Cosmic microwave background theory

J. RICHARD BOND*

Canadian Institute for Advanced Research Cosmology Program, Canadian Institute for Theoretical Astrophysics, 60 Saint George Street, Toronto, ON M5S 3H8, Canada

ABSTRACT   A long-standing goal of theorists has been to constrain cosmological parameters that define the structure formation theory from cosmic microwave background (CMB) anisotropy experiments and large-scale structure (LSS) observations. The status and future promise of this enterprise is described. Current band-powers in ε-space are consistent with a ∆T flat in frequency and broadly follow inflation-based expectations. That the levels are ∼(10^{-5})^2 provides strong support for the gravitational instability theory, while the Fair Infrared Absolute Spectrophotometer (FIRAS) constraints on energy injection rule out cosmic explosions as a dominant source of LSS. Band-powers at ε ≳ 100 suggest that the universe could not have re-ionized too early. To get the LSS of Cosmic Background Explorer (COBE)-normalized fluctuations right provides encouraging support that the initial fluctuation spectrum was not far off the scale invariant form that inflation models prefer: e.g., for tilted Λ cold dark matter sequences of fixed 13-Gyr age (with the Hubble constant H_0 marginalized), n_s = 1.17 ± 0.3 for Differential Microwave Radiometer (DMR) only; 1.15 ± 0.08 for DMR plus the SK95 experiment; 1.00 ± 0.04 for DMR plus all smaller angle experiments; 1.00 ± 0.05 when LSS constraints are included as well. The CMB alone currently gives weak constraints on Λ and moderate constraints on Ω_{tot} but theoretical forecasts of future long duration balloon and satellite experiments are shown which predict percent-level accuracy among a large fraction of the 10+ parameters characterizing the cosmic structure formation theory, at least if it is an inflation variant.

THE THEORETICAL AGENDA

Cosmic Microwave Background (CMB) as a Probe of Early Universe Physics

The source of fluctuations to input into the cosmic structure formation problem is likely to be found in early universe physics. We want to measure the CMB [and large-scale structure (LSS)] response to these initial fluctuations. The goal is the lofty one of peering into the physical mechanism by which the fluctuations were generated. The contenders for generation mechanism are (i) “zero point” quantum noise in scalar and tensor fields that must be there in the early universe if quantum mechanics is applicable and (ii) topological defects that may arise in the inevitable phase transitions expected in the early universe.

From CMB and LSS observations we hope to learn the following: the statistics of the fluctuations, whether Gaussian or non-Gaussian; the mode, whether adiabatic or isocurvature scalar perturbations, and whether there is a significant component in gravitational wave tensor perturbations; the power spectra for these modes, P_{θθ}(k), P_{uu}(k), P_{GW}(k) as a function of comoving wavenumber k. Sample initial and evolved power spectra for the gravitational potential P_{θθ}(k) (= dσ_{θθ}^2/d ln k, the rms power in each dln k band) are shown in Fig. 1. As the Universe evolves the initial shape of P_{θθ} is modified by characteristic scales imprinted on it that reflect the values of cosmological parameters such as the energy densities of baryons, cold and hot dark matter, in the vacuum (cosmological constant), and in curvature. Many observables can be expressed as weighted integrals over k of the power spectra and thus can probe both density parameters and initial fluctuation parameters.

The (linear) density power spectra, P_{θθ}(k) ∝ k^n P_{θθ}(k), are also shown in Fig. 1. In hierarchical structure formation models such as those considered here, the nonlinear wavenumber k_{NL}(z), defined by ∫_{k_{NL}}^∞ P_{θθ}(k)dk = 1, grows as the universe expands. k_{NL}(z) was in the galaxy band at redshift 3 and is currently in the cluster band. At k > k_{NL}(z), nonlinearities and complications associated with dissipative gas processes can obscure the direct connection to the early universe physics. Most easily interpretable are observables probing the linear regime now, k < k_{NL}(z). CMB anisotropies arising from the linear regime are termed primary; as Fig. 1 shows, these probe 3 decades in wavenumber. LSS observations at low redshift probe a smaller, but overlapping, range. We hope that z < 3 LSS observations, when k_{NL}(z) was larger, can extend the range, but gas dynamics can modify the relation between observable and power spectrum in complex ways. Secondary anisotropies of the CMB (see below), those associated with nonlinear phenomena, also probe smaller scales and the “gastrophysical” realm.

