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*VIRTUAL COULOMB EXCITATION IN NUCLEON TRANSFER**

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1. *Introduction.*—Attempts[1, 2] at the interpretation of early nucleon-transfer data suggested the possibility of participation of virtual Coulomb excitation (VCE) in nucleon transfer reactions. The early estimates[2] based on the Bethe-Levinger[3] sum rule were revised[4] downward making use of giant dipole resonance (GDR) data.[5] Although many reservations were made[2] regarding the accuracy of the estimates, the unexcluded possibility of appreciable VCE participation appeared to throw doubt on the nucleon-transfer data analyses for some years.[6] The large accumulation of new data[7] made it desirable to see whether the upper limit on the effect of VCE could reasonably be lowered. It proves possible to do so.

The discussion will be concerned mainly with the reaction $N^{14}(N^{14}, N^{13})N^{15}$ since it has received the most attention. The neutron transfer with which the experiments are mainly concerned is from the ground-state configuration of N^{14} to the ground state of N^{15}, with the residual N^{13} also in the ground state. The part of the GDR excitation that comes under consideration does not primarily consist therefore in the direct formation of a hole in a complete $1p_{3/2}$ shell and excitation to $1d_{5/2}$ or $1d_{3/2}$, although much of the general excitation may be caused[8] by such a formation of a "dipole state." On the extreme j-j coupling model the relevant VCE transition consists in a removal of a $p_{1/2}$ neutron. This cannot take place to $1d_{5/2}$ which is presumably the lower of the two $1d$ states. A detailed evaluation of the contribution of VCE to the transfer probability by means of a nuclear model is therefore complicated. Fortunately, a precise calculation is not necessary because of the entrance of strongly inhibitive factors. The absolute values of the (γ,n) matrix elements can be estimated well enough from photodisintegration data. The relative phases of effects in different parts of the GDR and some numerical coefficients representing the projection of the excited state onto the final one remain uncertain, but it is believed that the safety factors of the inequalities more than take care of this uncertainty.

The belief that the effect of the VCE transfer probability may have to be taken into account in data analysis came from an estimate of the probability of the VCE state for a relative position of the colliding nuclei A and B, corresponding to their separation $R = R_{\min}$, the distance of closest approach on a classical mechanics orbit, with A and B in their ground states. According to BE-II this probability may be appreciable even though for the excited state A^* of A the value $R_{\min}$ of R

is impossible in classical mechanics. In BE-II the ratio of the VCE transfer probability to that for tunneling was taken to be the product of the chance a_1 that the projectile is in the VCE state $R = R_{\min}$ and a_2, the ratio of tunnel penetration probability from the virtual state to that from the ground state. For a_2 the BE-II estimate used $\exp(-2\alpha G)$, where α is the reciprocal of the wave function decay length and G is the gap between the nuclear surfaces at $R = R_{\min}$. In the new estimates the large value of the excitation energy ΔE, the energy difference between A^* and A, is found to have a marked inhibiting effect on the transfer probability from the virtually excited A^*, the probability being affected by the largeness of ΔE not only through the decrease in the chance of A^* formation but even more by the rapid variation of the phase of the neutron wave function arriving at B from A^* as the positions of A^* and B are varied in the important region in the vicinity of the classical orbit.

2. *Calculations.*—The calculations reported were made directly for the continuum of states of which the GDR is composed. Replacing the continuum of states corresponding to neutrons emanating from A by a complete set of densely spaced discrete levels with eigenfunctions w_μ $(\mathbf{r}_n,k_n)$ vanishing on the surface of a quantizing sphere, the amplitude of the final state wave for the reaction products $A - n$, $B + n$ may be approximately represented by means of the transition matrix elements

