was given there because the patient has become more deaf at the higher pitches as shown by Curve 2, figure 1, since the first curve was taken.

It would seem, therefore, even in the case of the complete modification of the internal-ear mechanism that a person can hear in a normal manner, provided the tones are sufficiently intense and provided there has not been a complete destruction of the nerve endings or of the nerves themselves in which case we should expect complete deafness. Only one such case has as yet been found and in this case the patient was unable to hear at any pitch in either ear with as much as $5 \times 10^6$ times as much current passing through the receiver as was required by a normal ear.

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**THE QUANTITATIVE INFLUENCES OF CERTAIN FACTORS INVOLVED IN THE PRODUCTION OF CYANOSIS**

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Lundsgaard has shown that the appearance of cyanosis depends on the mean concentration of reduced hemoglobin ($C$) in the capillary blood. This concentration he estimated as

$$C = \frac{A + V}{2} \quad (1)$$

where $A$ is the arterial, $V$ the venous concentration of reduced hemoglobin. The effect of certain physiological factors contributing to $C$ is estimated as follows. We let $T$ represent the total hemoglobin concentration in the blood, $l$ the fraction of total hemoglobin passing in reduced form through the aerated parts of the lungs, $D$ the concentration of reduced hemoglobin formed by deoxygenation of the blood as it passes from arteries to veins through the tissue capillaries. In certain pathological conditions a fraction of the venous blood reaches the arteries without traversing aerated parts of the lungs and therefore carries reduced venous hemoglobin directly into the arterial blood. This fraction we designate as $\alpha$. $A$, $T$, and $C$ have the significance indicated above.

When $\alpha = 0$, only aerated blood entering the arteries, $A = l T$. When however, $\alpha$ has a positive value, the reduced hemoglobin of the arterial blood represents the sum of that in the fraction $\alpha$, of venous blood entering
the arteries, plus that of the fraction \((1 - \alpha)\) of arterial blood from the lungs. In such a case

\[
A = (1 - \alpha) l T + \alpha V
\]

\[
\alpha = \frac{A - l T}{V - l T}
\]

Substituting \(V - D\) for \(A\) in (3) and solving for \(V\),

\[
V = l T + \frac{D}{l - \alpha}
\]

Substituting \(A + D\) for \(V\) in (3) and solving for \(A\),

\[
A = l T + \frac{\alpha D}{1 - \alpha}
\]

We now substitute for \(A\) and \(V\) in (1) their values as expressed in (4) and (5), and obtain

\[
C = l T + \frac{(1 + \alpha)D}{2(1 - \alpha)}
\]

which indicates the effects of the four factors, \(l\), \(T\), \(\alpha\), and \(D\) on \(C\).

An interesting point brought out by (6), and in accord with observation, is that other factors normal about 40 per cent of venous blood may be mixed with arterial \((\alpha = 0.4)\) before the reduced hemoglobin concentration in the latter becomes great enough to cause cyanosis.

The effects on \(C\) of separate variations of \(l\), \(T\), \(D\), and \(\alpha\) are indicated by partial differentiation of (6).

\[
\frac{\partial C}{\partial l} = T \partial l
\]

\[
\frac{\partial C}{\partial T} = l \partial T
\]

\[
\frac{\partial C}{\partial \alpha} = \frac{D}{(1 - \alpha)^2} \partial \alpha
\]

\[
\frac{\partial C}{\partial D} = \frac{(1 + \alpha)}{2(1 - \alpha)} \partial D
\]

While the reduced hemoglobin concentration in the capillary blood may be accepted as the cause of cyanosis, there are various other factors which modify the resulting coloration. Such are local skin vasculature, pigmentation, thickness of epidermis; and also the fact that the mean capillary content of reduced hemoglobin \(\frac{1}{2} (A + V)\) only approximately represents the average content. With changing conditions the latter may instead of being midway between venous and arterial, approximate either more
nearly than the other. The effect of these modifying factors is to cause the value of $C$ at which cyanosis becomes perceptible to vary from 4 to 6 gms. of reduced hemoglobin per 100 cc. of blood, and perhaps sometimes even more widely, although it appears usually to lie near 5.