Cosmic Parameters

Even simple Gaussian inflation-generated fluctuations for structure formation have a large number of early universe parameters we would wish to determine (see next section): power spectrum amplitudes at some normalization wavenumber k_s for the modes present, [P_{θθ}(k_s), P_{uu}(k_s), P_{GW}(k_s)]; shape functions for the “tilts” [v_{θθ}(k), v_{uu}(k), v_{θθ}(k)], usually chosen to be constant or with a logarithmic correction—e.g., v_{θθ}(k), d v_{θθ}(k)/d ln k. [The scalar tilt for adiabatic fluctuations,

Abbreviations: CMB, cosmic microwave background; LSS, large-scale structure; COBE, Cosmic Background Explorer; MAP, Microwave Anisotropy Probe; DMR, Differential Microwave Radiometer; CDM, cold dark matter; ΛCDM, CDM with a cosmological constant; SCDM, the standard CDM model; TCDM, tilted CDM; HCDM, CDM with hot dark matter; OCDM, open (negatively curved) CDM models; COMBA, Cosmic Background Radiation Archive; FIRAS, Far Infrared Absolute Spectrophotometer; SZ, Sunyaev–Zeldovich; LDB, long-duration balloon; GW, gravity wave; HEMT, High Electron Mobility Transistor; Mpc, megaparsec.

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e-mail: bond@cita.utoronto.ca.
Fig. 1. The bands in comoving wavenumber $k$ probed by CMB primary and secondary anisotropy experiments, in particular by the satellites Cosmic Background Explorer (COBE), Microwave Anisotropy Probe (MAP), and Planck, and by LSS observations are contrasted. Mpc, megaparsec ($3.09 \times 10^6$ m). The width of the CMB photon decoupling region and the sound crossing radius ($\Delta r_{\text{dec}}$, $c s_{\text{dec}}$) define the effective acoustic peak range. Sample (linear) gravitational potential power spectra [actually $P_{g_0}(k)$] are also plotted. The region at low $k$ gives the 4-yr Differential Microwave Radiometer (DMR) error bar on the $\Phi$ amplitude in the COBE regime. The solid data point in the cluster-band denotes the $k$ constraint from the abundance of clusters, and the open data point at $10h^{-1}$ Mpc denotes a $\Phi$ constraint from streaming velocities (for $\Omega_{\text{tot}} = 1$, $\Omega_{\Lambda} = 0$). The open squares are estimates of the linear $\Phi$ power from current galaxy clustering data by ref. 1. A bias is “allowed” to (uniformly) raise the shapes to match the observations. The corresponding linear density power spectra, $P_{s_0}(k)$, are also shown rising to high $k$. Models are the “standard” $n_s = 1$ cold dark matter (CDM) model (labeled $\Gamma = 0.5$), a tilted ($n_s = 0.6$, $\Gamma = 0.5$) CDM ($TCDM$) model, and a model with the shape modified ($\Gamma = 0.25$) by changing the matter content of the Universe.

$v_s(k) = d \ln P_{g_0}/d \ln k$, is related to the usual index, $n_s$, by $v_s = n_s - 1$. The transport problem (see below) is dependent upon physical processes, and hence on physical parameters. A partial list includes the Hubble parameter $h$, various mean energy densities [$\Omega_{\text{tot}}$, $\Omega_{\Lambda}$, $\Omega_{\text{baryon}}$, $\Omega_{\text{cdm}}$], and parameters characterizing the ionization history of the Universe—e.g., the Compton optical depth $\tau_c$ from a reheating redshift $z_{\text{reh}}$ to the present. Instead of $\Omega_{\text{tot}}$, we prefer to use the curvature energy parameter, $\Omega_k = 1 - \Omega_{\text{tot}}$, thus zero for the flat case. In this space, the Hubble parameter $h = (2\Omega_{\Lambda})^{1/2}$, and the age of the Universe, $t_0$, are functions of the $\Omega_k^{-1/2}$. The density in nonrelativistic (clustering) particles is $\Omega_{\text{nr}} = \Omega_B + \Omega_{\text{cdm}} + \Omega_{\text{elm}}$. The density in relativistic particles, $\Omega_{\text{r}}$, includes photons, relativistic neutrinos, and decaying particle products, if any. $\Omega_{\text{r}}$, the abundance of primordial helium, etc., should also be considered as parameters to be determined. The count is thus at least 17. Estimates of errors on a smaller 9-parameter inflation set for the MAP and Planck satellites are given in the final section.