$$(\Psi_f^{in}, H'\Psi_N^{out}) = \mathfrak{M}, \tag{1}$$

where Ψ_N^{out}, the Ψ_{CN} of BE-II, is the outgoing wave modification of the incident wave. It includes a part containing the $w_\mu(\mathbf{r}_n,k_n)$. According to Figure 2 of BE-I and the considerations in equations (18) to (19.4) in the case of elastic scattering, the common orbit of the ingoing final and outgoing initial orbit sets is surrounded by a region of stationary phase of $(\Psi_f^{in})^* \Psi_N^{out}$. The same holds for a reaction with $Q = 0$. Since the $N^{14}(N^{14},N^{13})N^{15}$ reaction is of the greatest immediate interest and since for it Q is very small, the considerations below will be made for $Q = 0$. The extension to $Q \neq 0$ is easily inferred. In BE-I the stationary phase relationship was used to express the quantum-mechanical transition probability in terms of a line integral over the common orbit. The generalization of this procedure to the transition probability from the Coulomb excited state $A^* + B$ to the final $(A - n) + (B + n)$ rearrangement may be conveniently seen in terms of the slight generalization in the use of the action function of classical dynamics and its adaptation[9] to quantum-mechanical (QM) problems. Neglecting the last term in equation (3.8) or the related equations (3.12) and (3.13) of reference 9, the equations have the form of those in the semiclassical treatment (SCT), even though the quantity called t is not the true but only a fictitious "time" defined in equation (3.7). In this approximation the transition matrix element $\mathfrak{M}$ of equation (1) is contributed to by VCE through the presence in Ψ_N^{out} of parts containing the $w_\mu(\mathbf{r}_n,k_n)$ as factors. Their coefficient may be used to define the classical probability amplitudes called a_j in reference 9. The higher the excitation energy, the more rapid is the oscillatory variation of the a_j along the classical orbit, the characteristic frequency of the usual SCT that enters the nonadiabaticity parameter usually denoted by ξ being of the order $\Delta E/h$. Although this parameter is usually introduced in the restricted sense of the SCT, reference 9 shows that the rapid variation of the phases of the a_j with the classical time t implies a similar variation of the a_j of QM along the orbital paths. The VCE part of Ψ_N^{out} in equation (1) is therefore varying rapidly in the region of stationary phase of Ψ_f^{in}. On the other hand, the perturbing energy operator H' responsible for neutron capture, while also varying rapidly on account of the largeness of k_n, does not synchronize effectively with the variations of the a_j. Accordingly the integrand of the part of $\mathfrak{M}$ attributable to VCE is oscillatory and the ratio of the VCE to the tunneling contribution to $\mathfrak{M}$ is much smaller than the a_2 of BE-II, the relatively long collision time of the SCT implying long orbital paths over which the integration in equation (1) is extended and thus offering good opportunities for cancellation.

The second of the two estimates of the VCE part of Ψ_N^{out} contained in BE-II was made for high neutron energies. The result may be understood as follows. The equation to be solved is

$$[\Delta - 2/(\bar{a}R) - \beta^2]\psi_{-\mu}^{(1)} = (-)^\mu(2m/\hbar^2)(K/R^2)Y_{1\mu}\psi^c \qquad (2)$$

where $$\beta^2 = 2m(E_w - E)/\hbar^2, \qquad \bar{a} = \hbar^2/(Z_1Z_2me^2), \qquad (2.1)$$

which is equation (6.4) of Be-II. Here $H^{(0)}$ is the Hamiltonian of relative motion of the heavy aggregates A, B with the Coulomb potential taken into account; E_w is the energy of states w_μ measured with respect to the ground states v_i of A; E is the total energy in the center of mass system; $\psi_\mu^1(\mathbf{R})$ is the coefficient of w_μ in the wave function of the whole system; R is the distance between the centers of mass of A and B; ψ^c is the Coulomb wave in the space of $\mathbf{R} = \mathbf{R}_B - \mathbf{R}_A$; K is a constant proportional to the electric dipole matrix element introduced in equation (6.1) of BE-II in which r_p should be replaced by r_n and an appropriate change of sign should be made in the case of neutron excitation; $\psi_{-\mu}^{(1)}$, $Y_{1\mu}$, and ψ^c are functions in the space of $\mathbf{R}$. Neglecting $2/(\bar{a}R)$ in comparison with β^2 on the left-hand side of equation (2), neglecting variations of the right-hand side within distances $|\mathbf{R} - \mathbf{R}'|$ of order $1/\beta$, and requiring that $\psi_{-\mu}^{(1)}$ vanish at $R = \infty$, employment of the three-dimensional Green's function $-[\exp(-\beta|\mathbf{R} - \mathbf{R}'|)]/[4\pi|\mathbf{R} - \mathbf{R}'|]$ of $\Delta - \beta^2$ gives

