who gave statistical advice but who should not be held responsible for the exact procedures which we utilized.

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AN EXAMPLE OF A NONNORMAL DISTRIBUTION WHERE THE QUOTIENT FOLLOWS THE CAUCHY LAW

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Let x and y be two independently and normally distributed variates with zero means and the common variance; it is then well known that the quotient x/y follows the Cauchy distribution symmetrical about zero. It has been conjectured that this is a unique property of the normal distribution and that it is possible to obtain a characterization of the distribution by this property. In the present note we show that this conjecture is not true, and we give below an example of a nonnormal distribution where the quotient has the Cauchy distribution. This example, of course, does not rule out the possibility of characterizing a wider class of distribution laws by this property of the quotient.

EXAMPLE: Let x and y be two independently and identically distributed random variables, each having an absolutely continuous distribution function symmetrical about zero. Let the probability density function of x be given by

$$f(x) = \frac{\sqrt{2}}{\pi}\cdot\frac{1}{1+x^4} \qquad (-\infty < x < +\infty). \tag{1}$$

We introduce the polar transformation $x = r\cos\theta$, $y = r\sin\theta$ and can easily deduce that the joint probability density function of r and θ has the form

$$c\frac{r}{(1+r^4\cos^4\theta)(1+r^4\sin^4\theta)} \qquad (0 < r < \infty, 0 < \theta < \pi) \qquad (2)$$

where c is a constant.

Now, in order to find the distribution of θ, we have to integrate (2) with respect to r over the range $(0, \infty)$.

Making the substitution $r^2 = u$, the probability density function of θ is given by

$$\frac{c}{2}\int_0^\infty \frac{du}{(1+u^2\cos^4\theta)(1+u^2\sin^4\theta)}. \qquad (3)$$

From (3) it can be easily verified that θ has a uniform distribution in the interval $(0, \pi)$.

Finally, substituting $x/y = \cot\theta$ in the distribution of θ, it is proved that the quotient x/y follows the Cauchy distribution.

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SPIN, STATISTICS, AND THE TCP THEOREM

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The recent experimental discoveries relating to space parity and charge symmetry in the so-called weak interactions have emphasized the need for a clearer recognition of the role played by the continuous Lorentz subgroup of proper, orthochronous Lorentz transformations. This refers, in particular, to some work of the author,[1] in which the general requirement of Lorentz invariance for the quantum theory of fields is used to deduce the connection between the spin and the statistics of particles, and to a converse statement (now known as the "TCP theorem") in which the acceptance of the spin-statistics connection implies invariance under a combined space, time, and charge-reflection operation.[2] It is the intention of this note to make explicit the fact that the general dynamical structure of the quantum theory of fields, together with the specific assumption of invariance under the proper orthochronous Lorentz subgroup, and the existence of a lowest-energy state (the vacuum) for any physically realizable system, implies both the connection between the spin and statistics of particles and the TCP theorem.

The dynamical principle of the theory of fields, governing the development of the system between two space-like surfaces, is the differential action principle,

$$\delta\langle\sigma_1|\sigma_2\rangle = \langle\sigma_1|\delta\{i\int_{\sigma_2}^{\sigma_1}(dx)\mathcal{L}[\chi]\}|\sigma_2\rangle,$$

where

$$\mathcal{L}[\chi] = {}^1/_4\,(\chi A^\mu\partial_\mu\chi - \partial_\mu\chi A^\mu\chi) - \mathcal{H}(\chi)$$