(6) There was no material difference in temperature between a depth of 6 inches and one of 7 inches, while there was a distinct difference of temperature between a depth of 4 inches and one of 6 inches, thus showing the unreliability of measurements of temperature at a depth of less than 6 inches.

(7) The standing as compared with the lying position of the animal had hardly any effect on the body temperature, but there was some indication that the temperature was slightly affected when measured shortly after the change in position had been made.

(8) There was no difference in body temperature when measured before or after defecation.

(9) Daily fluctuations in body temperature depend to a great extent on the individuality of the cow.

(10) A variation of 0.8° in the rectal temperature of the same animal was observed, when measured at the same hour of the day under identical conditions and outside the influence of water or feed, while under the influence of water a difference of 1.3° was observed at the same hour on two consecutive days.

A detailed report of the results will appear in the Journal of Agricultural Research.

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ON THE STRUCTURE OF FINITE CONTINUOUS GROUPS WITH EXCEPTIONAL TRANSFORMATIONS

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Let $X_k = \sum_{i} f_{ik}(x) \frac{\partial}{\partial X_i}$ where $k = 1, 2, \ldots, r, r + 1, \ldots r + q$, be the differential symbols of a finite continuous group $G$ of order $r + q$, and let $E_k \equiv \sum_{j} \sum_{i} \alpha_i \gamma_{ji} \frac{\partial}{\partial \alpha_i}$ be the symbols of the operators of the adjoint of $G$. We then have

$$\left(X_i, X_j\right) = \sum_{k} c_{ijk} X_k$$

$$\left(E_i, E_j\right) = \sum_{k} c_{ijk} E_k$$

where the $c$'s are the so-called structural constants. We assume that
the group $G$ has $q$ ($\leq r$) exceptional infinitesimal transformations (Lie calls them “ausgezeichnete”), which for simplicity are taken to be

$$X_r + 1, X_r + 2, \ldots X_r + q.$$  

Then

$$(X_i, X_j) = 0 \quad (i = 1, 2, \ldots r + q; j = r + 1, \ldots, r + q).$$

The purpose of this paper is to find the conditions to be imposed on the group $G$ which would make the structural constants $C_{ijr+1}, \ldots, C_{ijr+q}$ all zero for $i, j = 1, 2, \ldots, r + q$.

Since the adjoint of $G$ has the same structure as $G$ itself, it follows that

$$(E_i, E_j) = \sum_{k=1}^{r} c_{ijk} E_k \quad (i, j = 1, 2, \ldots r).$$

Therefore we may say that if $G$ contains $q$ exceptional infinitesimal transformations, there exists another group, say $G'$, with $r$ essential parameters having the same structure as the group $G$.

We shall denote the operators of the adjoint of $G'$ by the symbols $M_k = \sum_{i=1}^{r} \sum_{k=1}^{r} \alpha_{ijk} \frac{\partial}{\partial \alpha_i}$ ($k = 1, 2, \ldots r$).

The condition we impose on the adjoint of $G'$ is that it shall have one invariant spread. From that follows that the nullity of the matrix formed by the coefficients of the $r$ differential operators is equal to one, i.e., at least one of the minors of the determinant of the coefficients of order $r-1$ is not zero. But every minor of that determinant is also a minor of the determinant formed by the coefficients of the differential operators of the adjoint of $G$, therefore in this larger determinant there will be at least one non-vanishing minor of order $r-1$. Furthermore, for $\alpha_r, \ldots, \alpha_{r+q}$ assigned, the symbolic equations

$$\left( \sum_{i=1}^{r+q} \alpha_i \frac{\partial}{\partial \alpha_i} \right) \left( \sum_{i=1}^{r+q} \beta_i \frac{\partial}{\partial \alpha_i} \right) = 0$$

have the following $q+1$ independent solutions:

$$\beta_1 = \alpha_1, \ldots, \beta_r = \alpha_r, \beta_{r+1} = \ldots = \beta_{r+q} = 0 \quad (1)$$

$$\beta_1 = \ldots = \beta_r = 0, \beta_{r+1} = 1, \beta_{r+2} = \ldots = \beta_{r+q} = 0 \quad (2)$$

$$\beta_1 = \ldots = \beta_{r+1} = 0, \beta_{r+2} = 1, \beta_{r+3} = \ldots = \beta_{r+q} = 0 \quad (3)$$

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$$\beta_1 = \ldots = \beta_{r+q-1} = 0, \beta_{r+q} = 1 \quad (q+1)$$

That shows that there will be in the large determinant at least one non-vanishing minor of order $r-q-1$. Therefore, the nullity of the matrix formed by the coefficients of the operators of the adjoint of $G$ is equal to $q+1$, which means that the adjoint of $G$ leaves invariant $q+1$
independent functions in \( r + q \) variables. It should be observed, however, that one of those functions will also be invariant to the adjoint of \( G' \) and will consist of the first \( r \) variables alone. The following theorem may therefore be stated:

*If the adjoint of \( G' \) has one invariant, the adjoint of \( G \) has \( q + 1 \) independent invariants, one of which is the invariant of the adjoint of \( G' \).*

We assume first that \( q \) of those invariants, not involving the one common to both, \( G \) and \( G' \), are all flats of order \( r + q - 2 \) in the \( r + q - 1 \) space and that their common intersection is an \( r - 1 \) flat. It is then proven that if that flat does not pass through any of the points of the space of the adjoint of \( G \), corresponding to the exceptional transformations of \( G \), we can form new operators, such linear functions of the old ones, of which \( r \), not involving the exceptional operators, form an invariant subgroup of \( G \) of order \( r \). This means that the structural constants, whose last subscripts are greater than \( r \), will all become zero. If, however, those \( q \) invariants are not flats but algebraic spreads, not necessarily of the same order, none of which passes through the points of the space of the adjoint of \( G \) described above, then by considering the polars of the \( q \) invariant spreads we prove that the structural constants which have for their last subscripts numbers greater than \( r \) can also be made all zero. If, finally, each of the \( q \) invariant spreads passes through only one of those points, not necessarily the same for all spreads, then the structural constants described above can also be made all zero.

Detailed proofs and references are given in a paper accepted for publication by the *Annals of Mathematics*. 