suffice.

**Present Progress.**—The construction of a satisfactory Manerkon was possible in the case of Galton's classical stature-study because Galton had
collected 1028 stature-measures for both $M$ the adult mid-parent, and $R$ the adult child, and had corrected the data satisfactorily for sex differences. Galton, however, did not apply the present probability-prediction methods to the analysis of his data; these principles were discovered and made available in 1933; Galton's classical data were collected in 1884–1889.

In researches on the genetics of racing capacity in the Thoroughbred horse, a number of Manerkonic formulae have been constructed—one for each selected relationship between a specific ancestral blood-kinship and the foal. For each Manerkon a few more than one thousand cases of $M$ the measured racing capacity in the given ancestor, and of $R$ the measured racing capacity in the foal, were secured as basic data. These particular Manerkonic studies on the specific quality of racing capacity in the Thoroughbred horse cover the following kinships: 1. Sire–Offspring; 2. Dam–Offspring; 3. Sire's Sire–Offspring; 4. Sire's Dam–Offspring; 5. Dam's Sire–Offspring; 6. Dam's Dam–Offspring; 7. Full Sib–Full Sib; 8. Half Sib–Half Sib; 9. Foal–Parent; 10. Grandchild–Grandparent; 11. Uncle or Aunt–Nephew or Niece; 12. First Cousin–First Cousin.

**Note on the q-Value.**—In probability-prediction work, the specific prediction must be that a certain pre-selected individual thing or function will, in reference to the particular measured trait or quality, fall not exactly on a line with no width, but will fall within a certain measured class-range between two no-width lines. The probability of the thing-predicted falling exactly on the line is zero because the line has no $R$-width.

In the Manerkonic formula the letter $q$ stands for the width of the prediction-class in $R$-units. The value of $q$ is an arbitrary matter; it is determined by the practical necessities of the case. For instance, in the present human stature study $q = 1$ inch or 1 $R$-unit, since the stature-scale is in terms of inches, and the range of normal stature in the particular population consists of "ten or fifteen" such 1-inch classes. If the data were fewer or coarser the prediction-class would have to be wider—that is, $q$ would have to equal more than 1 inch, and the whole study would be more crude. If, however, the data were more numerous and more accurate, then the $q$-class could be narrowed down to say a $1/4$-inch range. In this case the resultant would be somewhat more refined in reference to its particular trait and population than is the case with the Manerkon as pictured, which was worked out with the one-inch $q$-value.

In the study of racing capacity in the Thoroughbred horse the $q$-class range was taken at "5 pounds." Within the group of racing horses studied in these researches the measure of racing capacity ranges from 65 to 140. In the future it would be desirable to perfect this particular research so that the Manerkonic or prediction-formula could be based on a 1-pound or a 2-pound $q$-class range, and other refinements also included.
The Number of M and R Classes.—The number of these classes is determined by the natural range of the subject-trait or quality within the subject population, and the arbitrarily but sensibly and practically selected \( q \)-value. “Ten or fifteen” such classes are in general a satisfactory number for the computation of a reliable Manerkonic formula out of a thousand data. For each Manerkon the thousand or more cases of specific relationship are duly “cross-sectioned or pigeon-holed.” For the ordinary Manerkon it will be found that each of about three-fourths of the cross-sectioned-area-classes will be represented by one or more out of the thousand individuals which comprise the sample. The vacant classes or areas are those in the “uncorrelated class corners” of the \( M \) and \( R \) relationship.

While in building the Manerkon the \( M \)-values are of necessity thrown into class-ranges, when the Manerkon is completed and the \( M \)-classes smoothed into a continuous surface, \( M \)—as a practical prediction-basis—can henceforth be taken at any exact point. But \( R \)—the thing-predicted—is never predicted as a point; \( R \) must always remain an active class-range—bounded by a definite \( R \)-point \( \pm \frac{q}{2} \).

One Piece of Prediction-Evidence.—Each specific Manerkon \( K = f(M, R) \) provides, by its cross-section at the selected \( M \)-point, one piece of constituent probability-prediction evidence for use (in connection with several—that is \( n \)—similarly appropriate cross-sections, one-each-from-a-total-of-\( n \)-different-Manerkons) in the computation of the probability-resultant, \( K_p = (M_1, M_2, \ldots M_n, R) \), of the \( n \) pieces of independent evidence. All evidence in the shape of independent Manerkonic cross-sections, which bear upon the same \( R \) or Thing-predicted, can be used in the computation of the probability-resultant. The better the evidence, that is, the more accurate, the more extensive, and the more independence in reference to prediction-basis, the better will be the prediction—that is, the narrower and taller will be the probability-resultant curve. A tall thin probability-area shows good prediction; a low flat cross-section shows poor prediction; but both are equally definite and real in pointing toward the truth. In reference to its prediction-basis each is an accurate mathematical picture of Nature’s behavior, and a definite piece of evidence for equitable use in computing the probability-resultant.

Source of Data.—In making a specific probability-prediction-formula the principal task ahead is to secure a thousand or more first-hand quantitatively measured data which cover as a fair sample the entire range of both the prediction-basis and the thing-predicted. These data are then adjusted with the minimum amount of mathematical smoothing to give, in the Manerkonic formula and its mathematical model, an accurate mathematical picture of Nature’s behavior in the specific case.

When such a Manerkon is completed it becomes of permanent reference
value for the particular measured trait or quality for the particular population sampled. With the specific Manerkonic formula in hand, one can arbitrarily select any $M$-value and any $R$-value, and can then compute $K$, that is, the probability that the ancestor with the selected $M$-value will be associated with the pre-selected $R$-value in the particular pre-selected offspring.

Future Progress.—Many such Manerkonic prediction-formulae should be worked out in the fields of plant, animal and human genetics, just as these first Manerkons were worked out for stature in a section of the British people and for racing capacity in the Thoroughbred horse. One of the principal tasks of the Eugenics Record Office of the future is to prepare in its laboratories, and to preserve for ready and permanent reference in its archives, many Manerkonic prediction-formulae for specific measurable human qualities—physical, mental and temperamental.

In this field of research, in reference to a given trait or quality the order of most effective attack is first, yard-stick invention; second, pedigree-analysis of the thing which is first quantitatively measured.

Similarly, other research laboratories of human anthropometry and psychology would serve science if they should work out, and save for reference, accurate Manerkonic formulae for specific measurable structural and functional qualities in definite racial and family-stocks, and other population groups of mankind. Also the laboratories and experiment stations for plant and animal breeding would find it profitable to work out specific Manerkonic formulae of heredity, and to preserve them for their own reference and for exchange with other institutions and research workers. Because the task is too big for any one institution or small group of laboratories, the promotion of agreement on the particular subjects for quantitative yard-stick invention and for Manerkonic analysis by different members would constitute a valuable service for research associations.

In other sciences besides biology, whenever there is a definitely measurable relationship between $M$ the Prediction-basis, and $R$ the Thing-predicted, practical use can be made of the Manerkonic formula, $K = f(M, R)$—that is, $K$ or Probability, is a function of $M$ the Prediction-basis and of $R$ the Thing-predicted. The specific Manerkonic formula finds what this specific function is, and presents it for practical use.