With the aid of equations (72) and (76) we can determine the horizontal displacement \( q(t) \), using its operational representation given in equation (65).

4. Summary.—An exact solution is given for the motion of the surface of a uniform elastic half-space produced by a point pressure pulse situated at a depth \( H \) below the surface. The time variation of the pressure pulse is assumed to be given by the Heaviside unit function. The singularity of the point source is specified by the condition that the integral of the difference of the normal stress over a horizontal plane passing through the source is finite. This source excites both \( SV \)-waves and \( P \)-waves. The operational representation of the vertical component of displacement at the surface is given in equation (6), and its interpretation can be carried out with the aid of equations (51), (52), (57), (58), (63), and (64). The operational representation of the horizontal component of the displacement \( q(p) \) is given by equation (65), and its interpretation can be made by using equations (72)–(76). The interpretation is accomplished by transforming the paths of integration in equations (6) and (65) from the real axis to the paths \( OAB \) and \( OCD \) shown in Figure 1.

The motion of the surface is different in the case \( r < H/\sqrt{2} \) from the case \( r > H/\sqrt{2} \), where \( r \) denotes epicentral distance. When \( r > H/\sqrt{2} \) (for a medium in which the elastic constants \( \lambda \) and \( \mu \) are assumed to be equal), the \( P \)-wave is followed by a diffracted \( P \)-wave, derived from the \( SV \)-wave, which arrives before the direct \( SV \)-wave.\(^3\) This ray picture is illustrated in Figure 2. This refracted (diffracted) \( P \)-wave is represented in our solution by the integrals (58) and (74).

The numerical evaluation of the motion of the surface due to the buried source for various distances from the source will be given in a subsequent publication.

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**ON HERMITIAN OPERATORS OVER THE CAYLEY ALGEBRA**

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Let \( \mathfrak{g} \) be the vector space of all \( n \)-tuples \( X = (x_1, \ldots, x_n) \) with co-ordinates \( x_i \) in the Cayley algebra \( \mathfrak{C} \) of dimension \( 8 \) over the field \( \mathbb{R} \) of all real numbers. Then \( \mathfrak{g} \) is a vector space of dimension \( 8n \) over \( \mathbb{R} \). The algebra \( \mathfrak{C} \) has an involution \( x \mapsto x^* \), and we define an inner product in \( \mathfrak{g} \) by
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for all vectors $X = (x_1, \ldots, x_n)$ and $Y = (y_1, \ldots, y_n)$ in $\mathfrak{g}$. We seek to determine the set $\mathfrak{A}$ of all Hermitian operators $T$ on $\mathfrak{g}$, that is, the set of all linear transformations $T$ over $\mathfrak{g}$ of $\mathfrak{g}$ such that

$$ (XT, Y) = (X, YT). $$

It is easy to verify that $\mathfrak{A}$ is a special Jordan algebra over $\mathfrak{g}$. We shall derive the following result.

**Theorem.** The algebra $\mathfrak{A}$ is isomorphic to the special Jordan algebra of all $n$-rowed real symmetric matrices.

For, consider the equation

$$ (xT)y^* = x(yS)^*, $$

for all $x$ and $y$ of $\mathfrak{C}$, where $S$ and $T$ are linear transformations over $\mathfrak{K}$ of $\mathfrak{G}$. Equation (2) reduces to equation (3), with $S = T$ when $n = 1$. The algebra $\mathfrak{C}$ has a unity element $e = e^*$, and equation (3) implies that

$$ t = eT = (eS)^*, \quad xT = xt, \quad (yS)^* = ty^*. $$

But then equation (3) is equivalent to

$$ (xt)y^* = x(ty^*) $$

for every $x$ and $y$ of $\mathfrak{C}$. It follows that $t$ is in the nucleus of $\mathfrak{C}$, and so $t = \tau e$, where $\tau$ is real,

$$ T = \tau I = S. $$

We now consider equation (2). The transformation $T$ may be regarded as being an $n$-rowed square matrix $T = (T_{ij})$ whose elements $T_{ij}$ are linear transformations on $\mathfrak{G}$. Then equation (2) is equivalent to

$$ \sum_{i,j=1}^{n} (x_iT_{ij})y_j^* = \sum_{i,j=1}^{n} x_i(y_jT_{ji})^*. $$

Take $X = (0, \ldots, 0, x_6, 0, \ldots, 0)$, $Y = (0, \ldots, 0, y_7, 0, \ldots, 0)$, and see that equation (7) becomes

$$ (x_iT_{ij})y_j^* = x_i(y_jT_{ji})^*. $$

This is equation (3), with $T = T_{ij}$, $S = T_{ji}$. Then $T = S = \tau \delta_j^i = \tau_{ij}I$ by equation (6), and so $T = (\tau \delta_j^i)$ is a real symmetric matrix. The converse is trivial, and we have proved our theorem.