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THE THEORY OF MICRO-METEORITES.* PART I. IN AN ISOTHERMAL ATMOSPHERE

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Introduction.—In 1946, during the great Giacobinid meteor shower, H. E. Landsberg[1] collected several small magnetic particles that apparently were associated with the shower. Since some of these particles, a few microns in length, were extremely angular in shape (wedge-shaped and opaque), it seemed unlikely that they could have been the end-products of vaporizing meteors. Landsberg concluded that they must have been stopped by the atmosphere *without being heated above their melting-points.* As a result of his suggestion I have developed the present theory to investigate the process whereby temperature radiation can dissipate the energy gained by encounters with atmospheric molecules sufficiently rapidly to permit finite meteoric particles to be stopped without melting. Some basic concepts of this theory have been discussed by E. Öpik[2] and an application made in the case of an isothermal atmosphere.

The term *micro-meteorite* appears to be an appropriate designation for one of these small particles.

In every sense the micro-meteorites represent the lower extreme to the ascending sequence embracing *meteor, fireball* and *meteoritic crater formation.* Hence, the theory is a limited meteor theory, partially applicable to the unobservable beginning of a meteor.

We may now assume and later (Part II) prove that interaction between the air molecules striking and those leaving the micro-meteorite may be neglected. The molecular mean free paths, even after correction for the relatively slow velocity of air molecules thermally emitted, are greater than the linear dimensions of the micro-meteorite.

Let us suppose that the micro-meteorite presents a certain surface area, A, to the atmosphere, which it encounters with velocity, V. This surface may be an average frontal area in case the body is rotating, or a fixed area in case the body does not rotate. If $A(\phi, \theta)$, a function of the ordinary spherical coordinates, represents the actual cross-sectional profile of the micro-meteorite as seen from direction ϕ and θ, then the average frontal area, A_1, is given by

$$A_1 = \frac{1}{4\pi} \int_{\theta = 0}^{\pi} \int_{\phi = 0}^{2\pi} A(\phi, \theta) \sin \theta \, d\phi \, d\theta. \tag{1}$$

This average area, A_1, may or may not equal the frontal area A. The thermal radiating area, B, however, will be specific for a particle of a given shape; it is given by

$$B = 4A_1. \tag{2}$$

Let us assume that the temperature of the micro-meteorite is at all times uniform over the area B. This assumption is equivalent to an assumption that the heat conductivity is infinite or that the heat capacity is zero. We shall first develop the theory on the assumption that the heat capacity is negligible and later investigate the nature of the error made.

The air molecules impinge on the forward surface with relative velocity, V, because, by definition, the mean free path of the outgoing molecules relatively to the moving body is larger than its linear dimensions. To evaluate the energy transfer to the surface of the body we may make use of the concept of the accommodation coefficient, α, which is defined in terms of the kinetic energy of the air molecules, referred to the coordinate system of the moving body. The accommodation coefficient is, then, the actual loss of kinetic energy by the air molecules, as a result of the encounter, divided by the loss if all of them were momentarily to adhere to the surface and be re-emitted at the thermal velocity corresponding to the surface temperature. Since the thermal energies, both original and at the surface temperature, are relatively small compared to the kinetic energy at velocity, V, these energies may be neglected with an error less than 1% (minimum V = 11.2 km./sec.). Hence α represents the fraction of the molecular energy at velocity, V, that is transmitted to the micro-meteorite.

Since the air molecules will encounter the body with relative energies of the order of 8 to 800 electron volts, while the work function of the surface will be only a few volts, the molecules will certainly penetrate the surface for several molecular layers except at the lowest velocities. We must conclude that few of them will leave with high velocities; the losses by dissociation, excitation and ionization can be only a few volts. Hence α must be nearly unity at most velocities.

