Hubble's Law and the expanding universe

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In one of the most famous classic papers in the annals of science, Edwin Hubble's 1929 PNAS article on the observed relation between distance and recession velocity of galaxies—the Hubble Law—unveiled the expanding universe and forever changed our understanding of the cosmos. It inaugurated the field of observational cosmology that has uncovered an amazingly vast universe that has been expanding and evolving for 14 billion years and contains dark matter, dark energy, and billions of galaxies.

It is difficult to imagine that only 90 years ago, we did not know about the existence of most of the universe around us. From today's perspective, the reality of a very large, old, expanding universe, filled with billions of galaxies that are receding from each other as the cosmic space expands from an initial "Big Bang" billions of years ago seems so obvious that we expect it must have been known for centuries. Not so. It was Edwin Hubble's seminal 1929 PNAS paper, "A relation between distance and radial velocity among extra-galactic nebulae" (1), that led to a turning point in our understanding of the universe. In his short paper, Hubble presented the observational evidence for one of science's greatest discoveries—the expanding universe. Hubble showed that galaxies are receding away from us with a velocity that is proportional to their distance from us: more distant galaxies recede faster than nearby galaxies. Hubble's classic graph of the observed velocity vs. distance for nearby galaxies is presented in Fig. 1; this graph has become a scientific landmark that is regularly reproduced in astronomy textbooks. The graph reveals a linear relation between galaxy velocity (v) and its distance (d)

$$v = \mathrm{H_o} \times d.$$

This relation is the well-known Hubble Law (and its graphic representation is the Hubble Diagram). It indicates a constant expansion of the cosmos where, like in an expanding raisin cake that swells in size, galaxies, like the raisins, recede from each other at a constant speed per unit distance; thus, more distant objects move faster than nearby ones. The slope of the relation, $\mathrm{H_o}$, is the Hubble Constant; it represents the constant rate of cosmic expansion caused by the stretching of space-time itself. Although the expansion rate is constant in all directions at any given time, this rate changes with time throughout the life of the universe. When expressed as a function of cosmic time, $H(t)$, it is known as the Hubble Parameter. The expansion rate at the present time, $\mathrm{H_o}$, is about 70 km/s/Mpc (where 1 Mpc = 10^6 parsec = 3.26×10^6 light-y). The inverse of the Hubble Constant is the Hubble Time, $t_H = d/v = 1/\mathrm{H_o}$; it reflects the time since a linear cosmic expansion has begun (extrapolating a linear Hubble Law back to time $t = 0$); it is thus related to the age of the Universe from the Big-Bang to today. For the above value of $\mathrm{H_o}$, $t_H = 1/\mathrm{H_o} \sim 14$ billion years.

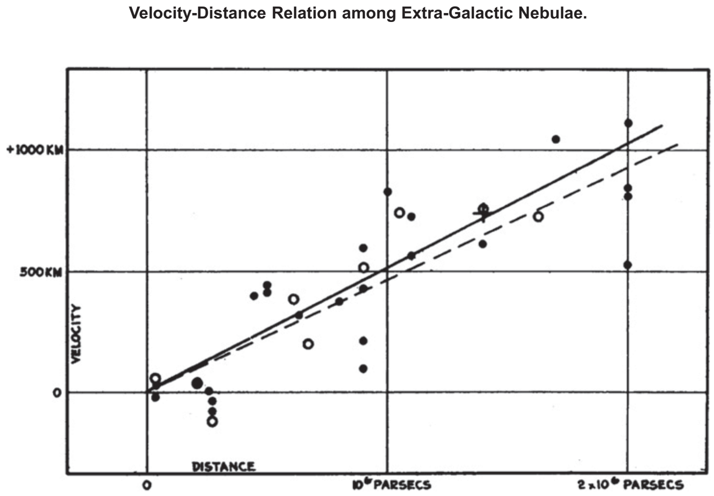
Velocity-Distance Relation among Extra-Galactic Nebulae.

Fig. 1. Velocity–distance relation among extragalactic nebulae (1). "Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually" (1). (Note: Velocity units should be in kilometers per second.)

