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$H = W$

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Let R^n be the real Euclidean space of points $x = (x_1,\ldots,x_n)$ with $|x| = (\sum x_i^2)^{1/2}$, and let Ω be an open set in R^n. Two concepts which are often used in the theory of partial differential equations and the calculus of variations are the so-called H spaces and W spaces. More specifically, if m is a nonnegative integer and $1 \leq p < \infty$, we have the definitions,

$W^{m,p}(\Omega)$ = Banach space of all complex valued functions $u = u(x)$ on Ω with distribution derivatives up to order m in $L^p(\Omega)$, with norm

$$||u||_{W^{m,p}(\Omega)} = \sum_{|\alpha| \leq m} ||D^\alpha u||_{L^p(\Omega)},$$

while

$$H^{m,p}(\Omega) = \text{closure in } W^{m,p}(\Omega) \text{ of } C^\infty(\Omega) \cap W^{m,p}(\Omega).$$

The derivatives of a function in W are frequently referred to as weak derivatives, while derivatives of functions in H are called strong derivatives.

We observe that $H \subset W$, by the very definitions. Conversely, Friedrichs proved that locally a weak derivative is strong. The belief persists, however, that in the large this is not the case; authors using the spaces H and W will often distinguish between them and point out that they are the same *provided that* the boundary of Ω satisfies certain smoothness conditions. Here we shall show that they are *always* the same.

THEOREM. $H \equiv W$.

This result is an immediate consequence of the following more general proposition.

LEMMA. *If $u = u(x)$ has distribution derivatives up to order m in Ω which are locally in L^p, then for each $\epsilon > 0$ there exists a function $v = v(x)$ in $C^\infty(\Omega)$ such that $u - v$ is in $W^{m,p}(\Omega)$ and $||u - v||_{W^{m,p}(\Omega)} \leq \epsilon$.*

Proof: Let Ω_ν be the open set defined by

$$\Omega_\nu = \{x | x\epsilon\Omega, |x| < \nu, \text{distance } (x, \partial\Omega) > 1/\nu\},$$

where $\nu = 1, 2, \ldots$. For convenience we also define Ω_0 and Ω_{-1} to be null sets. Now let $\sum\psi_\nu \equiv 1$ be a partition of unity on Ω such that

$$\text{supp } \psi_\nu \subset \Omega_{\nu+1} - \Omega_{\nu-1}, \qquad \nu = 1, 2, \ldots.$$

Also, let $K_\nu = K_\nu(x)$ be a C^∞ mollifier satisfying the two conditions

$$\text{supp } K_\nu \subset \{x|\ |x| < 1/(\nu + 1)(\nu + 2)\}$$

and

$$||K_\nu * \psi_\nu u - \psi_\nu u||_{W^{m,p}(\Omega)} \leq \epsilon/2^\nu,$$

where $\nu = 1, 2, \ldots$. Evidently, we have supp $K_\nu \psi *_\nu u$ contained in the set $\Omega_{\nu+2} - \Omega_{\nu-2}$, so that the series

$$v = \sum_1^\infty K_\nu * \psi_\nu u$$

is trivially convergent and defines a function v in $C^\infty(\Omega)$. Finally, choosing $k = 1, 2, \ldots$ we have

$$||u - v||_{W^{m,p}(\Omega_k)} = ||\sum_1^{k+1} (K_\nu * \psi_\nu u - \psi_\nu u)||_{W^{m,p}(\Omega_k)}$$

$$\leq \sum_1^{k+1} ||K_\nu * \psi_\nu u - \psi_\nu u||_{W^{m,p}(\Omega)} \leq \epsilon.$$

We now let $k \to \infty$, and the result follows from the Lebesgue monotone convergence theorem.

Remarks.—Since it is now shown that $H = W$, the distinction drawn above between the concepts of weak and strong derivatives need no longer be maintained. We are of the opinion that the phrase "strong derivative" is the appropriate terminology to retain, following the present practice of most mathematicians.

It should be pointed out in conclusion that an occasional author will define the H spaces by taking the closure of functions which are smooth up to the boundary of Ω. In this case the two spaces H and W certainly are not the same unless some smoothness conditions are imposed on the boundary.