The arena in which CMB theory battles observation is the anisotropy power spectrum in multipole space, Fig. 2, which shows how primary $C_\ell$ values vary with some of these cosmic parameters. Here $C_\ell = \ell (\ell + 1) ((\Delta T/T)_{\text{rms}}^2)/(2\pi)$. The $C_\ell$ values are normalized to the 4-yr DMR($53 + 90 + 31$) (A + B) data ($4 - 7$). The arena for LSS theory is the $P_{\delta}$ of Fig. 1.

For a given model, the early universe $P_{\delta}$ is uniquely related to late-time power spectrum measures of relevance for the CMB, such as the quadrupole $C_2^{\ell_0}$ or averages over $\ell$-bands $B$, $\langle C_\ell^{\ell_0} \rangle$, and to LSS measures, such as the rms density fluctuation level on the $8 h^{-1}$ Mpc (cluster) scale, $\sigma_8$, so any of these can be used in...
place of the primordial power amplitudes in the parameter set. In inflation, the ratio of gravitational wave power to scalar adiabatic power is \( \mathcal{P}_g/\mathcal{P}_s \approx -100/9 \nu_t/(1 - \nu_t/2) \), with small corrections depending upon \( \nu_t \approx \nu_s \) (5, 8). If such a relationship is assumed, the parameter count is lowered by one.

**Freedom in Inflation**

Many variants of the basic inflation theme have been proposed, sometimes with radically different consequences for \( \mathcal{P}_g(k) \sim k^{-1 - n_{\text{dil}}} \), and thus for the CMB sky, which is used in fact to highly constrain the more baroque models. A ranking order of inflation possibilities: (i) adiabatic curvature fluctuations with nearly uniform scalar tilt over the observable range, slightly more power to large scales \( \left(0.8 < n_t < 1\right) \) than “scale invariance” \( (n_s = 1) \), gives a predictable nonzero gravity wave contribution with tilt similar to the scalar one, and tiny mean curvature \( \left(\Omega_{\text{tot}} \approx 1\right) \); (ii) same as i, but with a tiny gravity wave contribution; (iii) same as i but with a subdominant isocurvature component of nearly scale-invariant tilt (the case in which isocurvature dominates is ruled out); (iv) radically broken scale invariance with weak to moderate features (ramps, mountains, valleys) in the fluctuation spectrum (strong ones are largely ruled out); (v) radical breaking with non-Gaussian features as well; (vi) “open” inflation, with quantum tunneling producing a negatively curved (hyperbolic) space which inflates, but not so much as to flatten the mean curvature \( (d_c \sim (Ha)^{-1}, \text{not} \gg (Ha)^{-1}) \), where \( d_c = H_i^{-1} \left| \Omega_i \right|^{-1/2} \); (vii) quantum creation of compact hyperbolic space from “nothing” with volume \( d^3_i \) that inflates, with \( d^3_i \sim (Ha)^{-1}, \text{not} \gg (Ha)^{-1} \), and \( d_f \) of order \( d_c \); and (viii) flat \( (d_c = \infty) \) inflating models that are small tori of scale \( d_f \) with a few \( (Ha)^{-1} \) in size. It is quite debatable which of the cases beyond ii are more or less plausible, with some claims that iv is supersymmetry-inspired, others that vi is not as improbable as it sounds. It is the theorists’ job to push out the boundaries of the inflation idea and use the data to select what is allowed.

**LSS Constraints on the Power Spectrum**

We have always combined CMB and LSS data in our quest for viable models. Fig. 1 shows how the two are connected. DMR normalization precisely determines \( \sigma_8 \) for each model considered; comparing with the \( \sigma_8 \sim 0.6 \Omega_m^{0.5} \) target derived from cluster abundance observations severely constrains the cosmological parameters defining the models. In Fig. 1, this means the COBE-normalized \( \mathcal{P}_g(k) \) must thread the “eye of the needle” in the cluster-band.

