observed which contain 8, 10 and 12 primordia while others have as many as 36 and 38 primordia. These figures are interesting since Kerkis showed that the ratio of the volumes of the larger (male) gonads to the smaller (female) gonads in newly emerged larvae was of the order of \(3^{1/2}\) to 1. It can also be understood why Kerkis had difficulty in determining the dimensions of female gonads in young larvae for so very few cells are contained in a female gonad in a late embryo. Even should one or two mitotic divisions occur during the first larval hours such gonads would still be extremely small and most difficult to measure.

Summary.—The germ cells have been traced from the time of their origin to their inclusion in the primitive gonads. Many germ cells which appear in the early embryo never reach the gonads. Size differences in the gonads have been observed and it is concluded that no indifferent stage in the development of the gonad exists.

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**ON THE INDEPENDENCE OF LINE INTEGRALS ON THE PATH**

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The investigations of Menger\(^1\) and Fubini\(^2\) have renewed the interest in the question of the independence of the line integral on the path. I shall offer here a necessary and sufficient condition that is purely local and contains the classic conditions for Cauchy’s theorem as well as those for Green’s formula. For the sake of simplicity we restrict ourselves to two variables, \(x, y\).

A differential \(dA\) shall be any function of the four variables \(x, y, dx, dy\), and the \(\int_C dA\) along a curve \(C\) shall be defined the obvious way (if it exists). We shall assume that the integral changes the sign if we invert the direction of the curve \(C\). (It would be easy to give conditions for that, too.)

**Definition.**—The differential \(dA\) is called regular at a point \(P\) if for any sequence \(\Delta_n\) of similar triangles with areas \(a_n\) of limit 0 we have

\[
\lim_{n \to \infty} \frac{1}{a_n} \int_{\Delta_n} dA = 0
\]
No uniformity in the limit as to the shape of the triangles is requested.

**Theorem 1.** In a simply connected domain we have \( \int \mathcal{J} \, dA = 0 \) for any triangle of the domain if and only if \( dA \) is regular at every point of the domain.

**Proof.**—The condition is obviously necessary. For sufficiency we follow the well-known proof of Cauchy's theorem. A sequence \( \Delta_n \) of triangles similar to \( \Delta \) is constructed, each half the size of the preceding and contained in it, so that

\[
| \int \Delta \, dA | \leq 4^n | \int \Delta_n \, dA |.
\]

Let \( a \) be the area of \( \Delta \) and therefore \( a/4^n \) the area of \( \Delta_n \). If \( P \) is the common point of all \( \Delta_n \) we have for sufficiently large \( n \) that \( | \int \Delta_n \, dA | \leq \frac{a}{4^n} \varepsilon \).

This proves the theorem.

It contains as special case:

**Theorem 2.** Let \( P(x, y) \) and \( Q(x, y) \) have differentials in the sense of Stolz at the point \( x_0, y_0 \) and in addition to that \( \partial P/\partial y = \partial Q/\partial x \) at \( x_0, y_0 \). Then \( Pdx + Qdy \) is regular at \( x_0, y_0 \).

**Proof.**—Our assumptions lead to an approximation of \( P \) and \( Q \) by linear functions such that the coefficient of \( y \) in \( P \) equals that of \( x \) in \( Q \). In a sufficiently close neighborhood of \( x_0, y_0 \) the error will be less than \( \varepsilon \delta \) where \( \varepsilon > 0 \) is given in advance and \( \delta \) is the distance of \( x, y \) and \( x_0, y_0 \). The integral along \( \Delta_n \) over the linear function yields zero, the integral over the error is of magnitude \( \varepsilon a_n \).

**Theorem 3.** If \( \partial P/\partial y \) and \( \partial Q/\partial x \) exist in a certain neighborhood of \( x_0, y_0 \) are continuous at that point and equal in \( x_0, y_0 \), then our differential is again regular.

**Proof.**—This time we approximate \( P(x, y) = P(x, y_0) + (y - y_0) \times (\partial P/\partial y) (x, y_0) \) by a function of the form \( \phi(x) + a(y - y_0) \) where \( \phi(x) \) depends on \( x \) alone and \( a \) is a constant, and \( Q(x, y) \) by \( \psi(y) + a(x - x_0) \). The error is of the same order of magnitude as before. Since \( \int_{\Delta_n} \phi(x) \, dx = \int_{\Delta_n} \psi(y) \, dy = 0 \) and \( \int \gamma \, dx + \int \gamma \, dy = 0 \) we have again to consider merely the error term.
