(see reference 3)

\[ \frac{1}{2} \{ \varphi (\zeta) \} = - \int_{\Sigma} [\varphi \Lambda \ast (\partial \theta_{\rho}) - (\varphi \theta_{\rho}) \ast \Lambda \varphi]. \]  

This is a Cauchy's formula for fields \( \phi \) on \( B_0 \) satisfying (4.1), the kernels being \( \partial \theta_{\rho} \) and \( \varphi \theta_{\rho} \).

We have seen that \( B_0 \) always possesses a singularity corresponding to complex-analytic forms of type \( (\rho, 0) \), and formula (4.2) therefore provides a Cauchy representation for such forms in which \( \varphi \theta_\rho = 0 \).

Since by (4.2) a closed Kähler manifold has a singularity \( \theta_\rho \) for \( \Box = \frac{1}{2} \Pi \Delta \) if and only if all forms satisfying (4.1) vanish identically, the restriction to submanifolds with boundary is obviously necessary.

* Apart from unimportant numerical factors. Dr. E. Calabi has recently called attention to a paper of Guggenheimer in which some of the operators were defined on symplectic manifolds.


SPECTRAL SENSITIVITY OF THE CONES IN THE DARK ADAPTED HUMAN EYE*

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The rods of the human eye exist in great numbers in the peripheral retina, and the cones exist primarily, although not exclusively, in the centrally placed fovea. The cones mediate day (or bright light) vision and the rods, night (or dim light) vision.

The two types of receptor exhibit different sensitivities to the wavelengths of the visible spectrum. Sensitivity for a given wavelength, or luminosity, is defined as the reciprocal of the energy of that wavelength required to produce a constant visual effect. Rod visibility is maximum at 505 to 510 m\( \mu \) and decreases toward the violet and red ends of the spectrum.

The cone luminosity curve has been determined by four methods classi-
fiable on the basis of the criterion of constant effect. The first method involves a direct brightness comparison of different wave-lengths against a constant standard.\(^3, 8, 18, 23\) The second method makes use of the step-by-step (or small step) procedure, in which one wave-length is matched to a constant brightness with a neighboring wave-length, a third is matched with the second, etc.\(^12, 6, 4, 2\) The third method involves an intensity adjustment to extinguish flicker. When two colors of different brightness are presented in rapid alternation, they flicker; when they stop flickering (due to proper adjustment of intensity) they are considered equally bright.\(^12, 15, 3, 17, 19, 14\) The fourth method is characterized by the determination of thresholds.\(^22, 21\)

In 1924 the International Commission on Illumination made a survey\(^16\) of the existing luminosity data obtained by the first three methods and adopted a set of adjusted average values which have served as an international standard. The ICI curve is a smooth, symmetrical function and has a peak at 555 m.\(\mu\). It represents cone function and is called the photopic curve.

In recent years some doubts have arisen concerning the detailed reliability of the ICI curve. On examination, the earlier curves of Gibson and Tyndall\(^8\) and of Coblentz and Emerson\(^3\) show considerable deviations from the standard in the blue end of the spectrum and seem to have a definite hump near 460 m.\(\mu\). Recently Stiles\(^20\) has reported a luminosity curve with a hump near 460 m.\(\mu\). Wald\(^22\) produced a cone sensitivity curve, much extended in spectral range, that also shows a hump near 460 m.\(\mu\). According to Wald the change in curve direction near 460 m.\(\mu\) is due to absorption by the macula lutea (or yellow pigment) which covers the central fovea. Stiles\(^21\) has published three individual cone luminosity curves but does not think that the effect at 460 m.\(\mu\) can be completely explained by macular absorption. An additional experiment\(^21\) shows that the effect is accentuated at a location 5° from the center of the fovea.

