as converging waves to the region of their origin, or an adjacent region, and following waves will behave in a similar manner. The above difficulty concerning the values of the epochal parameter will thus be removed.

It is conceivable that series of recurrent gravitational disturbances exist in the universe, each with its own epochal parameter, in addition to the one contemplated above. But hopefully this will not be true since it would complicate the problem of epochal classification.

5. Other Theories.—While the theory presented here is of the evolutionary type, it differs in its essential aspects from previous evolutionary theories, notably the original cosmological theory of Lemaitrë and the later theory due to Gamow. It is moreover incompatible with the steady-state theories as formulated by Bondi and Gold and by Hoyle, although it does contain the possibility of the more or less continuous creation of particles by a mathematically substantiated process as mentioned in the Remark at the end of section 3. We predict that these steady-state and foregoing evolutionary theories of the universe will eventually be discarded since they obviously provide no basis for the explanation of the revolutionary discovery of the discretization in galactic structure.

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AN ENERGY-MOMENTUM TENSOR FOR COLLAPSING STARS*

By R. d'E. ATKINSON†

CALIFORNIA INSTITUTE OF TECHNOLOGY

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It is well known that the energy-momentum tensor for a perfect fluid can be expressed in the form

\[ T_{\tau\tau} = -g^{\tau\tau}p + \frac{dx^\tau}{ds} \frac{dx^\tau}{ds} (p + \rho), \]  

(1)

where \( p \) and \( \rho \) are the proper pressure and proper macroscopic energy-density in the fluid, and the \( dx/ds \) are the components of the fluid's velocity in the coordinate system in use. In the static case, when the three spatial components are zero and \( g_{\mu}(dt/ds)^2 = 1 \), the mixed tensor \( T_{\tau\tau} \) associated with (1) reduces at once to \( T_{\tau}^1 = T_{\tau}^2 = T_{\tau}^3 = -p \), \( T_{\tau}^4 = \rho \), with all other components zero; and if we have spherical
symmetry (in the velocities as well as in the space), it is not difficult to write down similar components for the nonstatic case also. Taking the equation for the line-element as

$$ds^2 = -e^\lambda dr^2 - e^\mu r^2(d\theta^2 + \sin^2 \theta d\phi^2) + e^\nu dt^2$$ \hspace{1cm} (2)

(where $\lambda$, $\mu$, and $\nu$ are functions of both $r$ and $t$) and putting $d\theta/dt = d\phi/dt = 0$ and $dr/dt \equiv v$, we have at once

$$\frac{dt}{ds} = e^{-\nu/s} \kappa \hspace{1cm} (3)$$

$$\frac{dr}{ds} = e^{-\nu/s} v \kappa \hspace{1cm} (4)$$

with

$$\kappa \equiv (1 - e^\lambda - r^2)^{-1/2}$$ \hspace{1cm} (5)

and so, again lowering one index, and noting that $e^{\lambda/2 - \nu/2} v \kappa = \sqrt{k^2 - 1}$,

$$T_1 = -\kappa^2 p - (e^\lambda - 1)\rho \hspace{1cm} (6)$$

$$T_2 = T_3 = -p \hspace{1cm} (7)$$

$$T_4 = (e^\lambda - 1) p + e^{\lambda/2} \rho \hspace{1cm} (8)$$

and

$$e^{\lambda/2 - \nu/2} T_1^4 = -e^{\nu/2 - \lambda/2} T_4^4 = \kappa^{\sqrt{k^2 - 1}} (p + \rho). \hspace{1cm} (9)$$

These equations are sufficient for a spherically-symmetrical collapse, so long as the flux of escaping energy can be ignored.

