A SET OF INDEPENDENT POSTULATES FOR CYCLIC ORDER

By Edward V. Huntington

DEPARTMENT OF MATHEMATICS, HARVARD UNIVERSITY

Received by the Academy, October 16, 1916

There are four types of order which are important in geometry and other branches of mathematics: (1a) linear order with a definite sense along the line (theory of *serial order*); (1b) linear order without distinction of sense (theory of *betweenness*); (2a) circular order with a definite sense around the circle (theory of *cyclic order*); and (2b) circular order without distinction of sense (theory of *separation of pairs of points*).

The present note is concerned with type (2a): cyclic order.

Let us consider a *class* K of elements A, B, C, . . . , and a *triadic relation* $R(ABC)$. The class K may be said to be *cyclically ordered* by the relation R if the following postulates are satisfied:

I. *If A, B, C, are distinct, then ABC implies BCA.*

II. *If A, B, C are distinct, then at least one of the orders ABC, BCA, CAB, CBA, ACB, BAC is true.*

III. *If A, B, C are distinct, then ABC and ACB cannot both be true.*

IV. *If ABC is true, then A, B, and C are distinct.*

V. *If A, B, X, Y are distinct, and XAB and AYB, then XAY.*

From these postulates it follows that any three distinct elements A, B, C, in the order ABC, divide the class into three sections, such that (1) the three sections, together with the dividing elements, exhaust the class; (2) no element belongs to more than one section; (3) if X, Y, Z are elements taken one from each section, so that AXB and BYC and CZA, then XYZ. The details of the proof will be given in a later paper.

It will also be shown that the postulates are independent of each other.

From I, II, and V we can prove, as a theorem,

VI. *If A, B, X, Y are distinct, and AXB and AYB, then either AXY or YXB.*

But if we should replace postulate I by the following postulate:

I′. *If A, B, C are distinct, then ABC implies CBA,*

then VI would become an independent postulate, and the set of six postulates, I′, II, III, IV, V, VI, would then define not cyclic order, but betweenness, and would be, in fact, identical with one of the sets of independent postulates for betweenness obtained in a forthcoming paper by E. V. Huntington and J. R. Kline. The transition from the theory of cyclic order to the theory of betweenness may thus be made by merely interchanging two letters in the first postulate; postulates II–VI are true in both theories.

A NEW METHOD OF STUDYING IDEATIONAL AND ALLIED FORMS OF BEHAVIOR IN MAN AND OTHER ANIMALS[1]

By Robert M. Yerkes

PSYCHOLOGICAL LABORATORY, HARVARD UNIVERSITY

Received by the Academy, October 20, 1916

Despite widespread interest in the evolution of reasoning, the comparative study of ideational behavior has been neglected. Only a few methods of research have been devised, and these have seen scant service.

Thorndike[2] is responsible for the problem or puzzle-box method (used by him in the study of cats, dogs, and monkeys); Hamilton,[3] for the method of quadruple choices (by which he has studied cats, dogs, horses, monkeys, rats, gophers, and men); Hunter,[4] for the method of delayed reaction (applied by him to rats, raccoons, dogs, and children).

I have perfected and applied a new method—that of multiple choices —for the detection of reactive tendencies and the study of their rôle in the attempted solution of certain types of problem. The method involves the presentation to the subject of a problem or series of problems whose rapid and complete solution depends upon ideational processes.

The apparatus consists of twelve, or, in some forms, nine identical reaction-mechanisms, of which any number may be used for a given experimental observation. In the type of apparatus originally used for human subjects, these mechanisms are simple keys; in that which has been used for lower animals, they are boxes arranged side by side, each with an entrance door at one end and an exit door at the other, which may be raised or lowered at need by the experimenter through the use of a system of weighted cords. Under the exit door of each box is a receptacle in which some form of reward for correct reaction may be concealed until the door of the appropriate box is raised by the experimenter.

It is the task of the subject to select from any group of these boxes whose entrance doors are raised that one in which the reward (food, for example) is to be presented. The experimenter in advance defines the