Similar constrictions arise from galaxy–galaxy and cluster–cluster clustering observations: the shape of the linear power spectrum \( P(k) \) is roughly compatible with an allowed range \( 0.15 < \Gamma + \nu_t/2 \lesssim 0.3 \), where \( \Gamma \approx \Omega_m h^2 \Omega_{\text{tot}} / (1.681) \left| 1/2 - \Omega_m h^2 \Omega_{\text{tot}} \right|^{1/2} \) characterizes the density transfer function shape. The standard CDM (SCDM) model has \( \Gamma \approx 0.5 \). To get \( \Gamma + \nu_t/2 \) in the observed range one can: lower \( h \), lower \( \Omega_{\text{tot}} \) (ACDM, OCDM), raise \( \Omega_{\text{tot}} \), the density parameter in relativistic particles (1.68l), with three species of massless neutrinos and the photons)—e.g., as in \( \tau \text{CDM} \), with a decaying \( \nu \) of lifetime \( \tau \) and \( \Gamma = 1.08 \Omega_m h (1 + 0.966 (m_{\nu}/\text{keV}) \gamma)^{1/2}; \) raise \( \Omega_m \), tilt \( \nu_s < 0 \) (TCDM), for standard CDM parameters—e.g., \( 0.3 \approx \nu_s < 0.7 \) would be required. Adding a hot dark matter component gives a power spectrum characterized by more than just \( \Gamma \).

**Cosmological Radiative Transport**

Cosmological radiative transfer is a firm theoretical footing. Together with a gravity theory (invariably Einstein’s general relativity, but the CMB will eventually be used as a test of the gravity theory) and the transport theory for the other fields and particles present (baryons, hot, warm, and cold dark matter, coherent fields—i.e., “dynamical” cosmological “constants”, etc.), we propagate initial fluctuations from the early universe through photon decoupling into the (very) weakly nonlinear phase, and predict primary anisotropies, those calculated using either linear perturbation theory (e.g., for inflation-generated fluctuations), or, in the case of defects, linear response theory. The sources driving their development are all proportional to the gravitational potential \( \Phi \); the “naive” Sachs–Wolfe effect, \( \Phi/3 \); photon bunching (acoustic), \( 1/3 (\delta\rho_\gamma/\rho_\gamma) \), responsible for the adiabatic \( \nu/3 (\delta\rho/\rho) \) effect and the isocurvature effect; linear-order Thompson scattering (Doppler), \( \sigma T_\gamma \), with \( T_\gamma \) the Thomson cross section, \( \nu_c \), and the electron velocity and density, and \( q \) the photon direction; the (line-of-sight) integrated Sachs–Wolfe effect, \( -2 f_{\text{los},\Phi} \); there are also subdominant anisotropic stress and polarization terms. For primary tensor anisotropies, the sources are the two polarization states of gravity waves, \( \nu/3 (\delta\rho/\rho) \) again there are subdominant polarization terms.

Spurred on by the promise of percent-level precision in cosmic parameters from CMB satellites (see last section of this paper), a considerable fraction of the CMB theoretical community with Boltzmann transport codes compared their approaches and validated the results to ensure percent-level accuracy up to \( \sim 3 \times 10^{-5} \) (Cosmic Background Radiation Archive [COMBA]1). An important goal for COMBA was speed, since the parameter space we wish to constrain has many dimensions. Most groups have solved cosmological radiative transport by evolving a hierarchy of coupled moment equations, one for each \( \ell \). Although the equations and techniques were in place prior to the COBE discovery for scalar modes, and shortly after for tensor modes, to get the high accuracy with speed has been somewhat of a challenge. There are alternatives to the moment hierarchy for the transport of photons and neutrinos. In particular, the entire problem of photon transport reduces to integral equations in which the multipoles with \( \ell > 2 \) are expressed as history-integrals of metric variables, photon-bunching, Doppler, and polarization sources. The fastest COMBA-validated code uses this method (9).

**Secondary Anisotropies**

Although hydrodynamic and radiative processes are expected to play important roles around collapsed objects and may bias the galaxy distribution relative to the mass (gastro-physics regime in Fig. 1), a global role in obscuring the early universe fluctuations by late time generation on large scales now seems unlikely. Not too long ago it seemed perfectly reasonable, based on extrapolation from the physics of the interstellar medium to the pre-galactic and intergalactic medium, to suppose hydrodynamical amplification of seed cosmic structure could create the observed Universe. The strong limits on Compton cooling from Far Infrared Absolute Spectrophotometer (FIRAS) (10), in energy \( \delta E_{\text{Compton}} \sim 4xy < 6.0 \times 10^{-5} \) (95% confidence limits), constrain the product \( f_{\text{exp}} R_{\text{exp}}^2 \) of filling factor \( f_{\text{exp}} \) and bubble formation scale \( R_{\text{exp}} \), to values too small for a purely hydrodynamic origin. If supernovae were responsible for the blasts, the accompanying presupernova light radiated would have been much in excess of the explosive

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energy (more than a hundredfold), leading to much stronger restrictions (e.g., ref. 5).