$$\psi_{-\mu}^{(1)} \approx (-)^\mu(2m/\hbar^2)KY_{1\mu}\psi^c/(\beta R)^2, \qquad (2.2)$$

reproducing (11.3) of BE-II. The effect of $2/(\bar{a}R)$ being positive and comparable to β^2 in the important region of $R \approx R_{\min}$ is in the direction of making (2.2) overestimate $|\psi_{-\mu}^{(1)}|$. Similarly the effect of the smallness of ψ^c in most of the region inaccessible to classical-mechanics orbits is to increase the value of $|\psi_{-\mu}^{(1)}|$ as given by (2.2) in the proximity of the parabolic caustic. These effects tend to overestimate the VCE contribution to the transfer probability and suggest that (2.2) is not accurate. This is borne out by substitution into (2) of $\psi_{-\mu}^{(1)}$ by means of (2.2), resulting in disagreement of the two sides of the equation by amounts comparable with either. On the other hand,[9] the SC calculation of Coulomb excitation gives a first approximation to $\psi_{-\mu}^{(1)}$ even if the excitation is virtual, and such a calculation also gives essentially (2.2) as shown in BE-II. It thus appears reasonable to use equation (2.2) in an approximate calculation of an upper limit of the VCE effect, but without claims of accuracy.

The calculations were made for very low incident energies because in this case the upper limit of VCE transfer to tunneling transfer as estimated in BE-II is largest. In this limit the electric field component of B at A is dominantly along AB. The effect of the other components was therefore neglected. The absolute values of matrix elements for dipole absorption were expressed in terms of cross sections available from photodisintegration data. The phase relations between the matrix elements were assumed to be those of the single level with constant background resonance theory. The VCE wave functions emanating from the continuum of states A^* were analyzed in terms of functions with definite values of the orbital angular momentum $l\hbar$ around B. The result of integrating over a resonance was compared with the corresponding effect caused by tunneling in terms of the following ratio:

$$\left|\frac{\int_{-\infty}^{+\infty} \Sigma_{k_n}\psi_{ji}(\mathbf{r}_n,k_n)w_j(\mathbf{r}_n,k_n)dt}{\int_{-\infty}^{+\infty} \psi_{\text{comp}}^{\text{tun}}\, dt}\right| \cong \mathcal{A}\left(\frac{\alpha}{k_n}\right)^{1/2}\left| \sin\left(k_nR_m + \frac{\pi}{4} + \delta_2{}^0\right) - \frac{(1/8) + \epsilon/(1+\epsilon)}{k_na'\epsilon}\cos\left(k_nR_m + \frac{\pi}{4} + \delta_2{}^0\right) + \ldots\right|, \quad (3)$$

where

$$\mathcal{A} \cong \left(\frac{5}{6}\right)^{1/2} \times \left|\frac{(Z_Be/R_m{}^3)\sin(k_nR_m + \delta_2{}^0)[\sigma(E_\gamma)]^{1/2}\Gamma(c/v_n)^{1/2}F_1(k_nb)/(k_nb)}{[(\alpha a)^2/(1+\alpha a)]\mathcal{R}(a)[F_1(i\alpha b)/(\alpha^2R_mb)]E_\gamma{}^{3/2}\exp[-\alpha(R_m - b)]}\right|_r. \quad (3.1)$$