Hoyle and Fowler\(^1\) have recently considered the possibility that very massive stars ($M \approx 10^8 M_\odot$) might exist and might collapse in a fairly short time, releasing very large amounts of energy indeed; if the so-called "Schwarzschild limit" is approached, the total amount that might be liberated is much greater than can be obtained by atomic transmutations, and in fact it approaches $mc^2$ for a mass $m$. There is the qualification that the last part can only be released very slowly, since the red shift has now slowed down all processes severely; in addition, the curvature of the null-geodesics is now such that photons and neutrinos will be seriously hampered in their escape, even if their actual production is quite uninhibited. In empty space outside a highly collapsed star, the (coordinate) radius of curvature of a null-geodesic at periastron is equal to the radial coordinate itself (so that this geodesic becomes a circular orbit round the star) when the "red-ratio," $\nu/v$, falls to $1/\sqrt{3} \approx 0.578$. (This figure is independent of whether we use the Schwarzschild coordinate system or the "isotropic" one, though the two $r$-values themselves differ, at this point, by a factor of 1.61.) Beyond this point, neutrinos will be increasingly deflected back into the dense core, where some form of "internal solution" holds, and will not actually escape until they succeed in acquiring a velocity that is nearly all radial. On both counts, therefore, it is unlikely that more than about half the total energy could get out, even in the form of neutrinos, in a really short time; but this first half, roughly, should not be seriously hampered on purely relativistic grounds. Iben\(^3\) has shown, it is true, that if the final state is to be stable, there may be little
or no energy available for an escaping flux at all; but the fact that no stable state is in prospect if the energy escapes will not in itself prevent it from escaping, and if a stage can in fact be reached where neutrinos are produced in sufficiently large numbers before the red shift, \((v - \nu)/v\), has risen to 40 per cent or so, the collapse should proceed at something like the rate of free fall, always assuming, of course, that it is not held up by rotation, magnetic fields, etc. Under these circumstances, the mass of the flux itself (as opposed to the mass of the trapped radiation) may become considerable, and it will be necessary to modify (1).

The modification will certainly involve a provision for a radiation-flux, observable even by a proper observer stationary in the matter itself, and we may allow at the same time for a possible anisotropy in the pressure (gas plus trapped radiation). We therefore assume that the energy-momentum tensor in proper coordinates can be written

\[
\tau_{\alpha\beta} = \begin{pmatrix}
0 & 0 & f \\
0 & p_r & 0 \\
f & 0 & \rho
\end{pmatrix}
\]

(10)

where we have orientated our proper coordinate system with the 1-axis parallel to the flux. The pressure parallel to the flux has been labeled \(p_r\), in anticipation of the subsequent application to the radial direction in the spherically-symmetrical case, but this restriction on the symmetry does not actually involve equations (11) to (15).

Since (10) applies, by definition, to an observer who feels no gravitational forces (and is momentarily at rest with respect to the local "matter"), the problem of relating its components to the physical quantities measurable by this observer is primarily a non-Relativistic one, or at most a problem involving only the Special Theory. It is, however, a very difficult one, for which no rigorous general solution has yet been found, and it is not the purpose of the present Note to attack it. Our purpose is primarily the restricted one of deriving from (10), as it stands, a more general tensor to replace (1) in Einstein's equations \(1/\rho R^{rr} - R^{rr} = 8\pi T^{rr}\); but before we proceed to this it may perhaps be appropriate to outline the major difficulties facing any real attack on the physical problem.

The most important of these, it seems likely, are bound up with the fact that any flux is incompatible with local thermodynamical equilibrium. In the classical theory of stellar interiors, where the temperature is sensibly constant over a distance of one atomic mean free path, it is of course an extremely good approximation to consider a "box" whose dimensions are small multiples of that length, and to define a (kinetic) temperature on the basis of the mean energy of any kind of atom in that box. It is ordinarily possible to do much the same for radiation also, and in that case the radiation is isotropic and "black," and its temperature is equal to the other; also, the two pressures then simply add to produce a "total pressure," and similarly with the two densities (though the actual density of the radiation is usually negligible). This "total pressure" is all really available for keeping the gas distended, and the necessary condition for this evidently is that compressing the
matter (e.g., by a sound-wave, or a shock-wave) really will compress the radiation too. If, however, the absorption-coefficient becomes extremely small (as is likely if a collapse is to occur), a temperature-gradient which was insignificant in the "kinetic" work may now mean that radiation passing through the box in one direction is coming from regions of very appreciably higher temperatures than that of the box, while radiation passing through in the opposite direction is coming from regions appreciably cooler; also (ex hypothesi) neither stream now interacts much with the atoms in the box. The pattern of atomic velocities can therefore still be almost Maxwellian, while the distribution of radiation is not nearly Planckian, nor even isotropic, and the temperature and total pressure cannot be defined in the usual ways. The conditions become somewhat like those in a photosphere, and to a certain extent the same kind of treatment may be appropriate. But the creation of neutrinos will intensify the difficulties. Their energies will in general be completely different from those of the atoms in the box; they may have been on their way (on curved null-geodesics passing repeatedly through the core of the star) for a time long enough to allow an appreciable collapse since their formation; and their "pressure" is almost totally useless for supporting the matter and resisting collapse.