In air of density, ρ, the micro-meteorite will meet in time, dt, an air mass, dm_a, given by

$$dm_a = A\rho V\,dt. \tag{3}$$

The corresponding energy gain, dE_g, amounts to

$$dE_g = \frac{\alpha}{2} V^2\,dm_a = \frac{\alpha}{2} A\rho V^3\,dt. \tag{4}$$

Part of this energy will be utilized in raising the temperature of the meteoroid, part radiated by black-(or gray)-body radiation, part used in dissociation, excitation and ionization and perhaps part used in disengaging material from the surface. If the accommodation coefficient is defined to include the dissociation, excitation and ionization and if vaporization is negligible, we may deal here explicitly with only the heating and radiation terms.

We may assume that the meteoroid was previously in temperature equilibrium with the night side of the Earth (or Sun and Earth during the day) at temperature T_0. With a gray-body emissivity coefficient of β, the loss of energy by radiation, dE_r, of a surface at temperature T_s is

$$dE_r = \beta B\sigma\,(T_s^4 - T_0^4)dt, \tag{5}$$

where σ is the Stefan-Boltzmann constant.

If the meteoroid is small ($s \ll 1$ cm.), of mass, m, and if the coefficient of heat conductivity is at all comparable to that of ordinary rocks, the internal temperature should differ negligibly from the surface temperature in time intervals somewhat smaller than one second. The permissible limits to this assumption will be discussed later. If, then, the heat capacity per gram is C_s, the temperature will vary as

$$mC_s dT_s = dE_g - dE_r. \tag{6}$$

By equations (4) and (5), equation (6) becomes

$$mC_s \frac{dT_s}{dt} = \frac{\alpha}{2} A\rho V^3 - \beta B\sigma(T_s^4 - T_0^4). \tag{7}$$

The precise conditions under which the heat capacity in the left member of equation (7) can be neglected are not apparent *a priori*. We can, however, easily determine a resultant rough limit to the dimensions of the micro-meteorite and later study the question more thoroughly. The maximum temperature, T_m, to which a micro-meteorite can be heated without appreciable vaporization is just below the melting point of the least refractory material in the meteoroid, approximately 1200°K. to 1700°K. for typical stones.[3] Iron, iron oxides and silica also fall within this range. The temperature rise from the equilibrium temperature at the Earth to T_m is relatively large. Generally this rise will occur over a considerable distance through the atmosphere since the radiation varies as T_s^4 while the heating is proportional to atmospheric density. We will find that the heat capacity is negligible for fast micro-meteorites because they are so small. For slow micro-meteorites an atmospheric density increase of a factor of two will require roughly a half-second of atmospheric traverse at normal incidence.

Hence, if the heat radiation in an interval of the order of half-second near maximum temperature is large compared to the heat required to raise the temperature from T_0 to T_m we may safely neglect the heat capacity in equation (7). The condition just defined is

$$2mC_s\,(T_m - T_0) \ll \beta B\sigma\,(T_m^4 - T_0^4). \qquad (8)$$

The limiting radius, s, for a spherical meteor of density, ρ_s, is, from equation (8)

$$s \ll \frac{3\beta\sigma}{2C_s\rho_s}\,(T_m^3 + T_m^2T_0 + T_mT_0^2 + T_0^3). \qquad (9)$$

The right member of equation (9) is of the order of 0.01 cm. for an iron or stony meteorite. Hence we may safely ignore the heat capacity of micro-meteorites of radii less than 10 microns.

With our current assumptions then, we may set the left-hand member of equation (7) equal to zero and determine the surface temperature of a micro-meteorite as a function of its velocity and the atmospheric density. The result is

$$T_s^4 - T_0^4 = \frac{\alpha A\rho V^3}{2\beta B\sigma}. \qquad (10)$$

While the temperature of the micro-meteorite is rising, its velocity is being reduced by atmospheric resistance. We may define the drag coefficient, D, by the equation

$$mdV = -\frac{AD\rho V^2}{2}dt. \qquad (11)$$

The drag coefficient will include a major component, of the order of 2 numerically, if we assume that air particles momentarily adhere to the meteoroid and are reemitted isotropically. A smaller additional term will arise from the non-isotropic reemission according to the hypothesis of Tsien[4] and Miss Heineman.[5] No term will arise from evaporation of material, however, since we assume (perhaps erroneously at high velocities) that evaporation is negligible.

