ON THE RATE OF PASSIVE SINKING OF DAPHNIA

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The resistance encountered by a body falling in a viscous liquid is given approximately by dimensional analysis as

\[ R = k d^8 \rho^8 \mu^{8-q} \]  

(1)

where \( d \) is an appropriate linear dimension, \( v \) the velocity of the body, \( \rho \) the density of the medium and \( \mu \) the viscosity of the medium. When the resistance exactly balances the force of gravity the velocity becomes constant and is given by

\[ v^q = K d^3 \rho' (\rho' - \rho) \rho^{1-q} \mu^{q-2}, \]  

(2)

where \( K \) is constant for bodies of the same shape and \( \rho' \) is the density of such bodies.

For small bodies falling slowly in media of high viscosity, the resistance will be determined by the viscosity of the medium but not by the density. In this case \( q = 1 \) and

\[ v \propto d^2 (\rho' - \rho) \mu^{-1}. \]  

(3)

This is the general form of the relationship deduced just a century ago by Stokes for a small sphere falling slowly.

For large bodies falling so rapidly that viscosity is of no consequence in comparison with the inertia of the liquid, \( q = 2 \) and

\[ v \propto \sqrt{d (\rho' - \rho) \rho}. \]  

(4)

This is the general form of the relationship deduced by Newton late in the 17th century, and which holds with fair accuracy for some cases of large rapidly falling bodies.

In general Stokes' law corresponds to laminar flow and Newton's law to turbulent flow of the liquid past the body. It is generally conceded that Stokes' law holds with considerable accuracy when Reynolds' Number, \( N \), < 0.5, where

\[ N = \frac{d v \rho}{\mu}. \]  

(5)

Newton's law, however, does not apply immediately when \( N \) exceeds 0.5, and over a certain fairly wide interval there is empirical evidence that \( q = \frac{3}{2}, \) and therefore
In this expression the linear dimension $d_0$ is given by

$$d_0 = d - \xi d_*$$  \hspace{1cm} (7)

where $d_*$, the so-called critical diameter, is given, according to Allen, by

$$d_* = \frac{3}{2} \frac{9\mu^2}{2g\rho (\rho' - \rho)}$$  \hspace{1cm} (8)

and $\xi$ is a constant depending on the shape of the falling body.

Little data exists on the rate of passive sinking of freshwater planktonic organisms. Such information as has been recorded suggests that in the cases of bacteria, protozoa, unicellular algae and nearly all rotifers $N \leq 0.5$ and the rate of falling is therefore likely to be directly proportional to the square of the appropriate linear diameter, and to the density difference, and inversely proportional to the viscosity. For somewhat larger organisms the less well-known relationship derived by Allen may be expected to hold, the velocity being directly proportional to the appropriate linear diameter less a small correction, to the two-thirds power of the density difference and inversely to the cube root of the viscosity and density of the medium. It is unlikely that Newton's law is of any interest in the study of the plankton. Bowkiewicz who alone has attempted to go beyond Stokes' law in the study of sinking speeds unfortunately made two mistakes, one in the interpretation of his data and one in supposing that the reciprocal of the square root of the viscosity entered into Newton's formulation.