Nonetheless significant “secondary anisotropies” are expected. These include: linear weak lensing, dependent on the two-dimensional (projected) tidal tensor, $\bar{\epsilon}_{AB}$; the Rees–Sciama effect, $2I_{\text{los.}} \Phi_{\text{NL}}$, dependent upon the gravitational potential changes associated with nonlinear structure formation; nonlinear Thompson scattering, $\sigma_T \bar{n}_{\text{e}} \nu_c^2 q$, dependent upon the fluctuation in the electron density $\bar{n}_{\text{e}}$ as well as $\nu_c$ and responsible for the quadratic-order (Vishniac) effect and the “kinematic” Sunyaev–Zeldovich (SZ) effect (moving cluster/galaxy effect); the thermal SZ effect, associated with Compton cooling, $I_{\text{los.}} \phi_S(x) \delta(n_{\text{e}} T_e)$, where $\phi_S(x)$ is a function of $x = E_S/T$, passing from $-2$ on the Rayleigh–Jeans end to $x$ on the Wien end, with a null at $x = 2.83$ (i.e., 1.863 $\mu$m or 161 GHz); pregalactic or galactic dust emission, $\sim f_{\text{los.}} \phi_{\text{dust}}(x) T_d$ dependent upon the distribution of the dust density $\rho_d$ and temperature $T_d$ through a function of $x_d = E_S/T_d$.

Secondary anisotropies may be considered as a nuisance foreground to be subtracted to get at the primary ones, but they are also invaluable probes of shorter-distance aspects of structure formation theories, full of important cosmological information. The $\ell$-space range they probe is shown in Fig. 1. The effect of lensing is to smooth slightly the Doppler peaks and troughs of Fig. 2. $\ell$ values from quadratic nonlinearities in the gas at high redshift are concentrated at high $\ell$, but for most viable models they are expected to be a small contaminant. Thomson scattering from gas in moving clusters also has a small effect on $\ell$ (although it should be measurable in individual clusters). Power spectra for the thermal SZ effect from clusters are larger (11); examples in the top panel of Fig. 2 are for cluster-normalized HCDM and TCDM models, with $\tilde{y} \sim (0.5 - 1) \times 10^{-6}$. OCDM and ACDM models have slightly higher values, but still small—compare the FIRAS constraint. Although $\ell^2$ may be small, because the power for such non-Gaussian sources is concentrated in hot or cold spots the signal is detectable, and has been for two dozen clusters now at the $>5 \sigma$ level. $\ell$ for a typical dusty primeval galaxy model is also shown, the smaller angle contribution associated with clustering. Dusty anisotropies are very likely to be observable with new instrumentation on submillimeter telescopes (e.g., SCUBA on the James Clerk Maxwell Telescope on Mauna Kea). Similar shot-noise spectra are expected for other extragalactic point sources—e.g., radio galaxies.

CMB PARAMETER ESTIMATION, CURRENT AND FUTURE

The Theorists’ Phenomenology of CMB Experiments

We have progressed from the tens of pixels of early $\Delta T/T$ experiments through thousands for DMR (4) and SK95 (12), soon tens of thousands for long-duration balloon (LDB) experiments and eventually millions for the MAP (home page http://map.gsfc.nasa.gov) and Planck (ref. 13; home page http://astro.estec.esa.nl/SA-general/Projects/Cobras/cobras.html) satellites. Finding nearly optimal strategies for data projection, compression, and analysis that will allow us to disentangle the primary anisotropies from the Galactic and extragalactic foregrounds and from the secondary anisotropies induced by nonlinear effects will be the key to realizing the theoretically possible precision on cosmic parameters and so to determine the winners and losers in theory space. Particularly powerful is to combine results from different CMB experiments and combine these with LSS and other observations. Almost as important as the end-product is the application of the same techniques to probing the self-consistency and cross-consistency of experimental results.

Much phenomenology is done using few-parameter local models of $\ell_{\text{c}}$—e.g., one with a broad-band power $\ell_{\text{cB}}$ and a broad-band tilt $\nu_{\text{AB}}$ used for COBE. It has been usual in the COBE literature to denote $1 + \nu_{\text{AB}}$ by $n_{\ell}$, inviting confusion with the index for the primordial adiabatic fluctuation power spectrum, $n_{\ell} = 1 + \nu_{\ell}$; for SCDM models with $\Omega_m = 0$ and $\Omega_{\lambda} = 0$, $\nu_{\text{AB}} = 0.15 + \nu_{\ell}$ over the COBE band. It is evident from Fig. 2 that a single power law is a poor fit to the data when the $\ell$-range is enlarged. $\ell_{\text{c}}$-sequences such as the scalar-only tilted CDM sequence with variable $n_{\ell}$ shown in the second panel of Fig. 2 cover the rise in power suggested by the SK95 data (with $n_{\ell} \approx 1$) while still matching the COBE data well and are therefore of phenomenological as well as obvious theoretical interest.

