Math tools send legislators back to the drawing board

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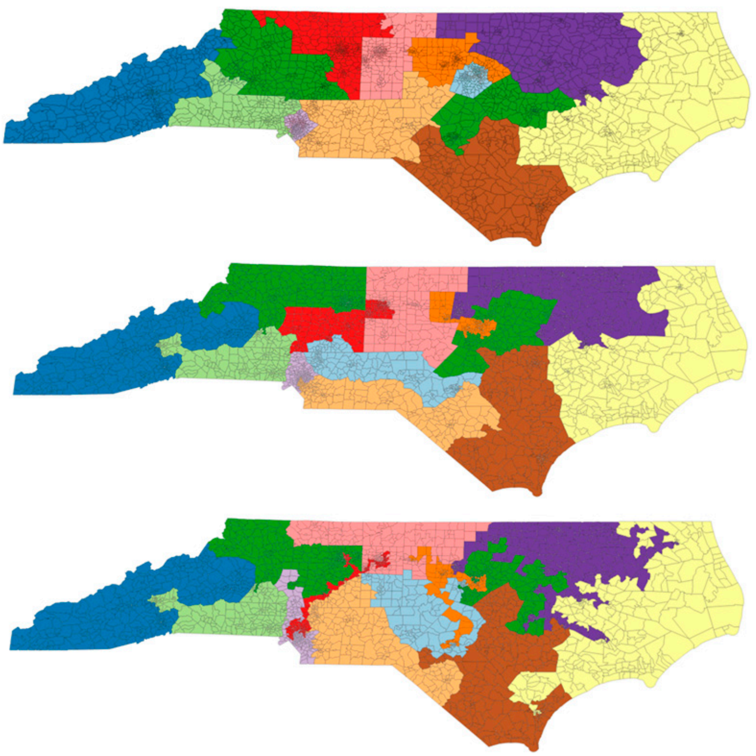
UPDATE: On June 18, 2018, after this article went to press, the Supreme Court of the United States ruled on two high-profile cases related to partisan gerrymandering. In effect, the rulings sidestepped the issue of when partisan gerrymandering is unconstitutional. Both cases—one concerning voting districts in Wisconsin, the other in Maryland—were sent back to lower courts. On June 25, the SCOTUS ruled on two other cases—in Texas and North Carolina—that will mostly let stand the use of purportedly gerrymandered maps.

On January 9, 2018, a trio of federal judges made history when they ruled that the boundaries of North Carolina's congressional voting districts gave an unfair advantage to Republican candidates. It was the first case in the nation in which a federal court had declared congressional maps unconstitutional because of intentional bias in favor of one party. The case was all the more remarkable because the court decision relied in part on mathematical tools that can probe the practice of gerrymandering—the drawing of voting districts to give an intentional advantage to one party.

It would be one of many gerrymandering cases. In February, the US Supreme Court rejected a request by Republican lawmakers to stay a lower court's decision—which drew on mathematical evidence—that Pennsylvania's congressional maps were partisan-gerrymandered and required redrawing. The Pennsylvania Supreme Court released its new map later the same month. This summer, the US Supreme Court is also expected to decide whether Wisconsin legislators must return to the districting drawing board to create new state legislative districts; the transcript of oral arguments made in the case include tens of pages of math exposition. In March, US Supreme Court justices heard oral arguments about districts in Maryland. Increasingly, social scientists, mathematicians, and statisticians have been drawn into debates over whether existing districts accurately represent the will of the voters.

Past US Supreme Court decisions have addressed cases of racial gerrymandering, which suppresses the political power of racial and ethnic populations. Showing partisan gerrymandering has been trickier, partly because the judges had not chosen a way to recognize when partisan unfairness in mapmaking becomes unconstitutional. In 2004, US Supreme Court justice Anthony Kennedy noted that no "workable standard" existed that judges could apply fairly and consistently to partisan-gerrymandering cases. "That no such standard has emerged in this case should not be taken to prove that none will emerge in the future," he wrote (1).



Changing the boundaries of voting districts can have a significant impact on an election outcome. North Carolina's 2012 election districting map (bottom) led to Republicans winning nine seats and Democrats winning four. North Carolina's redistricting for the 2016 election (middle map) led to Republicans winning 10 seats and Democrats winning three seats. But an arguably fairer redistricting plan (top), produced by retired judges, would have led to Republicans winning seven seats and Democrats the other six in 2012; it would have been nine seats for Republicans and four for Democrats in 2016. Image courtesy of Jonathan Mattingly (Duke University, Durham, NC).

If Kennedy's statement was a challenge, mathematicians and social scientists are now offering answers. The mathematical tools behind recent cases combine rigor with judicial applicability. Such tools include indices that measure district-divvying practices meant to produce unfair elections and mathematical methods for determining whether a district's squiggly boundary tips the scale. Algorithms can compare existing maps against large sets of other possible maps to identify cases of suspicious deviation. These tools will not likely produce a universally accepted yes-or-no gerrymandering test for every district. But they are bringing a modicum of mathematical precision into the political arena.