Nonetheless, it would seem reasonable to claim that there must still in principle be a tensor to be equated to $\nabla \cdot R - R^*_p$ (which will force its divergence to vanish), and that it must (in principle) be capable of being rigorously evaluated even for the most violent stellar collapse; it also seems clear that its contravariant associate must reduce to the form (10) for an unaccelerated observer at rest in the local matter (baryons), under the symmetry conditions we have adopted (e.g., that the flux is in the same direction at all frequencies, that all directions perpendicular to this are equivalent, and so on), whatever the physical significance of the several components may be. We may perhaps suggest, tentatively, that $p$, and $p_1$ (which may indeed be identical) will include all of the gas-pressure, and such part of the radiation-neutrino pressure as would actually resist the compression of the (box of) gas under consideration (with the corollary that its components in the $+x$ and $-x$ directions, at any one $x$, are numerically equal); that $f$ will include all of the flux-energy, whether or not it interacts with matter, with the understanding that the product of $f$ and the absorption coefficient (integrated over all "wavelengths") will contribute, along with the usual difference of $p$, between opposite faces of the box, to the net forces acting on the matter there; and that $p$ will include every kind of energy density. (It is clearly inadmissible to discuss the dynamics of stellar material on the basis of an "ideal" box whose walls would be impervious to neutrinos, $\gamma$-rays, etc.; if the radiation cannot actually push the matter, it must not be "ideally" provided with a tool for doing so.) The difficulties seem to be primarily those of devising suitable equations of state, absorption coefficient, energy generation, etc., or of defining adequate quantities to use in them; to this extent they may be irrelevant to a purely relativistic transformation, and we shall not here discuss them any further. Since, however, Einstein's equations do in fact yield important conclusions about the star as a whole (for example, its total gravitational mass, at least in the static case), and imply important conditions of conservation and so on, it may even transpire that they can themselves be applied to infer, "in reverse" so to speak, some of the physical significance of the invariants $p$, $f$, and $p$. We
proceed, then, to derive the formal extensions of (1), and of (6) to (9), which follow from (10) in any case.

The general tensor $T^{\sigma\tau}$ may be obtained from (10) by the procedure used (e.g., ref. 3, p. 217) to derive (1). The transformation-equation defining a second rank contravariant tensor, namely,

$$T^{\sigma\tau} = \frac{\partial x^\sigma}{\partial x_0^\alpha} \frac{\partial x^\tau}{\partial x_0^\beta} T_0^{\alpha\beta}$$

reduces in the present case to

$$T^{\sigma\tau} = \frac{\partial x^\sigma}{\partial x_0^\alpha} \frac{\partial x^\tau}{\partial x_0^\alpha} p_\tau + \frac{\partial x^\sigma}{\partial x_0^\alpha} \frac{\partial x^\tau}{\partial x_i^\alpha} p_i + \frac{\partial x^\sigma}{\partial x_0^\alpha} \frac{\partial x^\tau}{\partial x_0^\alpha} \rho +$$

$$\left(\frac{\partial x^\sigma}{\partial x_i^\alpha} \frac{\partial x^\tau}{\partial x_i^\beta} + \frac{\partial x^\sigma}{\partial x_j^\alpha} \frac{\partial x^\tau}{\partial x_j^\beta}\right) f,$$  \hspace{1cm} (12)

or, since $g^{\sigma\tau} = \frac{\partial x^\sigma}{\partial x_0^\alpha} \frac{\partial x^\tau}{\partial x_0^\beta} g^{\alpha\beta}$, with $g^{\alpha\beta} = (-1, -1, -1, 1)\delta_\alpha^\beta$,

$$T^{\sigma\tau} = -g^{\sigma\tau} p_\tau + \frac{\partial x^\sigma}{\partial x_0^\alpha} \frac{\partial x^\tau}{\partial x_0^\alpha} (p_i - p_i) + \frac{\partial x^\sigma}{\partial x_0^\alpha} \frac{\partial x^\tau}{\partial x_i^\alpha} (p_i + \rho) +$$

$$\left(\frac{\partial x^\sigma}{\partial x_i^\alpha} \frac{\partial x^\tau}{\partial x_i^\beta} + \frac{\partial x^\sigma}{\partial x_j^\alpha} \frac{\partial x^\tau}{\partial x_j^\beta}\right) f.$$  \hspace{1cm} (13)