A rigorous inclusion of the reemission drag term would involve the surface temperature of the micro-meteorite. Since the effect is rather small, we may include it approximately after a solution has been made with D constant.

The simultaneous solution of equations (10) and (11) will provide a relation among the temperature, velocity and time, if the atmospheric density can be expressed as a function of the time. A complicated, variable and poorly known relation between atmospheric density and height actually exists at the atmospheric heights concerned. In the following sections the fundamental equations will be solved for certain types of such relations.

The Solution in an Isothermal Atmosphere.—Let us assume that the atmosphere is isothermal and that the mean molecular weight of the air is also constant. Then the atmospheric density, ρ, varies with the height, h, above sea level according to the relation

$$\rho = \rho_0 e^{-bh}, \tag{12}$$

where b, the logarithmic density gradient, is a positive constant and ρ_0 is constant.

Let us neglect the curvature of the Earth. Then for a particle entering the atmosphere with velocity, V, from an apparent radiant at zenith distance, Z, the time and height are related by the equation

$$dh = -V \cos Z \, dt. \tag{13}$$

Since meteoric velocities are so large we may neglect the effect of the Earth's gravity, assuming that the pertinent part of the trajectory is a straight line, and that the velocity is unaffected by gravity.[6]

Equations (11), (12) and (13) then lead to the following differential relation between velocity and height:

$$\frac{dV}{V} = \frac{AD\rho_0}{2m \cos Z} e^{-bh} \, dh. \tag{14}$$

We have already assumed that all of the quantities in equation (14) except V and h may be treated as constants. Hence we may integrate equation (14) between the limits of V_∞, the initial velocity at great heights essentially infinity, to any pertinent velocity, V, and height, h. The integral of equation (14), when combined with equation (12) to eliminate the height explicitly, yields the following relation between the velocity and atmospheric density:

$$\log (V/V_\infty) = -\frac{AD\rho}{2bm \cos Z}. \tag{15}$$

Now the micro-meteorite can traverse the atmosphere undamaged only if the velocity is reduced sufficiently to prevent the surface temperature from exceeding a critical temperature, T_m, somewhat below the melting point of the meteoritic materials. This restriction can be imposed if we eliminate the density, ρ, between Equations (15) and (10) and then impose a maximal condition on T_s. The first step gives

$$T_s^4 - T_0^4 = -\frac{\alpha b m \cos Z}{\beta \sigma B D} V^3 \log (V/V_\infty). \tag{16}$$

To determine the maximum temperature from equation (16) we must make assumptions as to the functional forms of the accommodation coefficient, α, and the emissivity factor, β. In view of our lack of knowledge of the detailed physical and chemical structure of the micro-meteorites and, even with this knowledge, the uncertainty in the physical laws for α and β, we may as well adopt constant values for these quantities as well as for D. Hence, from the derivative of equation (16) the maximum temperature is reached at the critical velocity, V_c, given by

$$\log (V_c/V_\infty) = -\frac{1}{3}, \tag{17a}$$

or

$$V_c = V_\infty/e^{1/3} = 0.7165\ V_\infty, \tag{17b}$$

where e is the base of the natural logarithms.

We may now solve for the ratio of the mass to the effective surface area, m/B, of the micro-meteorite from equation (16) for the maximum temperature, T_m. The resulting critical ratio is

$$m/B = \frac{3e\beta\sigma D(T_m^4 - T_0^4)}{\alpha b \cos Z V_\infty^3}. \tag{18a}$$

Equation (18a) represents the maximum value of m/B for a micro-meteorite that is to be stopped, undamaged by the atmosphere. The maximum radius for a spherical particle of radius, s (sphere) and density, ρ_s, is then

$$s\ (\text{sphere}) = \frac{9e\beta\sigma D(T_m^4 - T_0^4)}{\alpha \rho_s b \cos Z V_\infty^3}, \tag{18b}$$

a result corresponding closely to Öpik's Equation (53).[2]

It is of interest to note from equations (18) that the maximum particle dimension depends directly upon the critical temperature to the fourth power and upon the inverse cube of the original velocity. Low velocity and high melting point strongly favor the passage of such a particle through the atmosphere. Irregular or elongated shape favors the process for a given meteoric mass. Because of the cos Z term, larger masses may enter at lower angles of incidence.

