THE RECTANGULARITY LAW OF TRANSFORMERS

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1. Introduction and Results.—The geometry of a transformer is completely specified by the windows of the iron core and the copper coil. The optimal dimensions of these windows are determined by many factors, such as desired kv-a rating, acceptable losses, cost of iron, cost of copper, etc. In spite of these many factors influencing design, we have been able to deduce, from topological arguments alone, a simple relation between the rectangularity of the iron and copper windows for optimum geometrical design. Let \( a, b \) and \( a', b' \) represent the dimensions of the iron and copper windows, as defined in Figure 1. We introduce the rectangularity coefficients

\[ x = \frac{a}{b}, \quad x' = \frac{a'}{b'} \tag{1} \]

The relation we derive at optimum design is

\[ x + (\epsilon - 1)x' + x' = 1 \tag{2} \]

where the quantities \( \epsilon \) and \( \epsilon' \) depend upon the type of transformer construction, and are listed in Table 1. Relation (2) is plotted in Figure 2 for the case where both core and coil are doubly connected, and for the case where either the core or the coil is doubly connected, the other triply connected.

<table>
<thead>
<tr>
<th>Iron core</th>
<th>Copper coil</th>
<th>( \epsilon )</th>
<th>( \epsilon' )</th>
<th>( \epsilon' - 1 )</th>
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If one failed to take into consideration the corners, i.e., those regions of space where the magnetic flux or the electric current changes direction, one would have \( \epsilon = \epsilon' = 0 \). A solution of (2) would then be \( x = x' = 1 \), corresponding to square windows. The corners therefore play the dominating role in requiring rectangular windows at optimum design.

2. Analysis.—Our design problem may be formulated in a variety of ways. A typical way would be to require that the weight be a minimum subject to the condition that the transformer convert so many kv-a and to the condition that the power dissipated be not greater than a certain value. Another way would be to specify that the customer cost be a minimum, due account being taken of the present dollar worth of future losses, subject only to the condition of a given kv-a rating. Our topological argument and result is independent of how the design problem is formulated.

All quantities of interest, such as volume or cost of iron core and copper coil, and iron and copper losses, as well as kv-a rating, can be readily expressed as simple functions of \( L_{Fe}, L_{Cu}, A_{Fe}, A_{Cu} \), referring respectively to the mean flux path length in the iron core, the mean electrical path length in the copper coil, and the cross
section areas of the iron core and copper coil. These four quantities can be expressed as functions of the four adjustable design variables \(a, b, a', b'\). Thus,

\[
L_{Fe} = 2(a + b + ea') \\
L_{Cu} = 2(a' + b' + e'a) \\
A_{Fe} = a'b' \\
A_{Cu} = ab.
\]

The topological problem relates to the independency of variations in the vector \((L_{Fe}, L_{Cu}, A_{Fe}, A_{Cu})\).

Physical reasoning shows that this vector cannot suffer arbitrary variations in the immediate vicinity of the optimum design. All the undesirable quantities, such as weights or costs of iron core and copper coil, as well as power losses, contain \(L_{Fe}\) and \(L_{Cu}\) in the numerator. The kv-a rating contains neither \(L_{Fe}\) nor \(L_{Cu}\).
Suppose now we are at optimum geometric design. Then, if our vector could suffer arbitrary variations, we could lower all the undesirable quantities by simply decreasing the first two components, keeping the second two components fixed. Such a possibility is in contradiction to our assumption of being at a minimum. We conclude that in the vicinity of the optimum design the vector \((L_{Fe}, L_{Cu}, A_{Fe}, A_{Cu})\) cannot be varied in an arbitrary manner. In mathematical language, the appropriate Jacobian must vanish:

\[
\frac{\partial}{\partial (a, b, a', b')} (L_{Fe}, L_{Cu}, A_{Fe}, A_{Cu}) = 0.
\]  

(4)

Substitution of relations (3) into (4), and making use of the definitions (1), leads directly to our result (2).

\[\text{Note added in proof:}\] The authors are indebted to the engineers of our Power Transformer Division for constructive comments. Mr. J. H. McWhirter has pointed out that the rectangularity law may be generalized to the case of fixed clearances between the copper conductors within the iron window. Dr. Stein has pointed out that Reed has obtained (eq. (2)) for that particular design which minimizes the total transformer losses [Reed, E. G., Essentials of Transformer Practice (New York: D. Van Nostrand Co., 1927), pp. 94-96.]

\[\text{COMPETITION, HABITAT SELECTION, AND CHARACTER DISPLACEMENT IN A PATCHY ENVIRONMENT}\]

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It is well known that related species often differ in either habitat\(^1\) or size, and thereby avoid competitive elimination. The way in which they differ is related to the specialized ways they have of using resources, which ways in turn control numbers of coexisting species and other aspects of the evolution of the community. The detailed reasons for these assertions are given in the following paragraphs.\(^2\) Briefly, the argument is as follows. Among species which specialize on a single uniform resource, only the most effective one will survive and that species will be found wherever the resource occurs, in abundance determined by the density of the resource. Other such pure specialist species will be found, one to a uniform resource; these will normally differ in morphology, but will not in general be affected by one another's distributions. On the other hand, species which specialize on a particular proportion of mixture of two or more particular resources will be found only where their favored proportion is found, and will be replaced by other species in other habitats where the proportion of the mixture changes to one on which the new species are more effective. Of this mixed-resource type of species there can be as many\(^3\) as there are proportions of the resources which can be counted on from season to season—i.e., very many in stable climates and fewer in unpredictable climates.

To make these ideas more precise, we first consider an imaginary habitat in which there is a scattering of uniform units or grains of resource 1 and another