Hubble's remarkable observational relation was obtained using 24 nearby galaxies for which both measured velocities and distances were available. Most of the velocities were from the pioneering spectroscopic Doppler-shift observations by the famous astronomer Vesto Melvin Slipher (although no reference is given in Hubble's paper). The distances to these galaxies (an inaccurate determination in those days) had been measured by Hubble—with much greater accuracy than previously possible—from the apparent brightness of their stars and, for the four most distant galaxies in the sample, each located in the Virgo cluster (with recession velocity of ∼1,000 km/s), from their galactic brightness. This method uses the stars (or galaxies) as "standard candles"; it compares their known intrinsic luminosity (known from similar well-calibrated nearby objects) with their observed apparent brightness to yield the distance to each object. The farther away the object, the dimmer it appears. Hubble distance determinations were sufficiently good to sort out the nearer galaxies from the farther ones well enough to be able to detect this astonishing linear relation. In addition to plotting all of the individual 24 galaxies in the graph, Hubble also binned them into nine groups

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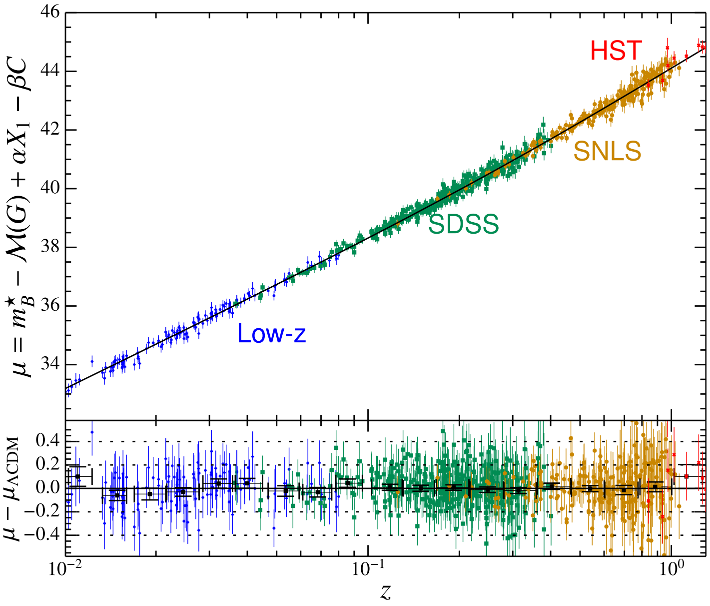

Fig. 2. The Hubble diagram of galaxies [distance vs. redshift (velocity)] from a large combined SNIa distance-indicator sample [reproduced with permission from ref. 14 (©) ESO]. A recent Hubble diagram of a large combined sample of galaxies using SNIa as standard candles for distance measurement. The graph presents distance (as distance modulus; proportional to log of distance) vs. redshift z (Doppler shift, proportional to velocity for small redshift: $v/c \sim z$). The different SNIa samples are denoted by different colors and are listed by name [low-z sample; Sloan SDSS sample; SN legacy survey, SNLS; and Hubble Space Telescope SNIa, HST; for detail and references, see Betoule et al.(14)]. The black line (that fits the data so well) represents the $d(z)$ relation expected for the current cosmology (a flat universe with mass density 30% and cosmological constant 70%) and a Hubble Constant of $H_o = 70$ km/s/Mpc. The slight deviation in shape at large distances is the evidence for acceleration. Hubble's 1929 graph (Fig. 1, plotted with reverse axes, v vs. d) will fit in a tiny spot near/below the origin of this diagram.