Since, further,

$$\frac{dx^\sigma}{ds} = \frac{\partial x^\sigma}{\partial x_0^\alpha} \frac{dx_0^\alpha}{ds} = \frac{dx^\sigma}{dx_0^4}$$

(14)

on account of $dx_0^\alpha/ds = (0, 0, 0, 1)$, we have

$$T^{\sigma\tau} = -g^{\sigma\tau} p_\tau + \frac{\partial x^\sigma}{\partial x_0^\alpha} \frac{\partial x^\tau}{\partial x_0^\alpha} (p_i - p_i) + \frac{dx^\sigma}{ds} \frac{dx^\tau}{ds} (p_i + \rho) +$$

$$\left(\frac{\partial x^\sigma}{\partial x_i^\alpha} \frac{dx^\tau}{ds} + \frac{\partial x^\sigma}{\partial x_j^\alpha} \frac{dx^\tau}{ds}\right) f.$$  \hspace{1cm} (15)

(It will be remembered that in these equations the partial derivatives express relations between the initial and final coordinate systems, while the total derivatives are components of the velocity of the continuous matter in the final system.)

Equation (15) has been derived here because it is a direct analogue of (1) whenever the radiation-flux is appreciable. It is, however, obviously a much less convenient equation than (1), and for the purpose of evaluating actual components of the tensor we may equally well omit it and proceed directly from (12).

We restrict ourselves now to the case of spherical symmetry, replacing $x^1, x^2, x^3,$ and $x^4$ by $r, \theta, \phi,$ and $t$, respectively; we also replace $x_0^1, x_0^2, x_0^3,$ and $x_0^4$ by $x_0, y_0, z_0$, and $t_0$. The surviving terms in the individual components of $T^{\sigma\tau}$ are then (from (12))

$$T^{11} = \left(\frac{\partial r}{\partial x_0}\right)^2 p_r + \left(\frac{\partial r}{\partial t_0}\right)^2 \rho + 2 \frac{\partial r}{\partial x_0} \frac{\partial r}{\partial t_0} f,$$  \hspace{1cm} (16)
\[ T^{22} = \left( \frac{\partial \phi}{\partial \eta} \right)^2 p_t, \tag{17} \]
\[ T^{33} = \left( \frac{\partial \phi}{\partial \omega} \right)^2 p_t, \tag{18} \]
\[ T^{44} = \left( \frac{\partial t}{\partial \phi} \right)^2 p_t + \left( \frac{\partial \phi}{\partial \omega} \right)^2 \rho + 2 \frac{\partial t}{\partial \phi} \frac{\partial t}{\partial \omega} f, \tag{19} \]
\[ T^{14} = T^{41} = \frac{\partial r}{\partial x_0} \frac{\partial t}{\partial x_0} p_r + \frac{\partial r}{\partial t_0} \frac{\partial t}{\partial t_0} \rho + \left( \frac{\partial r}{\partial x_0} \frac{\partial t}{\partial \omega} + \frac{\partial r}{\partial \omega} \frac{\partial t}{\partial x_0} \right) f, \tag{20} \]

and we have to evaluate the six partial derivatives which appear in these.

It is convenient and possible (ref. 3, p. 240, and ref. 4) to choose \( r \) and \( t \) so that there is no \( dr \cdot dt \) term in the expression for the line-element; subject to this condition, the most general spherically symmetrical expression is (2), and in order to pass from the proper coordinates of (10) (which, of course, have only a local and temporary validity) to a tensor valid throughout the space-time of (2), we introduce locally an intermediate coordinate system, designated by double subscripts, which is Cartesian and unconstrained (“freely-falling”) like the first, and is parallel to it, but is momentarily stationary in the final one. The first and intermediate systems are thus in relative motion with the velocity \( v_0 \) (say) in the \( x_0 \)-direction, and the ordinary Lorentz transformation applies, i.e., \( x_0 = \kappa_0(x_0 + v_0 t_0), y_0 = y_0, z_0 = z_0, t_0 = \kappa_0(t_0 + v_0 x_0) \), where \( \kappa_0 = (1 - v_0^2)^{-1/2} \). Thus, \( \frac{\partial x_0}{\partial x_0} = \frac{\partial y_0}{\partial y_0} = \frac{\partial z_0}{\partial z_0} = \frac{\partial t_0}{\partial t_0} = \kappa \), and all other partial derivatives vanish.

