WILLARD GIBBS ON SOARING FLIGHT

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In these days when so much depends on aerial supremacy it may interest some to see a hitherto unpublished letter of Gibbs to Langley dating from the very early infancy of aeronautical research in this country. The letter was furnished to me by Professor Ralph G. VanName, nephew of Gibbs.

On April 13, 1894, Langley wrote Gibbs asking for help in understanding soaring flight. He gave the usual Newton-type of formula for the wind pressure upon the aerofoil, proportional to the area, the square of the velocity and a function of the angle of attack which could be taken as proportional to that angle when it was small. Six weeks later Gibbs replied as follows:

New Haven, May 30/94

My dear Professor Langley

I do not know whether the following results—certainly very meager after so long a delay—will be at all to your purpose, but the discussion of a simple case may throw some light on the general question.

Let the velocity of the wind be

\[ V = A + a \sin nt. \]  \hspace{1cm} (1)

Let the horizontal velocity of the aeroplane (measured against the wind) be regulated by varying the inclination so as to be expressed by a similar function, say,

\[ v = B + b \cos nt. \]  \hspace{1cm} (2)

The relative horizontal velocity will be

\[ w = C + a \sin nt + b \cos nt, \]  \hspace{1cm} (3)

where \( C = A + B \). If \( \alpha \) is the tangent of the elevation of the edge (supposed small),
\[ \alpha = -\frac{dv}{gd} - \frac{Ew^2}{g}, \]  

(4)

where \( Ew^2 \) is the resistance of the air to the edge of the plane divided by its mass; and if \( z \) is the vertical height,

\[ \frac{dz}{dt} = \alpha w - \frac{g}{Pw} = \frac{w dv}{gd} - \frac{Ew^2}{g} - \frac{g}{Pw}, \]

(5)

where \( g/Pw \) is the rate at which the aeroplane would settle down when \( \alpha = 0 \). We shall treat \( P \) as roughly a constant. It is here assumed not only that \( \alpha \) is small, but also that \( w \) is large, and \( d^2z/dt^2 \) is small compared with \( g \), so that the variations of vertical momentum may be neglected in (4). Since by (2)

\[ \frac{dv}{dt} = -nb \sin nt, \]

(6)

(5), (6) and (3) give

\[ \frac{dz}{dt} = \frac{bn}{g} \sin nt \left( C + a \sin nt + b \cos nt \right) - \frac{Ew^2}{g} - \frac{g}{Pw}, \]

(7)

\[ \int_{t=0}^{t=2\pi/n} dz = \frac{\pi ab}{g} - \frac{2\pi}{n} \left[ \frac{Ew^2}{g} + \frac{g}{Pw} \right] \text{average value}. \]

(8)

Continued soaring is possible, if

\[ \frac{nab}{2g} > \left( \frac{Ew^2}{g} + \frac{g}{Pw} \right) \text{ave}. \]

(9)

Since \( C \) is the average value of \( w \), we shall err on the side of safety if we write for (9),

\[ \frac{nab}{6g} > \frac{EC^2}{g} + \frac{g}{PC}, \]

(10)

provided that \( C \) is three times as great as \( a \) and as \( b \), and provided also that \( \alpha \) and \( \frac{1}{g} \frac{d^2z}{dt^2} \) are sufficiently small to make our formulæ reasonably accurate.

To make \( \alpha \) small, \( nb \) must be small compared with \( g \). To make \( d^2z/dt^2 \) small compared with \( g \), we must have \( n^2bC \) small compared with \( g^2 \). To fix our ideas, we may make \( b = C/3 \). Then (10) becomes

\[ \frac{na}{18g} > \frac{EC^2}{g} + \frac{g}{PC^2}. \]

(11)

The left-hand member of this condition is given by the wind. If \( E \) is small enough and \( P \) large enough, it is easy to satisfy (11) by values of \( C \) which
are small compared to \( g/n \), which will make soaring possible, and if such values of \( C \) or any of them are greater than \( A \) (the average velocity of the wind), continued soaring \textit{against} the wind is possible.

The letter is taken from the handwritten copy which Gibbs kept and may not be identical with that which he sent; it closes without the customary "Yours truly" or equivalent and without signature, though undoubtedly both appeared on the letter sent to Langley.

Under date of June 9, 1894, Langley wrote: "I beg to return my very best thanks for your highly valued communication of the 31st ultimo. Not being certain when I should hear from you, I had asked the independent assistance of one or two other gentlemen, whose analytical skill I thought might be more trusted than my own.

"I am chiefly surprised at the entirely different ways in which the problem can be looked at and treated, and yours is certainly original and distinctive. It has arrived too late, I regret, to be used in a communication which is just going to France, but I shall probably make use of it later, with due public acknowledgment of your valued aid and great kindness."

About six months later (January 25, 1895), Langley wrote again:

"You will be interested in the discussion of a mathematical problem in aerodynamics, prepared at my request by Mr. de Saussure independently of the discussion that you were good enough to send me, which I have preserved, and may ask your permission to make use of it at some future time. Mr. de Saussure's article appeared as a supplement to the French translation of my paper on The Internal Work of the Wind in the \textit{Revue de l'Aeronautique}.

"I have sent to you, under a separate cover, a copy of the paper in question, and, should your leisure permit, I shall be glad to hear any criticism upon it that you may make.

"Renewing my thanks for your assistance, I am,"

So far as I have been able to ascertain Langley never took occasion to use Gibbs's material, nor Gibbs to accept the invitation to comment on de Saussure's article—there is certainly no evidence of either in the meager papers Gibbs left at his death.