For a rapidly spinning micro-meteorite in the shape of a right circular cylinder of length l and radius s (cyl.), the maximum radius in terms of the comparable sphere (equation 18b) of the same density and m/B ratio is given by

$$s \text{ (cyl.)} = s \text{ (sphere)} \times \frac{2}{3}\left(\frac{s+l}{l}\right) \text{ (cyl.)}. \quad (18c)$$

If the rapidly rotating cylindrical meteorite is to have the same radius as the spherical one, then the length must be equal to the diameter of the sphere. An extremely long cylindrical meteorite ($l \gg s$) will be stopped when its radius is 2/3 or less the corresponding spherical radius (equation 18b). The maximum permissible dimensions of micro-meteorites with other shapes or orientations can be determined by means of equations (1), (2) and (18).

In applying equations (18) approximately to an atmosphere of variable temperature it is necessary to obtain an appropriate value of the logarithmic density gradient, b. The critical altitude is at the point of maximum temperature for the micro-meteorite. From equations (15) and (17) we find the critical density, ρ_m, at maximum temperature T_m as follows:

$$\rho_m = \frac{2bm \cos Z}{3AD}. \quad (19)$$

If the frontal area of the meteorite, A, is approximated by the average area, A_1, equations (2) and (18a) transform equation (19) into

$$\rho_m = \frac{8e\beta\sigma(T_m^4 - T_0^4)}{\alpha V_\infty^3}. \quad (20)$$

The value of b and the height corresponding to ρ_m may be derived from some standard atmosphere. The fact that the shape and mass of the micro-meteorite and cos Z do not enter equation (20) is rather surprising. In case there is reason to believe that the actual frontal area differs from the average cross-sectional area, a correction term can easily be applied for this factor in equation (20).

The drag coefficient, D, is a vital factor in equations (18) and (19). In case the air molecules impinging on the surface of the micro-meteorite are momentarily "captured" by the surface, but very quickly reemitted with energies corresponding to the temperature of the surface, D is equal to 2 with a small correction term for the Tsien-Heineman effect mentioned above. This situation corresponds to that when the accommodation coefficient, α, is exactly unity. M. L. Wiedman[7] finds that the accommodation coefficient for air on various metals is generally close to 0.9. At the higher energies here considered, there is little doubt that the air molecules will largely penetrate the surface molecular layers and be momentarily captured. Since only a few can combine chemically and since condensation is impossible at the temperatures considered, there appears to be little question that essentially all of the air molecules will be captured and then reemitted with an average velocity, v_T, corresponding to the temperature of the surface. The average velocity is given by

$$(v_T)^2 = \frac{8k_1T_s}{\pi\mu_1}, \tag{21}$$

where k_1 is the gas constant and μ_1 is the molecular weight of the gas.

If the micro-meteorite is assumed to be spherical and the reemission is uniform with respect to solid angle from a small area of the surface, the momentum transfer to a hemisphere per average molecule is $4\mu v_T/9$. Since the time lag of reemission must be small compared to a possible rotation period of the micro-meteorite, this momentum transfer will oppose the motion of the body. If we further accept $D = 2$ as the best approximation to the drag coefficient without including reemission, the value of D becomes:

$$D = 2[1 + 4v_T/(9V)]. \tag{22}$$

A numerical calculation shows that the v_T-correction in equation (22) amounts at most to a few per cent in the velocity and temperature ranges under consideration. Most of the deceleration of interest occurs fairly near the critical region of maximum temperature. In adopting a constant value of D, therefore, we may use v_T applying at the maximum temperature and compensate this error somewhat by adopting $V = V_\infty$ in equation (22).

