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ORBITING CLUSTERS IN ATOMIC NUCLEI

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Abstract.—As an alternative to their description as vibrational levels, the low excited states of even-even nuclei can be described as rotational states of a helion, dineutron, diproton, or other cluster about the rest of the nucleus, leading to reasonable values of the average distance between centers of the clusters. Some states involve rotational excitation of two or more helions or other clusters. The nature of the rotating clusters is determined by the relation of the neutron and proton numbers to the magic numbers.

I have noticed that many of the excited states of nuclei can be described as corresponding to rotational motion of small clusters of nucleons, usually spherons (helions, tritons, trelions, dineutrons, diprotons), about the rest of the nucleus. The principal criterion used in reaching this conclusion is that the radius of the orbit should have a reasonable value, ranging from about 6 fm for mass number $A = 40$ to about 9 fm for $A = 200$.

The short range of internucleonic forces leads to the requirement that the radial wave function for an orbiting spheron or other cluster not extend far beyond the surface of the nucleus, and that, for radial quantum number zero, the motion of the spheron would correspond to rotation in a nearly circular orbit, for which the energy equation for a rotator may be used.

For example, the titanium isotopes ${}^{46}_{22}Ti_{24}$ and ${}^{48}_{22}Ti_{26}$ may be described as having a helion (alpha particle), plus two or four neutrons, attached to the doubly magic and spherically symmetric cluster ${}^{40}_{20}Ca_{20}$. The lowest excited state is 2^+, at 0.889 MeV and 0.983 MeV, respectively. For a moving helion, with reduced mass 3.66, the values of the distance R from the center of the helion to the center of the large cluster, calculated with the rotator energy expression, are 6.2 fm and 5.90 fm, respectively.

The values 6.60 fm, 6.37 fm, 6.31 fm, 6.45 fm, 5.81 fm, and 5.67 fm are similarly obtained for the nuclei ${}_{24}Cr_{26}$, ${}_{24}Cr_{30}$, ${}_{26}Fe_{30}$, ${}_{26}Fe_{32}$, ${}_{30}Zn_{34}$, and ${}_{30}Zn_{36}$. The average value of R, 6.2 fm, corresponds to spheron-spheron distance about 3.5 fm for a spheron rotating about a two-layer cluster with one central helion.[1]

The first excited state, 2^+, for nuclei with $Z = 28$ or $N = 28$ lies about twice as high above the normal state: at 1.55 MeV, 1.434 MeV, 1.41 MeV, and 1.452

MeV for $_{22}Ti_{28}$, $_{24}Cr_{28}$, $_{26}Fe_{28}$, and $_{28}Ni_{30}$, respectively. The factor 2 indicates that the moving mass is only half as great, corresponding to a moving diproton or dineutron (reduced mass about 1.93); the corresponding values of R are 6.49 fm, 6.74 fm, 6.80 fm, and 6.69 fm, which average about 7 per cent larger than the moving-helion values quoted above.

Similar results are found for other nearly spherical nuclei. The values of the energy for the lowest excited state (2^+) of $_{48}Cd_{58}$ to $_{48}Cd_{66}$ and $_{52}Te_{68}$ to $_{52}Te_{74}$ lie between 0.560 MeV and 0.665 MeV, and correspond to the moving-helion values R = 7.18, 7.17, 7.03, 7.26, 7.63, 7.60, 7.58, 7.33, and 6.97 fm. For $_{50}Sn_{62}$ to $_{50}Sn_{74}$ the state 2^+ lies nearly twice as far above the normal state, at 1.13 to 1.30 MeV. The moving-dineutron values of R range from 7.01 fm to 7.51 fm, in good agreement with the moving-helion values for the adjacent nuclei.

The lowest excited state of the doubly magic nucleus $_{82}Pb_{126}$ is 3^- at 2.615 MeV. The helion-hole nucleus $_{80}Hg_{124}$ has 2^+ at 0.43 MeV, and the dineutron-hole nucleus $_{82}Pb_{124}$ has 2^+ at about twice the value, 0.803 MeV; these values lead to the distances 8.62 fm and 8.88 fm, respectively. Roughly equal values are found also for other moving-helion, moving-dineutron, and moving-diproton nuclei in this region. There is, however, indication of both a smaller effective mass (helion dissociation in $_{84}Po_{128}$) and a larger effective mass (moving 2α in $_{86}Rn_{134}$ and moving 3α in $_{88}Ra_{132}$, corresponding to incipience of pronounced permanent prolate deformation).