Besides the abrupt change in the curve near 460 m.\(\mu\), another one seems to exist near 600 m.\(\mu\). Two of Stiles' three individual cone luminosity curves show a "depression" of the curve near 600 m.\(\mu\) while the third does not. Sloan's earlier data,\(^18\) with low intensity and a small field (57 minutes), shows a similar effect, as do Granit's results\(^7\) on the photopic dominator curves for the cat. Finally, results obtained by Walters and Wright in 1943\(^23\) provide further evidence on the depression at 600 m.\(\mu\). It is probable that the effect at 600 m.\(\mu\) cannot be understood in terms of absorption by the ocular media.

Walters and Wright have suggested that the red receptors of the Young-Helmholtz theory tend to maintain their sensitivity over a greater range of intensity than do the green and blue receptors; and Forbes\(^4\) earlier made the same suggestion with respect to Sloan's data. Granit explains
the depression at 600 mμ for the cat's eye as due to a deficiency of red receptors. The red receptors raise the dominator curve in the orange-red region but are insufficient to provide high visibilities in the yellow-orange. Stiles is of the opinion that three maxima of luminosity curve are involved and perhaps represent the individual sensitivity curves of the three receptors.

Recently we have been investigating luminosity curves of color blind individuals. It became necessary, as part of this work, to determine the luminosity curve of the normal person for standardizing comparison purposes. The work is now going on, but because of the fact that our data on the normal person have considerable bearing on the general problem of the shape of the luminosity curve, we present some results here. Data on only one condition of observation are presented and it will remain for the future to vary other parameters.

We used the threshold method. Our test field was a circular one subtending a visual angle 42 minutes in diameter. The test field was viewed by the subject's dark adapted eye and appeared in the center of a fixation ring, i.e., a thin, dim ring of white light that ordinarily subtended a visual angle 3 degrees in diameter.

We were fortunate to have the double monochromator apparatus which Hecht, Shlaer and Pirenne used in their quantum study. We used it, except for the fixation ring, in its original arrangement. Light from a ribbon filament tungsten source (run on 6 volts 18 amperes D.C.) passes through a set of filters and wedges (Jena glass type), is focused on the entrance slit of the first monochromator and then passes to the second monochromator (Schmidt and Haensch type) by way of a middle slit. On leaving the exit slit the light floods the field lens, which brings the rays to a focus at an artificial pupil (2.5 mm. diameter) thus providing Maxwellian view. The fixation ring is projected through a hole in the side of the viewing tube onto a piece of thin glass set at 45° to the line of sight to the test field and centered at a distance of one inch from the pupil. The subject sees the ring in the dark surround of the test field and in the same frontal plane. An electrically controlled shutter is interposed between the two monochromators; it presents the test field to the eye for 4 milliseconds at each exposure. The subject presents himself with a single exposure (when he has good fixation) by releasing a key.

We made new wave-length and energy calibrations of the apparatus. A constant wave-length band of 5 mμ throughout the spectrum was used in all determinations. The filters and wedges were measured for their selective transmission in a Beckman spectrophotometer, and the data were corrected accordingly. The energy measurements were done with a photronic cell (Weston, Model 594RR, at the pupil position) and a moving coil galvanometer. The spectral response of the photronic cell was, twice calibrated by the Bureau of Standards in a period of 27 months.
At the beginning of each session of the subject became dark adapted for ten minutes, a sufficient time for the cones to gain full sensitivity. Then the spectral lights were presented to him by the method of limits; the threshold datum for each wave-length in any one session is the average for three ascending and three descending series that involved small steps of log energy. The subject responded by activating a bell to indicate whether or not he saw the light. Each sitting was restricted to an hour in order to minimize subject fatigue. A good deal of time was spent in making slit settings, and so only about eight wave-length positions could be covered in one sitting. Since determinations were made at nearly each 10 m\(\mu\) in most regions of the spectrum between 415 and 760 m\(\mu\), and since for each position the threshold was repeated two, three or even more times, the production of a complete curve for a single subject involved the combined effort of the experimenter and subject for a considerable period of time.