It would be desirable to make some correction in the drag coefficient for the fact that some of the impinging air molecules must deviate from the assumed process of penetration and thermal reemission, but we have no detailed information as to the surface characteristics of micro-meteorites. The relative error made in D by this omission is less than $1 - \alpha$.

The remaining undetermined constant in Equation (18) is the emissivity factor, β. Nothing precise can be known about this factor until micro-meteorites have been studied carefully in the laboratory. Probably β is very near unity. For want of better information we may set $\alpha = \beta \cong 1$, so that their effects cancel in equation (18). We may adopt $T_m = 1600°$K., and $T_0 = 300°$K. The remaining quantities to be specified in equation (18a) concern the atmosphere. The Tentative Standard Atmosphere of the National Advisory Committee for Aeronautics[8] represents good modern estimates of upper-atmospheric densities and will be used for the present calculation.

For a velocity of 23 km./sec., corresponding to the 1947 Giacobinid Meteor Shower, the atmospheric density at which a micro-meteorite of limiting dimensions attains maximum temperature ($\rho = 8.2 \times 10^{-10}$ gm./cm.^{3}) occurs near the 112-km. level, by application of equation (20). With an air temperature of 346°K. and $b = 1.1 \times 10^{-6}$ cm.$^{-1}$ the maximum radius of spherical iron particles for cos $Z = 0.45$ are then calculated to be about 4 microns. Long cylindrical particles might have a diameter as great as 6 microns, in good agreement with Landsberg's observations mentioned earlier.

Part II of this paper will concern the solution of limiting dimensions for micro-meteorites in an atmosphere of constant temperature gradient and in the general case. A critical discussion of the assumptions adopted here will be included as well as some elaboration of the numerical results.

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[1] *Pop. Ast.*, 55, 322 (1947).

[2] *Pub. Univ. Tartu*, 29, No. 5, 51 (1937).

[3] See Daly, R. A., *Igneous Rocks and the Depths of the Earth*, McGraw-Hill Book Co., 1933, p. 65.

[4] Tsien, Hsue-Shen, *J. Aero. Sci.*, 13, 653 (1946).

[5] Heineman, M., *Comm. Apl. Math.*, 1, 259 (1948).

[6] See, e. g., Whipple, F. L., *Proc. Am. Phil. Soc.*, 79, 499 (1938).

[7] See Tsien, *loc. cit.*

[8] Warfield, C. N., NACA, Tech. Note, No. 1200, Langley Field, Jan., 1947.

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*MERCAPTAN-INDUCED COAGULATION OF PROTEINS**

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In this paper it will be demonstrated that mercaptans possess the property of inducing coagulation of certain proteins at room temperature and neutrality. This effect throws light on the importance of —S—S— bonds in the intramolecular folding of several proteins in the native state.

While the dispersive action of mercaptans on keratin is well known, the coagulative action on soluble proteins apparently has not been described. Goddard and Michaelis[1] found that wool readily dissolves at room temperature in thioglycollate solutions at pH 10–13 because of reduction of disulfide bonds. The following reaction[1–4] occurs in the thiol-disulfide system:

$$\text{RS—SR} + 2\text{R}'\text{SH} \rightleftharpoons 2\text{RSH} + \text{R}'\text{S—SR}'$$

Since greater alkalinity was required in the experiments of Goddard and Michaelis than was needed for the reduction of cystine by thiols, they[1] postulated that an additional intramolecular bridge must be broken before disulfides of keratin can be reduced; they assumed that the second bridge was salt-like in character and that it was disrupted by removal of a proton from the amino group in alkaline solution. Jones and Mecham[5] observed that a variety of keratins were dispersed at pH 7 and 40°C., by adding urea or anionic detergents to the mercaptan solution. Mirsky and Anson[2] found that thioglycollate in excess completely reduces disulfide groups in