We have determined the rate of falling of a number of specimens of Daphnia sp. of various sizes from Bantam Lake, Conn., at temperatures between 21.7° and 27.2°C., corresponding to viscosities between 0.0085 and 0.0097 poise. All specimens were narcotized with ethyl urethane and had open antennae; they sank at various angles, the long axis being inclined with the head upward at angles from about 10° to about 80° above the horizon. The experiments were conducted in a cylinder of internal diameter 7.8 cms. For a sphere of diameter $d$ falling in such a cylinder the velocity must be raised by a factor of $\left(1 + \frac{2.1d}{7.8}\right)$ according to Ladenburg's correction for the effect of the walls of the vessel. In the case of our smallest animal if $d$ be taken as the length of the body the correction would amount to multiplication by 1.016, in the case of the largest to multiplication by 1.048. Actually a sphere of diameter $d$ has a larger volume than an animal of length $d$ and it is practically certain that such a correction would be excessive.
In figure 1 all the data, uncorrected for the effect of the walls, have been plotted against body length (exclusive of tail spine) on a double logarithmic grid. The maximum effect of the size of the vessel is indicated by the single open circle which is the corrected position of the point immediately below it, and which almost certainly represents an overcorrection. Although there is a fair amount of irregularity, it is evident that the points tend to fall along the line of slope 2, corresponding to direct proportionality between velocity and the square of the length. It is probable that most of the irregularity is due to differences in the density dependent on differences in the nutritive condition of the animal. In so far as the rather uncertain correction for the effect of the walls of the vessel is significant, it would raise the values of the velocity for the larger animals a little, improving the fit at the upper end of the curve to a very slight extent.

A few determinations of sinking speeds for a series of animals of almost identical lengths (0.86–0.89 mm.) were made at different temperatures and so at different viscosities. The results are plotted in the lower part of figure 4, from which it can be seen that in spite of the meagerness of the data there is an evident inverse relationship with the first power of the viscosity.

![Figure 1](image1.png)  
**Relation of sinking velocity of Daphnia sp. 1 to length (tail spine excluded), the open circle indicates the maximum effect of Ladenburg's correction. Inset shows an experimental animal in lateral view.**

![Figure 2](image2.png)  
**Relation of sinking velocity of Daphnia sp. 2 to length (tail spine excluded). Inset of an experimental animal in lateral view.**

![Figure 3](image3.png)  
**Relation of sinking velocity of Daphnia pulex to length, from the data of Eyden; solid circles measured individuals, open circles means of groups.**
The passive sinking of D. sp. 1 both in its relation to the linear dimensions of the animal and in its relation to the viscosity of the medium thus clearly obeys the generalized form of Stokes' law. It is probable that the largest individuals, for which the length is 1.79 mm., and the velocity of sinking 0.24 cm. per sec. when the viscosity is 0.0097 poise, must come very close to the upper limit for the validity of the law. In such a case Reynolds' Number, \( N \), can hardly be very much less than \( \frac{0.179 \times 1 \times 0.24}{0.0097} = 4.43 \).

In figure 3 the mean sinking speed of three narcotized specimens of D. pulex, falling with closed antennae, are plotted from Eyden's data. A fourth specimen which changed density owing to reproduction during the experiment has been excluded. The open circles represent the mean values, for the midpoint in the size ranges, of two experiments not reported in detail. The whole series of five points falls very close to a straight line of slope 1. According to Allen's law, the velocity varies as the appropriate linear dimension less a quantity \( \xi d_s \) as has been indicated above. In the present experiment if the density difference be taken as 0.02, \( d_s \) will be given by (8) as 0.028 cm. For a sphere \( \xi \) is 0.4, for an irregular body 0.28, a value for a falling Daphnia of 0.3 would seem reasonable, implying a correction of 0.08 mm. to the linear dimensions of the animals employed. The effect of such a correction would be to reduce the slope of the line of best fit in figure 3 very slightly below unity. It is, however, practically certain that the ratio of the length of the animal to the unknown diameter of the tube employed by Eyden was at least as great, and probably somewhat greater, in her experiments, than was the maximum ratio in those on D. sp. 1. The effect of the

![Figure 4](image_url)

Relation of the sinking velocity of Daphnia magna (from the data of Bowkiewicz) and of D. sp. 1, to viscosity. The crosses, open circles and solid circles indicate different individuals.
Ladenburg correction on Eyden's data would be to increase the slope of the line of best fit. The most reasonable interpretation of figure 3, therefore, is that these two small errors balance each other and that Allen's law holds, at least approximately.