The higher rotational levels indicate a reasonable amount of centrifugal stretching. The energy values 0.983, 2.30, and 3.34 MeV for 2^+, 4^+, and 6^+ of the moving-helion nucleus $_{22}Ti_{26}$, for example, lead to R = 5.90, 7.04, and 8.47 fm, respectively, and a similar trend is observed for many other nuclei with an extra helion or a helion hole.

The pattern of excited states for nuclei with two extra helions is, however, different, and the difference can be given a simple interpretation. Thus $_{34}Se_{42}$ has, in addition to 2^+ at 0.5593 MeV (R = 7.69 fm), a set of levels 0^+, 2^+, 4^+ at 1.118, 1.217, and 1.332 MeV, respectively. These levels can be ascribed to the excitation of two helions to 2^+. The two 2^+ vectors can combine to resultants $J = 0^+, 2^+$, and 4^+. The average energy for these two excited-helion states corresponds to R = 7.19 fm.

A similar pattern is observed for $_{28}Ni_{32}$, 2^+ at 1.332 MeV (R = 6.98 fm for moving dineutron) plus 2^+, 0^+, 4^+ at 2.158, 2.286, and 2.506 MeV, respectively. This 2^+, 0^+, 4^+ set of states, interpreted as resultants of two 2^+ dineutrons, leads to R = 7.39 fm, in reasonable agreement with the value for the first excited state.

The observed values of the first excited state show that $Z = 40$ and $N = 40$ are not found as completed shells ($L + N$) in the normal states of nuclei with $A = 72$ and 92. For $_{42}Mo_{52}$ the 2^+ level at 0.87 MeV leads to R = 6.14 fm. The values of 2^+ for $_{40}Zr_{52}$ and $_{40}Zr_{54}$ are 0.93 MeV and 0.916 MeV, respectively. These values lead to the unacceptably high values R = 8.30 fm and 8.37 fm when calculated for a moving dineutron, as expected for an $L + N$ structure for 40 protons. The values calculated for a moving helion, 5.94 fm and 5.99 fm, are acceptable. For $_{32}Ge_{40}$, the 2^+ level at 0.835 MeV similarly indicates a moving helion (R = 6.30 fm), rather than a moving diproton.

The striking rotational bands in the regions for A between 150 and 190 and greater than 220 have been interpreted as resulting from permanent prolate deformation. A reasonable interpretation of the values of the energy levels can be given in terms of moving clusters. For example, ${}_{66}Dy_{98}$ may be described as the large spherical doubly magic cluster ${}_{50}Sn_{82}$ plus eight helions on its surface, seven in a centered hexagonal cap and the eighth in a circular groove at the edge of this cap. Five observed bands with $K = 0^+, 2^+, (2^-), (6^-)$, and (2^+) lead to the respective values 8.2, 8.8, 8.6, 8.2, and 8.3 fm for the distance between the center of the large cluster and the center of the helions of the cap. The energy of the lowest level of the first excited band ($K = J = 2^+$ at 0.7618 MeV) leads to 5.67 fm for the cylindrical radius of the helion rotating in the groove. These values and values for other nuclei in the region of permanent prolate deformation correspond to reasonable values of the radii of the clusters. Thus ${}_{62}Sm_{90}$ might be taken as another example. It would be described as the essentially spherical cluster ${}_{54}Xe_{82}$ plus four helions on its surface. Both the band beginning with the normal state, $K = J = 0^+$, and that beginning with the excited state $K = J = 2^+$ (at 1.087 MeV) correspond to the distance 8.5 fm from the center of the large cluster to the center of the helions in the cap, in agreement with the values of the distance, 8.5 ± 0.3 fm, found for the eight-helion cap of ${}_{66}Dy_{98}$. The value of the cylindrical radius of the helion rotating in the groove is, however, calculated from the energy of the 2^+ state to be only 5.16 fm, considerably smaller than the value 5.67 fm for ${}_{66}Dy_{98}$. This decreased cylindrical radius reflects the difference in size of a three-helion and a seven-helion cap.

The foregoing discussion of excited states of nuclei is an alternative to the usual discussion of these states as vibrational states or as rotational states for ellipsoidally deformed nuclei. It has the feature of providing a simple explanation of values of the energy of many excited states with use of a single variable, the dimensions of the clusters.

[1] Pauling, L., *Science*, **150**, 297 (1965).