(open circles in Fig. 1) based on their proximity in direction and distance; this was a good way to minimize the large scatter. Hubble used an additional 22 galaxies for which velocities were available (from Slipher measurements), but no individually estimated distances. For these, Hubble used the mean velocity of the 22 galaxies and estimated their mean distance from their mean observed brightness; this mean value, shown by the cross in Fig. 1, is nicely consistent with the rest of the data. Although there were hints of a possible relation between velocity and distance in previous work [Lemaitre (2) and Robertson (3), who laid out the theoretical foundation; see refs. 4–6 and references therein], Hubble's paper was the definitive work that convinced the scientific community of the existence of this observed relation and thus of an expanding universe. Hubble's work rested on the accumulated body of scientific data at the time, from the critically important velocities determined by Slipher to numerous attempts at distance measurements using a variety of standard candles and the greatly improved calibration offered by the observed period-luminosity relation of Cepheid stars [discovered by Henrietta Swan Leavitt in 1912 (7); Hubble used these to calibrate his distances]. Hubble was fortunate to use the most powerful telescope in the world at that time, the 100-in. Hooker telescope at Mount Wilson, which enabled him to identify individual stars in galaxies and thus reveal their distances. He was able to select and measure a consistent set of the best-determined distances for a select sample of galaxies and, despite a large systematic calibration error, had succeeded in unveiling convincingly this remarkable relation. Evaluating his data, Hubble concludes: "For such scanty material, so poorly distributed, the results are fairly definite."

Hubble's diagram of velocity vs. distance (Fig. 1) appears plain and simple. It shows a clear trend of increasing velocity with distance, despite a large scatter. What makes this plain-looking graph astonishing is the far-reaching implications of the observed trend: that we live in a large, dynamically evolving universe that is expanding in all directions. It is not the static universe that Einstein and others assumed in 1917. In fact, Einstein introduced a cosmological constant into his equations to keep the universe static, as it was then believed to be. Hubble results suggested otherwise; they suggested that the universe has been expanding for billions of years, from an early beginning to the present (and future) time. In fact, in 1922, Alexander Friedmann (8), the famed Russian cosmologist, derived the first solutions to Einstein's equations for an expanding universe (Friedmann Equations). In 1927, Georges Lemaitre (2) derived a nonstatic solution to Einstein's equations and coupled it to the then available observations to suggest a possible but inconclusive relation between velocity and distance, which would be expected for the nonstatic universe (see also refs. 5 and 6). Hubble's 1929 diagram proved that they were right. Unfortunately, Friedmann died young in 1925 and did not live to witness Hubble's results. Hubble himself, however, did not connect his results to these expanding universe solutions. The Friedmann (1922) and Lemaitre (1927) papers were not yet well known or broadly discussed in 1929. Instead, Hubble refers (in the last brief paragraph of his paper) to the possibility that his observed linear relation may relate to the then discussed—now long abandoned—de Sitter static model where the Doppler shifts arise mainly from slowing down of time at large distances rather than from an expanding universe. Shortly after Hubble's discovery, cosmologists, including Einstein, became aware of Lemaitre's (1927) paper; Hubble's observed relation provided the turning point for the expanding universe.

Over the decades since Hubble's discovery, numerous observations of the Hubble Law have been carried out to much greater distances and with much higher precision using a variety of modern standard candles, including Supernovae type Ia (SNIa) (9–14), and a greatly improved stellar/Cepheid distance indicator to the Virgo cluster (15), carried out with the Hubble Space Telescope, aptly named in honor of Hubble. Fig. 2 presents a recent compilation of the observed Hubble Diagram using SNIa as distance indicators (14) to galaxies at distances hundreds times greater than observed by Hubble; Hubble's original diagram fits into a tiny spot near the origin of this graph (corresponding to our immediate cosmic neighborhood). The beautiful linear relation observed to these distances is a remarkable triumph to Hubble's results. Hubble's values for his distances in 1929 were, however, wrong, by a large factor of ~7! This was mainly due to a wrong zero-point calibration of the standard candles used at the time. All distances were thus too small by a factor of 7, and the expansion rate H_o too large by the same factor. Hubble's value for H_o was 500 km/s/Mpc, whereas today's well-calibrated value is $H_o = 70$ (±~2) km/s/Mpc (15–20). However, despite this large difference and its major implications for the expansion rate and age of the universe, Hubble's fundamental discovery of the expanding universe is not affected; the underlying linear $v \sim d$ relation remains unchanged.