We now transform from this system to the final one. We have, in addition to (2),
\[ ds^2 = -(dx_0)^2 - (dy_0)^2 - (dz_0)^2 + (dt_0)^2, \tag{21} \]

and since these two systems are parallel in all four directions, we can equate the separate components, so that \( dx_0 = e^{\lambda/2} dr, dy_0 = e^{\mu/2} r d\theta, dz_0 = e^{\phi/2} r \sin \theta d\phi, dt_0 = e^{\nu/2} dt \); also, in this case, \( \frac{\partial r}{\partial x_0} = dr/dx_0 \) (and similarly for \( y_0, z_0, \) and \( t_0 \)), so that \( \frac{\partial r}{\partial x_0} = e^{-\lambda/2}, \frac{\partial \theta}{\partial y_0} = e^{-\mu/2}/r, \frac{\partial \phi}{\partial z_0} = e^{-\phi/2}/r \sin \theta, \frac{\partial t}{\partial t_0} = e^{-\nu/2} \). Further, again on account of the parallelism, \( \frac{\partial r}{\partial x_0} = \frac{\partial r}{\partial x_0} \cdot \frac{\partial z_0}{\partial x_0} = \frac{\partial r}{\partial x_0} \cdot \frac{\partial x_0}{\partial x_0} = 0 \) (i.e., not summed), etc.; and since \( v_0 \) is measured in “unconstrained” lengths and times we have \( v_0 = e^{\lambda/2-\nu/2} v \), where \( v = dr/dt \), and so also \( \kappa_0 = \kappa \) (see (5)). Collecting the various terms, we thus obtain
\[ \frac{\partial r}{\partial x_0} = e^{-\lambda/2} \kappa \tag{22} \]
\[ \frac{\partial \theta}{\partial y_0} = e^{-\mu/2}/r \tag{23} \]
\[ \frac{\partial \phi}{\partial z_0} = e^{-\nu/2}/r \sin \theta \tag{24} \]
\[ \frac{\partial t}{\partial t_0} = e^{-\nu/2} \kappa \tag{25} \]
\[ \frac{\partial r}{\partial t_0} = e^{-\lambda} v_0 \kappa = e^{-\nu} v_0 \kappa \tag{26} \]
and
\[ \frac{\partial l}{\partial x_0} = e^{-r/\nu K} = e^{-\lambda/\nu} - \nu. \]  
(27)

These can now be inserted, as appropriate, in equations (16) to (20), and the results are

\[ T^{11} = e^{-\lambda/\nu^2} p_r + e^{-\nu^2/\nu^2} p + 2e^{-\nu/\nu^2} f; \]  
(28)

\[ T^{22} = \frac{p_t}{e^{\nu^2}} \]  
(29)

\[ T^{33} = \frac{p_t}{e^{\nu^2} \sin^2 \theta}; \]  
(30)

\[ T^{44} = e^{\lambda - 2\nu^2/\nu^2} p_r + e^{-\nu^2/\nu^2} p + (e^{-\nu^2/\nu^2} - e^{\nu^2/\nu^2}) f; \]  
(31)

\[ T^{41} = T^{41} = e^{-\nu^2/\nu^2} (p_r + \rho) + (e^{\nu^2/\nu^2} - e^{-\nu^2/\nu^2}) f; \]  
(32)

or, lowering one index, and noting, as before, that

\[ e^{\lambda/\nu} - \nu = \sqrt{\kappa^2 - 1}, \]  
(33)

\[ T^i_1 = -\kappa^2 p_r - (\kappa^2 - 1) \rho - 2\kappa\sqrt{\kappa^2 - 1} f \]  
(34)

\[ T^i_2 = T^i_2 = -p_t \]  
(35)

\[ T^i_4 = (\kappa^2 - 1) p_r + \kappa^2 \rho + 2\kappa\sqrt{\kappa^2 - 1} f \]  
(36)

and

\[ e^{\lambda/\nu} - \nu T^i_4 = -e^{\nu} - \lambda/\nu T^i_4 = \kappa\sqrt{\kappa^2 - 1} (p_r + \rho) + (2\kappa^2 - 1) f. \]  
(37)

