

two lines, which coincide if

$$bh - b_0^2 = 0.$$

But this requires the Hessian to have a double point; therefore the general quartic cannot have a polar conic made up of two coincident lines. Also the polar cubic of $(0, 0, 1)$ is

$$a_2x_0^3 + 3lx_0^2x_1 + 3gx_0^2x_2 + 6nx_0x_1x_2 + 3c_0x_0x_2^2 + 3c_1x_2^2 + cx_2^3 = 0.$$

This can have a cusp only if

$$c_1l - n^2 = 0.$$

This has clearly nothing to do with $b = 0$, the condition that x_0 be a stationary line of the quartic. Therefore the cusps of the Steinerian do not lie on the stationary lines, as might be expected from their number—twenty-four. $n = 0$ is the condition that $x_2 = 0$ be the tangent to the Hessian; then the cusp cannot be obtained by making $c_1 = 0$, for then the Hessian has a double point. Putting $l = 0$ shows that the cusp tangent is also the tangent to the Hessian. Use of $n = 0$ also shows that the polar points of lines of the Cayleyan as to $(\xi)^6$ lie on the corresponding tangents to the Hessian.

A SEARCH FOR AN EINSTEIN RELATIVITY-GRAVITATIONAL EFFECT IN THE SUN

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From the equivalence principle of generalized relativity Einstein¹ concludes that the propagation of light is influenced by gravitation, and deduces two important consequences that can be subjected to the test of observation; namely, a train of light waves passing close to the edge of the sun is refracted so that the angular distance of a star appearing near the sun is increased by $1''.75$, and the Fraunhofer lines are displaced to the red in the solar spectrum by an amount equivalent to a velocity of recession of 0.634 km/sec. The amount depends only on the difference in gravitational potential between the gravitation field in which the radiation originates and the field where it is received. In the case of massive stars with density comparable to that of the sun the line displacement may be large, equivalent to 0.634 km/sec. $M^{2/3} d^{1/3}$, where M and d are in terms of the sun's mass and density.² Confirmation of either of these consequences would have not only an important bearing upon the establishment of the relativity principle but also upon the in-

terpretation of astrophysical data. The problem of determining stellar motions in the line of sight, a matter of fundamental importance, would be confronted with difficulties of a high order, depending as it does upon line displacement in stellar, relative to terrestrial spectra. Our knowledge of the motions, pressure, and many other phenomena in the solar atmosphere must be obtained from line displacements in the spectrum, but here it would be possible to apply definite corrections, this would in many cases, however, modify our interpretations.

The question of confirmation may be approached through direct observation of the displacement of a star near the sun³ but the conditions of observation are beset with great difficulties, statistical study of stellar masses and motions⁴ or the determination of displacements of solar lines⁵ under conditions that eliminate other possible causes.

Some results of an investigation bearing upon the relativity displacement of lines in the sun's spectrum are here communicated, a full account will appear later in a *Contribution from the Mount Wilson Solar Observatory*.

The substance of the investigation is the behavior of lines in the nitrogen (cyanogen) bands, λ 3883, at the center and at the limb of the sun, these lines owing to their freedom from pressure shift are in so far well fitted for such an investigation, but their compound character in the most important series and the frequent superposition of the lines of different series may introduce disturbing factors. The line density in the bands is high, above ten lines per angstrom. The possible occurrence of blends with metallic lines and the closeness of adjacent lines are important considerations in the selection of lines for observation. As the probability of blends is least for the narrowest lines and the precision of measurement is highest for lines showing no evidence of duplicity and sufficiently separated from adjacent lines,⁶ a greater weight is attributed to the lines of group A than to the broader lines of group B in the accompanying table.

The wave-lengths of the lines were measured in the arc and at the center and limb in terms of identical iron standards. The limb-center shifts were found by two independent determinations. The sun-arc displacements at the center were obtained by direct comparison and by three indirect methods. The 'Standard' ($R-I$) is the mean difference, Rowland minus International for lines whose wave, lengths in the sun and arc are equal, determined from the iron lines in this region by taking account of their sun-arc displacements.

The mean, zero, given by 43 lines for center-arc requires from the relativity point of view a radial movement in the solar atmosphere just

COMPARATIVE SOLAR AND TERRESTRIAL WAVE-LENGTHS OF LINES OF THE NITROGEN (CYANOGEN) BANDS

	GROUP A 25 LINES INTENSITY	GROUP B 18 LINES INTENSITY
<i>At center</i>	00—1	2—4
a. Direct comparison of sun and arc.....	0.000	+0.0013 A
b. λ at center minus λ in arc.....	0.000	+0.0026
c. (R—I) for band lines minus "Standard" (R—I)...	-0.003	+0.0010
d. (λ limb— λ arc) minus (λ limb— λ center).....	-0.001	+0.0008
Mean (Center—Arc).....	-0.001	+0.0014
	17 LINES INTENSITY	18 LINES INTENSITY
<i>At limb</i>	00—1	2—4
a. λ at limb minus λ in arc.....	0.000	+0.0037
b. (λ center— λ arc) + (λ limb— λ center).....	0.000	+0.0035
Mean (Limb—Arc).....	0.000	+0.0036

balancing the gravitational effect; at the limb where its influence would disappear, a displacement to the red of 0.008 A should be observed. For the lines of highest weight the displacement at the limb is zero, the mean for all lines +0.0018 A. Owing to the probable occurrence of blends in this region, the displacement to the red for the broader lines of group B may be in part at least attributed to this influence, as the effect of blends with metallic lines is to introduce systematic displacement to the red even for lines normally undisplaced at the limb.

The general conclusion from the investigation is that within the limits of error the measurements show no evidence of an effect of the order deduced from the equivalence relativity principle. Two other series of observations at Mount Wilson are at variance with the relativity principle, the equal wave-lengths of the H and K lines of calcium in the arc and at the sun's limb⁷ and a center-arc displacement of 0.004 A at λ 6300 for iron lines instead of 0.013 A required by the relativity principle.

¹ Einstein, A., *Ann. Physik, Leipzig*, 35, 1911, (898–908).

² Eddington, A. S., *Mon. Not. R. Astr. Soc., London*, 77, 1917, (377–382).

³ Dyson, F. W., *Observatory, London*, 40, 1917, (153–154); *Mon. Not. R. Astr. Soc., London*, 77, 1917, (445–447).

⁴ Freundlich, E., *Astr. Nachr., Kiel*, 202, 1916, (20–23).

⁵ Freundlich, E., *Physik Zs., Leipzig*, 15, 1914, (369–371); Schwartzschild, K., *Berlin, Sitz. Ber. Ak. Wiss.*, 1914, (1201–1213).

⁶ St. John, C. E., and Ware, L. M., *Mt. Wilson Contrib.*, No., 120; *Astroph. J., Chicago*, 44, 1916, (311–341).

⁷ St. John, C. E., *Mt. Wilson Contrib.*, No. 48; *Astroph. J., Chicago*, 32, 1910, (36–82).