Hubble's discovery inaugurated the field of observational cosmology and opened up a magnificent vast universe to be explored. Observations of the large-scale structure of the universe, clusters of galaxies, SNIa (used as standard candles to explore the evolution of the Hubble Law to large distances), and the cosmic microwave background radiation have revealed an amazing universe: a universe that is flat (zero spatial curvature) and contains 5% baryons (stars, gas), 25% exotic nonbaryonic dark matter, and 70% dark energy that causes the current expansion rate of the universe to accelerate. The astonishing result of cosmic acceleration was discovered in 1998 (9–12) using a distance indicator method similar to that used by Hubble, but using the very bright SNIa as accurate standard candles to measure the evolution of the expansion rate (the Hubble Diagram) at large distances (early cosmic times). The surprising result showed that the expansion rate has been speeding up in the last ~6 billion years. The nature of the mysterious dark energy that causes this acceleration is not yet known. Is it the cosmological constant, representing the energy density of the vacuum, or is it something else? This is one of the most fundamental questions in cosmology today. The quest to answer this question is currently underway. The Hubble Space Telescope, among others, is currently observing the Hubble Law to greater distances (using SNIa) to trace the precise evolution of the expanding universe. The linear relation observed at small distances starts deviating from linearity at large distances due to the specific cosmology of the universe, including the cosmic mass density (whose gravity decelerates the expansion) and the amount and nature of the dark energy (which accelerates the expansion). The small deviation from linearity, seen at large distances in Fig. 2, is indeed the observational evidence for the accelerating universe (9–14).

Hubble's discovery has opened up remarkable research in numerous other areas, such as the large-scale structure of the universe, the evolution and properties of galaxies and quasars, and the evolution of the universe as a whole. Using Hubble's Law enables the crucial determination of Hubble distances to galaxies and quasars (Hubble distances are those derived from Hubble's Law using the observed velocity of the object; these distances represent the true cosmic distance plus a small peculiar motion component). In turn, these distances enable the determination of the 3D location and distribution of millions of galaxies and quasars from their observed spectroscopic Doppler shift (redshift) velocities obtained from large redshift surveys of galaxies [such as the Sloan digital sky survey (21–23) and others]. Such surveys reveal a remarkable interconnected large-scale network of galaxies, clusters of galaxies, filaments, and voids (22, 23). Hubble distances are routinely used in astronomy to measure distances to galaxies from their (relatively) easily measured spectroscopic redshifts and even from their photometric redshifts (obtained from multiband imaging surveys). The evolution of galaxies and quasars from the young ~1-billion-year-old universe to today is enabled by measuring these distances. An accurate determination of the age of the universe has also been enabled from the precisely measured Hubble Constant, when combined with the cosmological parameters above, to be 13.8 ± 0.1 billion years (15–20). This age is nicely consistent with the age of the oldest stars. These are but a few examples of the all-encompassing applications of Hubble's discovery.

In his paper, Hubble concludes "The results establish a roughly linear relation between velocities and distances among nebulae for which velocities have been previously published, and the relation appears to dominate the distribution of velocities..... New data to be expected in the near future may modify the significance of the present investigation or, if confirmatory, will lead to a solution many times the weight." Indeed, Hubble and his colleague Milton Humason at the Mount Wilson Observatory expanded their investigation by measuring additional distances and velocities of galaxies in their follow-up work, extending to 20 times greater distances (24), and confirming the original results. Today, 85 years later, the Hubble Law is a given, measured with high precision to vastly larger cosmic scales than Hubble's first glimpse into our immediate cosmic neighborhood.

Hubble's discovery portrays an amazing scientific tale: his distances had a large systematic error by a factor of seven, his velocities came mostly from those measured by Slipher, he used a small sample of merely 24 nearby galaxies, and his interpretation of the results in terms of the then de Sitter kinematic model was wrong; yet, his main result of the velocity vs. distance relation changed the course of science by revealing the expanding universe. The Hubble Law, the Hubble Constant, the Hubble Time, and the more recent Hubble Space Telescope are but tributes to this awe-inspiring discovery.

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