Parameters from the CMB, Current State

Combining CMB anisotropy experiments probing different ranges in $\ell$-space improves parameter estimates because of the much extended baseline. Current band-powers, shown in the top panel of Fig. 2, broadly follow inflation-based expectations (the SCDM $\ell_{\text{c}}$ is shown underlying them) but may still include residual signals. Lower panels compress the information into nine optimal bandpower estimates derived from all of the current data by A. Jaffe, L. Knox, and me.

Jaffe and I (6) have undertaken full Bayesian statistical analysis of the 4-yr DMR (4), SP94 (14), and SK94/SP95 (12, 15) data sets, taking into account all correlations among pixels in the data and theory. Other experiments were included by using their bandpowers as independent points with the Gaussian errors shown in Fig. 2. We have shown this approximate method works reasonably well by comparing results derived for DMR + SP94 + SK95 with the full analysis with those using just their bandpowers (16).

With current errors on the data, simultaneously exploring the entire parameter space of the Cosmic Parameters section is not useful, so we restricted our attention to various subregions of $(\Omega_B h^2, \Omega_{\cdm} h^2, \Omega_{\text{baryon}} h^2, \Omega_{\text{cDM}} h^2, \Omega_\gamma h^2, \nu_c, \nu_r, \sigma_8)$, such as $(\sigma_8, n_s, h | \text{fixed } t_0, \Omega_B h^2)$, where $\Omega_B = 0$ and $\Omega_{\lambda} = 0$, $\Omega_B = 0$ and $\Omega_{\lambda} > 0$ is a function of $h t_0$. The age of the Universe, $t_0$, was chosen to be 11, 13, or 15 Gyr. A recent estimate for globular cluster ages with the Hipparcos correction is 11.5 ± 1.3 Gyr (17), with perhaps another Gyr to be added associated with the delay in globular cluster formation, so 13 Gyr is a good example. We considered the ranges $0.5 \leq n_s \leq 1.5$, $0.43 \leq h \leq 1$, and $0.03 \leq \Omega_B h^2 \leq 0.05$. Results are mostly shown for the old “standard” nucleosynthesis estimate $\Omega_B h^2 = 0.0125$. We assumed reheating occurred sufficiently late to have a negligible effect on $\ell_{\text{c}}$, by no means clear. $\ell_{\text{c}}$ values for selected parameter ranges are shown in Fig. 2. We made use of signal-to-noise compression of the data (by factors of 3) to make the calculations of likelihood functions such as $L(\sigma_8, n_s, h | \text{fixed } t_0, \Omega_B h^2)$ more tractable (without loss of information or accuracy).

The $n_s$ constraints are quite good. If $\sigma_8$ is marginalized for the tilted ΛCDM sequence with $H_0 = 50$, with DMR only, the primordial index is $n_s = 1.02^{+0.23}_{-0.08}$ for scalar perturbations only (no gravity waves, $\nu_r = 0$) and $1.02^{+0.21}_{-0.03}$ (with gravity waves and $\nu_r = \nu_c$), rather encouraging for the nearly scale-invariant models preferred by inflation theory. For the 13-Gyr tilted ΛCDM sequence and $H_0 = 50$ (and $\Omega_B = 0$) we get $1.18^{+0.07}_{-0.04}$ for DMR + SK95 + SP94, $0.99^{+0.04}_{-0.03}$ when all of the data are used but SK95 and SP94 (in a bandpower approximation), and $1.05^{+0.10}_{-0.03}$ when all of the current data are used; for $H_0 = 70$ (and $\Omega_B = 0.66$), the numbers are quite similar, with all of the data giving $1.00^{+0.04}_{-0.04}$. And marginalizing over $H_0$ gives $1.00^{+0.04}_{-0.04} H_0$ for fixed age is not that well determined by the
CMB data alone. After marginalizing over all $n_s$, we get $H_0 < 75$ at 1σ, but effectively no constraint at 2σ.