Solving for \( p_r, \rho, \) and \( f, \) we have also

\[ p_r = -\kappa^2 T^i_1 + (\kappa^2 - 1) T^i_4 - 2\kappa\sqrt{\kappa^2 - 1} \sqrt{T^i_4 T^i_4} \]  
(38)

\[ \rho = -((\kappa^2 - 1) T^i_1 + \kappa^2 T^i_4 - 2\kappa\sqrt{\kappa^2 - 1} \sqrt{T^i_4 T^i_4}) \]  
(39)

\[ f = \kappa\sqrt{\kappa^2 - 1} (T^i_1 - T^i_4) + \sqrt{T^i_4 T^i_4}. \]  
(40)

Equations (34) to (37) differ from equations (6) to (9) only by the presence of the terms in \( f, \) and by the fact that \( p_t \) appears in \( T^i_2, T^i_4, \) and \( T^i_4, \) while \( p_t \) appears in \( T^i_2 \) and \( T^i_4; \) contrary to what (15) suggests, there is no component in which \( p_r \) and \( p_t \) both appear. As in the flux-free case (1), \( T^i_1 + T^i_4 = \rho - p_r, \) (i.e., \( T = -p_r - 2p_t \) + \( \rho, \) and it is also readily verified that

\[ T^i_1 T^i_4 - T^i_4 T^i_1 = f^2 - p_r \rho; \]  
(41)

in fact, \( p_t^2(f^2 - p_r \rho) \) is the determinant of \( T^i_r, \) and this determinant is thus invariant for transformations which preserve the spherical symmetry (at least in so far as these do not introduce a term in \( dr \cdot dl \)).

Our results for the mixed tensor are independent of the factor \( e^\mu \) which appears in the "tangential" terms of the line-element (2); as in the static case, we may put \( \mu = 0 \) (i.e., "Schwarzschild") or \( \mu = \lambda \) (i.e., "isotropic") or may adopt any other permissible function for \( \mu, \) and equations (33) to (41) are formally unaffected.
Although equations (34) to (37) are mathematically compact, their form does perhaps tend to obscure their physical significance, since the obviously important velocity $v$ does not appear explicitly at all. We note, therefore, that this velocity (as (33) shows) is always present in the first power wherever the quantity $\sqrt{1 - \kappa^2}$ appears, and the positive sign of the root corresponds to the convention $v = dr/dt$, i.e., $v$ is the "velocity of rise" of the matter. We may substitute (33), and also (5) if we wish, into (34), etc., and the resulting forms will perhaps be more advantageous in some respects.

The velocity $v$ may take either sign; where the star really is collapsing, it is negative, and where the proper observer is stationary with respect to matter that is being blown away, it is positive. The matter may also be stationary, perhaps even over large regions, temporarily; but even if $v$ is zero everywhere, this does not return us to the completely static case; putting $\kappa = 1$ leads to $T^1_1 = -p$, $T^2_2 = T^2_2 = -p$, $T^4_4 = \rho$, and $\rho^{1/2} - \nu^{1/2}T^1_4 = -\nu^{1/2} - \lambda/2T^1_4 = f$, so that there may in principle still be a flux of energy. Whether there actually is one will depend, of course, on the equations of state (including that of energy-generation), and on the continuity-conditions implicit in Einstein's equations.

Summary.—The well-known relativistic energy-momentum tensor for a perfect fluid is modified by the inclusion of a term representing a radiation-flux, so as to make it more serviceable for a rapidly collapsing massive star. Provision is also made for a nonisotropic pressure. The components of the new tensor are obtained explicitly for the case of spherical symmetry, in terms which are formally independent of the particular radial coordinate ("Schwarzschild," "isotropic," etc.) that may be employed. The determinant of the mixed tensor is invariant for transformations which preserve the spherical symmetry. The physical significance, for a "proper" observer, of the symbols appearing in the tensor is briefly discussed.

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† On leave from the Royal Greenwich Observatory, Herstmonceux.
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