Partisan Symmetry

Claims about partisan gerrymandering in the United States are as old as the nation itself. In Virginia's first congressional election in 1789, critics accused founding father Patrick Henry of drawing district boundaries to favor his preferred candidate, James Monroe, over Monroe's opponent, James Madison. The term gerrymander is a portmanteau that emerged after critics accused Massachusetts Governor Elbridge Gerry, in 1812, of drawing biased voting districts. Those districts included one that curled around Essex county like a salamander (Gerry + mander = gerrymander).

In 1987, political scientist Gary King, who runs Harvard University's Institute for Quantitative Social Science, proposed a standard called "partisan symmetry." (2) The idea behind it was straightforward. For an individual voting district, whoever gets a plurality of the votes wins the seat. Extended to an entire state, this practice implies a type of symmetry: The map is fair only if each political party would win the same number of seats with the same share of votes. Since then, partisan symmetry, or something similar, has been a guiding principle for social scientists trying to measure fairness.

"Everybody thinks that's fair, it's so obvious that it's fair," says King, referring to notions of symmetry. But gerrymandered redistricting plans have ways to violate that symmetry.They typically take one of two forms. In "packed" districts, borders are drawn to concentrate voters who favor one party, so they have less influence in other districts. "Cracked" districts have boundaries drawn to distribute voters of one party across many districts to weaken their influence.

> "Anything is better than having one party do whatever they want."
>
> —Wesley Pegden

Gerrymandering can lead to decidedly asymmetric scenarios in which one party gets the majority of votes while the other gets the most seats. In 2012, for example, Republicans won control of the House of Representatives 234 to 201, but Democrats received 1.4 million more votes.

Wasted Voices, Wasted Votes

Over the last few decades, a variety of methods and metrics have been proposed to try to identify gerrymandering when it occurs. One of the most commonly cited, the "Polsby-Popper test," divides a district's area by the area of a circle with a circumference equal to the perimeter of the district (3). Fairer districts would have a ratio closer to one—because they'd be closer to circular—and unfair districts would end with a score closer to zero. The Polsby-Popper test follows from the idea that if districts with weirdly shaped boundaries are disallowed, then it will be more difficult for redistricting plans to violate partisan symmetry.

Districts with wandering, squiggly boundaries, which may have been drawn to include or exclude certain populations, end up with scores closer to zero. Oddly shaped districts, such as Maryland's Sixth, which follows the meandering Potomac River, also receive lower scores. Justin Solomon, a computer scientist at Massachusetts Institute of Technology (MIT), notes this measure is one "everybody likes to hate on," calling it "famously unstable." That's because it doesn't account for natural boundaries that might impose limits on the district shapes—such as the Potomac, a mountain range, or the ocean—and it is extremely sensitive to the irregular shapes of coastlines, rivers, and other geographic features.

Another gerrymandering test—one that's involved in the Wisconsin case—is called the "efficiency gap," and it's a measure of partisan symmetry. Imagine 10 people vote in an election where 8 vote for the Republican and 2 for the Democrat. Those two Democratic votes are considered wasted because they were cast in an election that the Democrats didn't win. And because the Republicans needed only six votes—a simple majority—to win, two Republican votes were also wasted.

To calculate the efficiency gap, sum up the wasted votes for each party from all districts. The difference between the number of wasted votes from one party and the other, divided by the total number of votes, gives the efficiency gap. Efficiency gaps greater than zero show that one party has more wasted votes than the other, which can indicate cases of packing and cracking. The efficiency gap was introduced in 2014 by Eric McGhee, a political scientist at the Public Policy Institute of California in San Francisco, and Nicholas Stephanopoulos, a law professor at the University of Chicago who represents the plaintiffs in the Wisconsin case (4).

Other mathematicians have developed tools that go beyond the efficiency gap to probe the mathematical underpinnings of the maps themselves. In 2017, mathematicians at Carnegie Mellon University in Pittsburgh and the University of Pittsburgh published a theorem that can be applied to demonstrate gerrymandering (5). The theorem involves making small and random tweaks to the entire boundary of an existing district and rerunning the election; if those small tweaks consistently make the district less partisan, then the district is gerrymandered, according to the test. Wesley Pegden, one of the mathematicians who worked on the theorem, testified before the Pennsylvania Supreme Court in that state's recent case.

Pegden has also devised a simpler solution that gets parties to amicably agree instead of pointing fingers with mathematical formulas. In November

2017, he and computer scientist Ariel Procaccia, also at Carnegie Mellon University, proposed a scheme in which one political party gets to draw the districts, and the other party gets to choose one first. Then the second party redraws the district map, and the first party gets to choose. The two parties continue until all the districts have been drawn and chosen. Together, the pair—along with student Dingli Yu, from China's Tsinghua University—mathematically proved that neither side gets an advantage under their system.