For the DMR4 + SK95 + SP94 data, the 15-Gyr sequence with $\Omega_0 h^2$ fixed at 0.0125 fits better than the 13-Gyr sequence, which in turn fits better than the 11-Gyr sequence for $n_s < 1$. $\Omega_0$ is not that well determined: if we consider just the 13-Gyr sequence with variable $\Omega_0$ for $n_s = 1$, $\Omega_0 \sim 1.17$ is preferred, if we allow $n_s$ to exceed 1, then a low $\Omega_0$ is preferred (and $n_s = 1.15$); but with all of the CMB data, low $\Omega_0$ is preferred for $n_s \geq 1$, high for $n_s < 1$. The strong dependence of the position of the acoustic peaks on $\Omega_0$ means that the OCMD sequence is better restricted; e.g., for the 11-Gyr sequence $H_0 \sim 65$ and $\Omega_{\text{tot}} \sim 0.6$ is preferred, with either $n_s$ marginalized or constrained to be 1. For the 13- and 15-Gyr sequences, smaller $H_0$s but similar $\Omega_{\text{tot}}$s are preferred.

### Parameters from the CMB + LSS, Current State

The first lesson of Fig. 2 is that, in broad brush stroke, smaller-angle CMB data (e.g., SP94, SK95) are consistent with COBE-normalized $\xi_0$ values for these models. Although the CMB data alone may soon be powerful enough to offer strong selection, this will definitely not diminish the role that combining LSS and CMB data will play. The approach we used in ref. 6 to add LSS information to the CMB likelihood functions was to design prior probabilities for $\Gamma + \nu/2$ and $\sigma_{\Omega h^2}$, reflecting the current observations, but with flexible and generous non-Gaussian and asymmetric forms to ensure the priors can encompass possible systematic problems in the LSS data. For example, our choice for $\sigma_{\Omega h^2}$ was quite flat over the 0.5 to 0.7 range, but fell off below 0.5, although some authors actually prefer such low values.

Using only the 4-yr DMR data and these priors we get $n_s = 0.72^{+0.05}_{-0.03}$ with gravity waves (GW), $n_s = 0.53^{+0.05}_{-0.04}$ without for $H_0 = 50$ TCDM, with values slightly increased when SK95 and SP94 are added. (With $\Omega_{\text{adm}} = 0.2$ of the dark matter in massive neutrinos, we get $n_s = 0.93^{+0.04}_{-0.03}$ with GW, $n_s = 0.89^{+0.07}_{-0.05}$ without.) For $H_0 = 70$ and $\Omega_0 = 0.66$, we get $n_s = 0.86^{+0.06}_{-0.04}$. The preferred Hubble parameter for the $n_s = 1$ sequence is $70 \pm 5$ (0.66), so the product is narrow, but with a small confidence limit, slightly better than other groups find since we used full map statistics. The constraint is not as strong if the repetition directions are asymmetric, $0.7(2H_0^{-1}) = 6600 h^{-1}$ Mpc from DMR for flat equal-sided three-tori at the 95% confidence limit, slightly better than other groups find since we used full map statistics. The constraint is not as strong if more general topologies are considered—e.g., the large class of compact hyperbolic topologies (18).

### Parameters from the CMB, Future

Quite an industry has developed forecasting how well future balloon experiments (Maxima, Boomerang, ACE, Beast, Top Hat), interferometers (VSA, CBI, VCA) and especially the satellites MAP and Planck could do in measuring the radiation power spectrum and cosmological parameters if foreground contamination is ignored (13, 19–23), and that is what is done here in Table 1, using the techniques employed in ref. 22; similar results have been obtained in ref. 23. Earlier forecasts made in refs. 21 and 13 were quite influential in the case for MAP and Planck funding. The expected error bars on the power spectrum from MAP and Planck in Fig. 2 show even quite small differences in the theoretical $\xi_0$ values, and thus the parameters can be distinguished.

We assume models have Gaussian-distributed temperature anisotropies. Among the >17 parameters of the Cosmic Parameters section, we use a restricted 9-parameter space, five densities, $(\Omega_0, \Omega_{\text{adm}}, \Omega_k, \Omega_T, \Omega_B) h^2$, the Compton depth $\tau_c$, the scalar tilt, $n_s$, the total bandpower for the experiment $(\xi_0)_{\text{tot}}$ in place of $\sigma_8(\Omega_h^2)$, and the ratio of tensor to scalar quadrupole powers, $r_{\text{ts}} = \sigma_8(\Omega_T^2)/\sigma_8(\Omega_S^2)$, in place of $n_t = r_{\text{ts}}$ is a sensitive function of $n_t$ but it also depends on $n_s$, $n_l$, $\Omega_\Lambda$, etc. (5). In this space, recall that $h^2 = \sum_j (\Omega_j h^2)$ is a dependent quantity.