Each of our 5 subjects had normal color vision. The subjects were tested with the Stilling test and the Shlaer anomaloscope before they participated in the experiment. The ages of the subjects varied between 20 and 30. With the exception of LM, all were males.

Figure 1 gives the individual cone sensitivity curves for five subjects; the topmost curve presents the average data.† The basic data of these curves are relative energies required for the cones to respond to the spectral lights at the absolute threshold; the logarithms of the reciprocals of these values (i.e., log sensitivity values) are here plotted. The peak of the average curve is arbitrarily set at zero (i.e., maximum sensitivity is set at unity). For purposes of clarity of presentation each successive curve is moved downward through one logarithmic unit. It will be noticed that the individual curves do not have peaks at the value log Sensitivity = 0. Since the peak of the average curve is assigned the value of zero, data for a given individual are either higher or lower than the average. By this de-
vice an indication of deviations of threshold values from the average are obtained.

All of the curves of figure 1 show abrupt changes in direction near 460 m\(\mu\) and 600 m\(\mu\); hence they support the results of other investigators who found similar effects. The effects seem to be characteristic of the cone sensitivity curve as obtained with a small field that is well within the dark adapted fovea.

The depression at 600 m\(\mu\) is not as prominent in the average curve as it is in individual curves. In this connection it should be noted that subject HL probably shows the effect at a wave-length nearer 610 m\(\mu\) than 600 m\(\mu\). The other subjects give data more nearly in accord with the averaged results.

The peaks of our individual curves are in the neighborhood of 555 m\(\mu\), or possibly slightly below. The peak of the ICI photopic luminosity curve is 555 m\(\mu\). Arndt\(^5\) and Fedorov, \textit{et al.},\(^4\) suggest that the peak should be around 565 m\(\mu\) at high intensities. The latter authors found it to be near 558 m\(\mu\) at a low intensity. Wald\(^22\) estimates that it may occur at 562 m\(\mu\). Of the three individual curves reported by Stiles\(^21\), probably none has its peak above 550 m\(\mu\). Walters and Wright \(^{23}\) and Sloan\(^18\) report that the peak is close to 540 m\(\mu\) for low intensities. For a 20 minute area, Wright\(^24\) found a peak near 560 m\(\mu\).

We have wondered whether fixation conditions might not have an effect on the position of the visibility peak. One of our subjects did, in fact, show a peak near 540 m\(\mu\) when the 3° fixation ring measured. When the ring was reduced to 2° and slightly raised in brightness she showed a peak near 550 m\(\mu\). The other subjects all showed a peak near 550 m\(\mu\) with the 3° ring.

Our curves exhibit higher luminosity values in the blue end of the spectrum than does the ICI standard. The difference between the log sensitivity values at 555 m\(\mu\) and 415 m\(\mu\) is, for our data, about 1.8 log units; for Wald's data, it is about 2.2 log units; and for Stiles' data it is about 1.7 log units. These values are to be contrasted with those of the ICI curve: 2.7 log units. As judged by the data of Wald, Stiles and ourselves, the ICI curve underestimates luminosity in the blue by a considerable amount.

It will certainly be important if further analysis indicates that the abrupt changes in curve direction found near 460 m\(\mu\) and 600 m\(\mu\) may be associated with three specifiable components. The relationship of such a finding to the Young-Helmholtz trichromatic theory is clear.

\textit{Summary}.—New data on cone sensitivity, based on threshold measurements, are reported. Sensitivity is at a maximum near 550 m\(\mu\). The spectral sensitivity curve shows two points of abrupt change in direction: one near 460 m\(\mu\) and the other near 600 m\(\mu\). These findings are in line with the results of some other experiments\(^{18, 21, 23}\).

† For the numerical data of the experiment, order Document 3423 from the American Documentation Institute, 1719 N St., N. W., Washington 6, D. C., remitting $1.00 for microfilm (images 1 inch high on standard 35-mm. motion picture film) or $1.00 for photocopies (6 × 8 in.) readable without optical aid.

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