

A NEW FINITE SIMPLE GROUP WITH ABELIAN 2-SYLOW SUBGROUPS

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Communicated by Saunders Mac Lane, January 25, 1965

Let J be the subgroup of $GL(7,11)$ generated by the following two matrices:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -3 & 2 & -1 & -1 & -3 & -1 & -3 \\ -2 & 1 & 1 & 3 & 1 & 3 & 3 \\ -1 & -1 & -3 & -1 & -3 & -3 & 2 \\ -1 & -3 & -1 & -3 & -3 & 2 & -1 \\ -3 & -1 & -3 & -3 & 2 & -1 & -1 \\ 1 & 3 & 3 & -2 & 1 & 1 & 3 \\ 3 & 3 & -2 & 1 & 1 & 3 & 1 \end{pmatrix}.$$

A 2-Sylow subgroup of J is elementary abelian of order 8 and J has no subgroup of index 2. If τ is an involution in J , then $C(\tau) = \langle \tau \rangle \times K$, where $K \cong A_5$.

Let G be a finite group with the following properties: (a) S_2 -subgroups of G are abelian; (b) G has no subgroup of index 2; and (c) G contains an involution t such that $C(t) = \langle t \rangle \times F$, where $F \cong A_5$. Then G is a (new) simple group isomorphic to J .

The order of G is $11(11^3 - 1)(11 + 1)$. An S_2 -subgroup of G is elementary abelian of order 8, and all odd order Sylow subgroups are cyclic (of prime order).

Every maximal subgroup of G is conjugate to one of the following groups: (1) 2-Sylow normalizer which is a holomorph of an elementary abelian group of order 8 by a noncyclic group of order 21, (2) 3-Sylow normalizer (which is also a 5-Sylow normalizer), and this is a holomorph of a cyclic group of order 15 by an elementary abelian group of order 4, (3) 7-Sylow normalizer which is a Frobenius group of order 42, (4) 11-Sylow normalizer which is a Frobenius group of order 110, (5) 19-Sylow normalizer which is a Frobenius group of order 114, (6) centralizer $\langle t \rangle \times F$ of an involution t , where $F \cong A_5$, and (7) the projective special linear group $PSL(2,11)$.

The group G has precisely one conjugate class of subgroups isomorphic to $PSL(2,11)$ and two conjugate classes of subgroups isomorphic to A_5 . If H is a subgroup of G isomorphic to $PSL(2,11)$, then two icosahedral subgroups H_1 and H_2 of H which are nonconjugate in H remain nonconjugate in G .

The outer automorphism group of G is trivial. The Schur's multiplier of G is trivial.

The group G has 15 conjugate classes of elements (which are all real) and hence 15 irreducible (complex) characters ψ_i ($i = 1, \dots, 15$). We have obtained the full character table of G , and in particular the degrees $\psi_i(1)$ are 1, 56, 56, 76, 76, 77, 77, 77, 120, 120, 120, 133, 133, 133, and 209.

The modular representation of G generated by matrices A and B (using $G \cong J$) is a faithful representation of G of the smallest possible degree in any field.

The group G does not have a doubly transitive permutation representation.

W. A. Coppel has shown that the group G is a subgroup of the Dickson's simple group $E_2(11)$ related to the simple Lie algebra of type (G_2) .

The author would like to express his thanks to T. M. Gagen and M. A. Ward who have computed the order of the matrix group J .

A FAMILY OF SPECTRAL OPERATIONS*

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Communicated by Saunders Mac Lane, January 25, 1965

1. *Introduction.*—Since the invention of the Steenrod squares, cohomology operations have been proved of vital use in algebraic topology. These operations are best handled by computing the cohomology of the universal example $K(\pi, n)$.¹ In this note, we shall exploit similar operations in spectral sequences by carrying out some computations on the corresponding universal example. Some operations have been found by Araki and Vazquez (see ref. 4) following the Steenrod approach of considering the cochain product.

A simplicial set (ref. 5, p. 233) is called *regular* if every nondegenerate simplex has nondegenerate faces. Let \mathcal{E} be the category whose objects are epic simplicial maps $f: X \rightarrow Y$, with Y regular, while a morphism $(g, \bar{g}): f \rightarrow f'$ in \mathcal{E} is a pair of simplicial maps g, \bar{g} such that the diagram

$$\begin{array}{ccc} X & \xrightarrow{g} & X' \\ f \downarrow & & \downarrow f' \\ Y & \xrightarrow{\bar{g}} & Y' \end{array}$$

is commutative. Given $f: X \rightarrow Y$ in \mathcal{E} , X is increasingly filtered with $F_p X = f^{-1}(Y^{(p)})$, where $Y^{(p)}$ is the p -skeleton of Y , consisting of all simplices of form $s_{i_1} \dots s_{i_r} y$, y a nondegenerate simplex of dimension $\leq p$. The cochain complex $C^*(X; G)$, with G any coefficient group, is decreasingly filtered with $F^p C^*(X; G)$ consisting of all those cochains which vanish on $F_{p-1} X$. This filtration gives rise to a spectral sequence $\{E_*^{*,*}(f; G)\}$, called the spectral sequence of f .

In this note, we study (single-valued or many-valued) natural transformations of functors $T: E_r^{p,q}(\ ; Z_2) \rightarrow E_t^{a,b}(\ ; Z_2)$, called spectral operations. For the functor $E_r^{p,q}(\ ; G)$ on \mathcal{E} , we shall construct a universal example which consists of an object $P_r^{p,q}(G): K_r^{p,q}(G) \rightarrow B$ and a fundamental class $\{\zeta\} \in E_r^{p,q}(P_r^{p,q}(G); G)$; and shall compute $E_*^{*,*}(P_r^{p,q}(Z_2); Z_2)$ for $r \geq q + 2$. Then every spectral operation $T: E_r^{p,q}(\ ; Z_2) \rightarrow E_t^{a,b}(\ ; Z_2)$ can be identified with $T\{\zeta\}$, which is a single element of $E_t^{a,b}(P_r^{p,q}(Z_2); Z_2)$ if T is single-valued and is a subset of the same group if T is many-valued. By the nature of the universal example (see Theorem 2.1), an element of $E_t^{a,b}(P_r^{p,q}(Z_2); Z_2)$ defines in general a single-valued or many-valued operation (since $\bar{\gamma}$ is not unique).

The result can be applied to the spectral sequences of fibrations. In fact, the category \mathcal{K} of Kan fibrations with regular base is a full subcategory of \mathcal{E} , hence every