Except for the integrated Sachs–Wolfe effect at low $\ell$, the angular pattern of CMB anisotropies now is a direct map of the projected spatial pattern at redshift $\sim 100$, dependent upon the cosmological angle–distance relation, which is constant along a line relating $\Omega_0 h^2$ and $\Omega_T h^2$ for fixed $\Omega_B h^2$. This defines a near-degeneracy between $\Omega_0$ and $\Omega_\Lambda$ broken only at low $\ell$, where the large cosmic variance precludes accurate determination of both parameters simultaneously, and other cosmological observables such as type I supernovae at high redshift are needed to break this degeneracy.

Error forecasts depend upon the parameters defining the correct underlying theory. In Table 1 and the lower panels of Fig. 2, untold SCDM was chosen as the target model, but the values shown are indicative. See ref. 22 for more cases. The table shows how the forecasts work for DMR, using the average noise in the 53 + 90 + 31 GHz map for $\sigma_\text{pix}$, 60 μK per 2.6° pixel. The channel-weighted noise power (first row) is $\xi_0 = (\sigma_\text{pix} \theta_{\text{pix}})^2$. In a 2-parameter tilted sequence, the forecast is for a bandpower with 7% accuracy and $n_s$ to ±0.20 accuracy, in agreement with the full analysis of the Parameters from the CMB section above. The number of orthogonal
combinations of parameters that could be determined to better than $\pm 0.1$ accuracy was determined by using the full 9-parameter sequences. The north central pole (NCP) numbers are SK95-like, for a 9° radius patch. In combination with DMR, the forecast is that two combinations can be obtained to $\pm 0.1$, in agreement with the Parameters from the CMB section.

The third column gives errors forecasted for an LDB experiment [using conservative numbers for the bolometer-based TopHat experiment (home page: http://cobi.gsfc.nasa.gov/msam-tophat.html) that took account of excess noise associated with foreground removal]. Other balloon bolometer experiments such as Boomerang (home page: http://astro.caltech.edu/med/boom/boom.html) and MAXIMA (Millimeter Anisotropy Experiment Imaging Array (home page: http://physics7.berkeley.edu/group/cmb/gen.html) should be able to do as well. High Electron Mobility Transistor (HEMT)-based LDB experiments, such as BEAST (home page: http://www.deepspace.ucsb.edu/research/Sphome. html) using 40-GHz HEMTs, might also achieve similar accuracy. The $\ell$-cuts shown for NCP and LDB reflect their limited sky coverage; adding DMR to extend the baseline diminishes the forecasted errors.

We adopt improved specifications especially in beam size, $\ell_s$, for MAP (http://map.gsfc.nasa.gov) and Planck (13) over the original proposal values; these are likely to evolve for Planck. Of the five HEMT channels for MAP, we assume the three highest-frequency channels, at 40, 60, and 90 GHz, will be dominated by the primary cosmological signal (with 30- and 22-GHz channels partly contaminated by bremsstrahlung and synchrotron emission). MAP also assumes 2 yr of observing. For Planck, 14 months of observing and current (proposal-modified) values are used. The HEMT-based LFI specifications are significantly improved; the 100-, 65-, and 44-GHz channels, but not the 30-GHz channel, were used. For the bolometer-based HFI, 100, 150, 220, and 350 GHz were used. Dust contamination will certainly affect the 550- and 850-GHz channels.

These idealized error forecasts do not take into account the cost of separating the many components expected in the data, in particular Galactic and extragalactic foregrounds, but there is currently optimism that the Galactic foregrounds at least may not be a severe problem—e.g., see MAP and Planck home pages. There is more uncertainty about the extragalactic foregrounds. We forecast that not far away there will be a day when phenomenological theorists will have optimally analyzed LDBs/VSA/CBI/VCA/MAP/Planck and delivered the power spectrum and cosmic parameters to wonderful precision. What will it mean? It may not be clear. Take inflation as an example. There will be parameters to wonderful precision. What will it mean? It may not be clear. Take inflation as an example. There will be
data will be called upon to guide us to a new theory of how fluctuations are generated.