Previously undefined symbols occurring here have the following meanings: Z_Be is the charge on nucleus B; ϵ is the eccentricity of the classical orbit; i and j designate respectively the lower and upper magnetic substates ($m_i = 0$, $m_j = 0$) between which the Coulomb excitation transitions take place; $\delta_2{}^0$ is the level shift defined such that the phase shift of the $l = 2$ neutron wave emerging from A has a phase shift $\delta_2{}^p + \delta_2{}^0 = \delta_2$, where $\delta_2{}^p$ is the pole part of δ_2; E_γ is the γ-ray energy needed to excite A to a state with neutron momentum $\hbar k_n$; $\sigma(E_\gamma)$ is the total (γ,n) cross section for excitation from the ground state of A by a photon with energy E_γ; 2Γ is the half-value breadth of the resonance; a and b are the radii of A and B, respectively; v_n is the classical-mechanics velocity of the neutron produced by a photon of energy E_γ; $F_1(\rho) = [(\sin \rho/\rho)\text{-}\cos \rho]/r$ is the regular solution of the radial equation for $l = 1$ with $\rho = kr$; $1/\alpha$ is the decay length of the radial neutron wave function in the ground state of A such that asymptotically $\exp(-\alpha r_n)$ is the dominant factor; $\mathcal{R}(a)$ is the radial neutron wave function normalized to unity on omission of integrations over the solid angle of the neutrons;[10] and subscript r indicates evaluation at the resonance energy. A factor $(6/5)^{1/2}$ may be included on he right-hand side of equation (3) to allow for the ratio $(v_0\alpha, (d_{3/2})_{1/2})/(u_0\alpha, (p_{1/2})_{1/2})$, where u_0, v_0 are respectively the orbital wave functions of p and d states and α,β are the Pauli spin functions. This factor is applicable if the excitation is definitely to the $d_{3/2}$ state. Since the exact nature of the excitation is uncertain and since the factor is numerically insignificant, it is omitted. Experience with the use of a rotating coordinate system in BE-I indicates that the error introduced cannot be serious.

For an incident laboratory energy $E_l = 10$ Mev and a head-on collision ($\epsilon = 1$) numerical substitution gave $\mathcal{A} = 3.4$. The value of the factor $(\alpha/k_n)^{1/2}$ occurring in equation (3) is nearly unity for the case under consideration. The value[11] of $\alpha = 0.693\ F^{-1}$ was used corresponding to the inclusion of the reduced mass effect for the relative motion of $n + N^{13}$. $\mathcal{A}$ is sensitive to α and is decreased appreciably by the inclusion of the reduced mass factor in the latter. The numerical evaluation made use of the single particle square well model as in BE-I but can easily accommodate the experimentally known cross section provided the latter is completely explicable by tunneling. The number quoted above corresponds to the inclusion of the larger of the two peaks shown in Figure 4 of King, Haslam, and Parsons,[12] the effect of which was approximately represented by $\sigma(E_r) = 3$ mb, $\Gamma = 2$ Mev. The effect of the other peaks and of the fine structure of the spectrum found by Kosiek, Maier, and Schluepmann[13] was not taken into account for reasons to be mentioned presently. The ratio of the VCE to tunneling effects as given by equation (3) is probably somewhat overestimated, except for the possibility of destructive interference between resonances for the evaluation of which there are no available data since δ_2^0 has probably different values for the different levels.

The estimate of equation (3) is larger than the smaller of the two earlier estimates.[4] This is partly caused by the inclusion of the effect of the greater length of path over which VCE is active as compared with tunneling, the amplitude of which is strongly influenced by the factor $e^{-\alpha R}$. However, equation (3) takes no account of the variation of the phase of the neutron wave arriving at nucleus B. The origin of the variation is twofold: (*a*) The phase of the VCE waves $\psi_{ji}(\mathbf{r}_n, k_n)$ depends on R in the vicinity of the classical orbit as previously described in connection with reference 9. (*b*) Since the neutron wave arrives at nucleus B from A, there is an additional phase k_nR which also varies rapidly since $k_nR \gg 1$ for the relatively large excitation energy of the GDR.