The I-cut-you-choose strategy is more straightforward than other metrics, but Pegden says it might not work in the real world because it's intended to address only partisan gerrymandering. It doesn't address deal-making between parties that has the aim of creating antidemocratic districting, he says. At the same time, it does offer a simple way for both parties to protect their interests. "Anything is better than having one party do whatever they want," he says.

Careful Calculations

Solomon at MIT sees devising mathematical tools as the first of three necessary steps toward making districts fairer. The second is to figure out which methods are the most stable and when they're likely to be useful, given the data available. The third step is to share those tools with the lawyers, judges, and policymakers who ultimately draw the maps. "Probably the most difficult part that's often neglected is, how the heck do you convince people to use [the mathematical tools] in practice?" says Solomon.

In the fall of 2017, Chief Justice John Roberts expressed discomfort in using statistical tools to justify or overturn election outcomes, calling them "sociological gobbledygook." Justice Elena Kagan, on the other hand, noted at one point that all the algorithms and other tools they'd heard about were pointing in the same direction, toward gerrymandering.

Courts don't need all the mathematical details, says mathematician and geneticist Eric S. Lander, head of MIT's Broad Institute in Cambridge, MA. They need to know that there are mathematical tools that answer Justice Kennedy's call for a workable standard to recognize excessive gerrymanders. In an amicus brief filed in September 2017 (6), Lander argued that mathematical tools can identify whether a state has chosen a redistricting plan that is an extreme outlier, which means its election outcomes differ significantly from the vast majority of possible plans.

Lander says he wanted the brief to bolster the court's confidence in these approaches by pointing to other situations that involve sampling from a distribution of outcomes. "Like figuring out when a bomb will explode and won't explode; whether a hurricane won't hit here," he says. "Sampling from a distribution is used for pretty damn important things." Even if the justices didn't follow the math, they should feel confident in the application. Lander pointed to the work of Jonathan Mattingly at Duke University in Durham, NC, and others for their work showing that it's possible to finger the outliers.

And such tools have shown promise and real-world utility. Mattingly led the development of a tool that he and his collaborators used to create an ensemble of 24,000 possible North Carolina district maps, all based on the legal requirements that districts had to be connected and include the same number of constituents. When he compared results from the 2012 and 2016 elections—using those redistricting maps—to results predicted by the ensemble of possible maps, he found that Democrats would have picked up more seats in the vast majority of outcomes (7). The algorithm showed what Mattingly has come to call the "signature of gerrymandering"—that the real-world election outcomes were significantly different from those of other possible arrangements. The existing districts were what Lander pointed to as "extreme outliers." In the fall of 2017, in the North Carolina case, Mattingly explained his methods and findings to the federal judges, who threw out North Carolina's maps. That decision was stayed by the US Supreme Court in January.

Efforts continue to try to link mathematics and mathematicians to laws grounded in fairness. The Metric Geometry and Gerrymandering Group, led by Tufts University mathematician Moon Duchin, brings together disparate areas of math—network theory, geometry, complexity—to better understand gerrymandering, both racial and partisan. Over the last year, Duchin has run a series of workshops and a summer camp. "We want to think about using the space of all possible maths as a tool to understand what's fair and unfair," she says. "Mathematicians haven't been showing up for the key court cases, and we've got something to say about some of the underlying ideas."

If the gerrymandering cases reveal new ways that science and the law can come together, Lander says, then they also highlight the friction between the two fields. The courts have turned to the math to better understand fairness, but the rigorous math alone can't dictate what's fair in the messy human world. "The tension," Lander says, "expresses something interesting about good law versus good science."

1 Vieth v. Jubelirer, 241 F2d 478, affirmed, 541 U.S. 267 (2004).

2 King G, Browning RX (1987) Democratic representation and partisan bias in congressional elections. *Am Polit Sci Rev* 81:1252–1273.

3 Polsby DD, Popper RD (1991) The third criterion: Compactness as a procedural safeguard against partisan gerrymandering. *Yale Law Policy Rev* 9:301–353.

4 Stephanopoulos N, McGhee E (2015) Partisan gerrymandering and the efficiency gap. *Univ Chic Law Rev* 82:831–900.

5 Chikina M, Frieze A, Pegden W (2017) Assessing significance in a Markov chain without mixing. *Proc Natl Acad Sci USA* 114:2860–2864.

6 Brief for Eric Lander as Amicus Curiae Supporting Appellees (2017) Gill v. Whitford. no. 16-1161. Available at www.campaignlegalcenter.org/case/gill-v-whitford. Accessed May 9, 2018.

7 Mattingly JC, Vaughn C (2014) Redistricting and the will of the people. arxiv.org/abs/1410.8796.