The upper part of figure 4 indicates the relationship between mean sinking velocity and viscosity in experiments by Bowkiewicz with three specimens of *D. magna*, about 2 mm. long. These animals were narcotized, but unlike Eyden's, had open antennae. They certainly were considerably denser than the *D. pulex* employed in her experiments. Bowkiewicz concluded that the sinking speed in this case varied inversely as the square root of the viscosity. He therefore supposed that Newton's law held in this case. Both conclusions are erroneous. Newton's law contains no viscosity term while examination of the figure clearly shows that at least for narcotized animals the sinking speed is inversely proportional to the cube root of the viscosity. Some of Bowkiewicz' experiments with fixed *Daphnia* diverge somewhat from this relationship but so far as living animals are concerned the relationship of the velocity to viscosity is exactly as would be expected from Allen's Law. In these experiments the upper limit of Reynolds' Number, calculated as before, using the length of the animal as the appropriate linear dimension, lies between 10 and 16.

In figure 2 we have given observations on a few specimens of *Daphnia* sp. 27 from Bantam Lake, Conn. These animals are somewhat larger than most specimens of *D. sp. 1* though the increment is mainly due to the taller helmet of *D. sp. 2*. The available evidence seems to suggest some intermediate condition between Stokes' and Allen's laws but it is uncertain whether there is a more or less abrupt transition between the two possible relationships or whether some intermediate value of *q*, between 1 and 1.5, is implied.

**Summary.**—The velocity of passive falling of narcotized specimens of small limnoplanktonic *Daphnia* (*D. sp. 1*) is proportional to the square of the linear dimensions and inversely proportional to the viscosity, as implied by Stokes' law; the velocity of passive falling of large pond *Daphnia* (*D. pulex, D. magna*) is nearly proportional to the linear dimensions and inversely proportional to the cube root of the viscosity, as implied by Allen's law.

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3 *Daphnia* sp. 1 would be called *D. longispina* var. *hyalina* Leydig 1860 form mendotae according to Birge (in Ward and Whipple's *Fresh Water Biology*, 1918). A redescription of this species is in preparation by one of us (J. L. B.).
4 Eyden, D., "Specific Gravity as a Factor in the Vertical Distribution of Plankton,"


7 This species has not been described. Woltereck, Trans. Wisc. Acad. Sci. Madison, 27, 487–522 (1932) gives an outline drawing of a Daphnia undoubtedly referable to this species as figure 26 (Plate XVI). He has named it D. longispina apicata form nasuta without any description. A description is in preparation by J. L. B.

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NATIONAL ACADEMY OF SCIENCES: MINUTES OF THE MEETING FOR ORGANIZATION, APRIL, 1863

BY EDWIN B. WILSON

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The Report of the National Academy of Sciences for the Year 1863 published in Washington by the Government Printing Office, 1864, contains only the briefest reference to the organization meeting of the Academy. It has seemed to the Home Secretary and the Chairman of the Committee on Revision of Constitution that the members of the Academy who have recently been engaged upon a reorganization of the Constitution and By-laws might be interested in having for reference the Minutes of the Meeting for Organization and a copy of the initial Constitution and Bylaws as considered at that meeting in Committee of the Whole and finally passed at the stated meeting on the 6th of January, 1864. The address of the Hon. Henry Wilson to which reference is made in the Minutes is also appended.

In presenting this material of historic significance to the members of the Academy and to scientists at large, it may be well to call attention to a few remarks of President A. D. Bache in the first annual report of the Academy as revealing the spirit in which the original members and officers approached their tasks.

"The want of an institution by which the scientific strength of the country may be brought, from time to time, to the aid of the government in guiding action by the knowledge of scientific principles and experiments, has long been felt by the patriotic scientific men of the United States. No government of Europe has been willing to dispense with a body, under some name, capable of rendering such aid to the government, and in turn of illustrating the country by scientific discovery and by literary culture.

"It is a remarkable fact in our annals that, just in the midst of difficulties which would have overwhelmed less resolute men, the 37th Congress of the United States, with an elevated policy worthy of the great nation which