These effects enter through the factor

$$\int_{-\infty}^{+\infty} (1/R^3) \sin (k_nR + \delta_2^0) \cos (\omega t)dt = \text{Im}(I_+ + I_-)/(2va'^2), \qquad (4)$$

where

$$I_s = \int_{-\infty}^{+\infty} (1 + \epsilon \cosh w)^{-2} \exp \{i\xi[\lambda(1 + \epsilon \cosh w) + s\xi(w + \epsilon \sinh w)]\} dw \qquad (4.1)$$

and

$$\xi = \omega a'/v = E^\gamma \eta/(2E), \qquad \lambda = k_nv/\omega = k_na'/\xi, \qquad s = \pm. \qquad (4.2)$$

The distance of closest approach for a classical orbit in a head-on collision is denoted here by $2a'$ and the relative velocity of A and B at $R = \infty$ by v. The parameter ξ is the usual one of Coulomb excitation theory applied here to virtual excitation and η is the Sommerfeld parameter for Coulomb collisions. Displacement of the path of integration for I_+ by $\pi i/2$ in the w plane and consideration of contributions from $-\infty$ to 0 separately from those from 0 to $+\infty$ gives

$$|I_+|/|I_+|_{\xi=0,\xi\lambda=k_na'} \lesssim (2\pi)^{-1/2}(1 + \epsilon)^2[(\lambda\xi\epsilon)^{-1/2} + (2\pi\lambda)^{1/2}e^{\xi\epsilon\lambda^2/2}]e^{-[(\pi/2)+\epsilon]\xi}. \qquad (5)$$

The derivation of the inequality presupposes that $\exp\{-\xi\epsilon[(w^2/2) - \lambda w]\} \ll 1$

when $\lambda w \ll 3$. Satisfying the latter by $\lambda w = {}^1/_2$ and giving λ its approximate value ${}^1/_4$ for the conditions for which the numerical estimate was quoted in connection with equation (3), this condition amounts to the requirement $\exp(-3\xi\epsilon/2) \ll 1$. For the GDR peak under discussion $\xi \approx 2\eta \approx 18$ and since $\epsilon \geq 1$, this condition is well satisfied. The dominant factor in equation (5) is the last one. For the conditions previously used for equation (3) with $\eta = 9.1$, it is $e^{-46.8} = 1/(2.0 \times 10^{20})$ when $\epsilon = 1$. Even for half this excitation which corresponds to very weak GDR effects the factor is $<10^{-10}$. The exponential factor in the second of the two terms in square brackets in equation (5) is large but is powerless in comparison with the last factor in that equation, especially at low excitation energies. The numbers arrived at are so small that it is not safe to use them directly for over-all upper-limit estimates. The contributions to $\mathfrak{M}$ from regions of **R**-space other than those dealt with above may not be negligible in comparison with the minute effects of the region normally responsible for major contributions to transfer. The omission of "dynamic reaction" terms and other imperfections of the incomplete theory used here may matter also.

3. *Conclusions.*—The apparent smallness of the VCE effect on transfer as compared with that of direct tunneling is in agreement with the lack of experimental indications[7, 11, 14, 15] for its presence. Although the treatment reported on has no pretensions to accuracy and although it is conceivable that regions of configuration space other than that paid attention to here may increase the VCE effect on transfer, it is highly improbable that the very strong influence of rapid phase variations can be overcome. It appears safe, therefore, to analyze available data in terms of tunneling theory.

Nucleon transfer experiments at energies below the Coulomb barrier are thus offering better possibilities of determining the reduced width like parameters of mechanically stable states than would be the case if the VCE produced appreciable complications. The inhibiting effects of rapid phase variations are especially pronounced at small angles corresponding to large impact parameters.

Establishment of true upper limit is not claimed. The contributions to VCE transfer previously feared have been shown, however, to be strongly inhibited.

Summary.—Estimates of the effect of virtual Coulomb excitation (VCE) on nucleon transfer reactions are made with particular attention to low-energy data for $N^{14}(N^{14}, N^{13})N^{15}$. A justification for making part of the calculation on an essentially semiclassical basis is formulated. If the effect of rapid phase variations of the Coulomb-excited wave function is omitted, the effect of VCE on neutron transfer takes on values similar to some early estimates. If the rapid phase variations are taken into account, this VCE contribution drops to values much smaller than those expected from the tunneling process. Data interpretation is therefore simplified. The reduction of contributions to the VCE nucleon transfer effect from the region of space contributing mostly to tunneling is so strong that the establishment of true upper limits is not claimed. The main reason for regarding VCE as a probable complicating influence has been removed, however.

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