

Pattern recognition applied to presidential elections in the United States, 1860–1980: Role of integral social, economic, and political traits

(algorithm of recognition/parametrization of qualitative problem/dynamics of society)

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ABSTRACT The outcome of American presidential elections in 1860–1980 follows certain regular patterns which can be described phenomenologically by simple integral parameters of “common sense” type.

Despite a vast literature on voters and elections, there is no adequate model of the process that leads to victory for the winners in American presidential elections. In lieu of such a model, we use pattern recognition to disclose empirical regularities that may contribute to a better understanding of the electoral process. We hypothesize that the outcome of elections follows the dynamics of certain simple, integral parameters that depict social, economic, and political circumstance. We show that two distinct types of situations diagnosed by these parameters preceded elections won by incumbent and by challenging parties, respectively. In the statistical test of our conclusions we reject the simplest competing hypothesis—that the outcome of an election is independent of our diagnosis of the situation. We neither claim that other parameters cannot be used for the same purpose nor suggest methods for predicting future elections.

Traditional approaches to presidential elections emphasize either the party identification of voters or the impact of particular issues. Yet party loyalty remains highly stable over long periods of time whereas the role of issues is continually changing. To understand the outcome of elections we need to examine societal traits that are more dynamic than party identification and yet transcend the decisions made by individual voters, the movement of voter blocs, the unique issues of an election, and the stratagems of campaigners. If successful, our analysis would show that the process of retaining or rejecting an incumbent party follows regular patterns independently of the turbulence of particular campaigns.

Given the small number of presidential elections, the relatively large number of potentially informative traits, and the absence of guiding theory, pattern recognition is an appropriate procedure for this study. As a heuristic device for ordering and condensing information, pattern recognition can disclose relationships that otherwise would be concealed from view (1–3).

DATA

The objects of our analysis are American presidential elections from 1860 to 1980. Elections, identified by their year, are divided into two classes: *I*, the Incumbent party gains a plurality of the popular vote (whether or not the incumbent president was a candidate for reelection); and *C*, the Challenging party gains a plurality of the popular vote.

For two elections, 1876 and 1888, the tally of electoral votes reversed the preference expressed by the popular vote. Therefore, we tabulated 1876 as belonging to class *C* (reflecting the popular vote plurality for challenging party candidate Samuel

J. Tilden rather than the electoral vote victory for incumbent party candidate Rutherford D. Hayes) and we tabulated 1888 as belonging to class *I* (reflecting the popular vote plurality for incumbent party candidate Grover Cleveland rather than the electoral vote victory for challenging party candidate Benjamin Harrison). In classifying the elections of 1880 and 1892, the party that actually gained the presidency was considered the incumbent party.

Each election year is described by a set of answers to a questionnaire (Table 1). The questions are the same for each election year, but the answers vary according to the circumstances prevailing in given years. The questionnaire can be answered *prior* to the coming election; most questions can be answered definitively by the time both major parties have selected their nominees. The answers to some questions may be altered during the course of a campaign—for example, the occurrence of social unrest or scandal. All questions are to be answered at the lowest possible level of resolution: “yes” or “no.”

The answer to each question reflects diverse features of complex situations that may be both causes and symptoms of the prospects for incumbent and challenging parties. Social unrest, for instance, may indicate dissatisfaction with the status quo while itself becoming a reason for rejecting the incumbent party; the incumbent party in turn may attempt to exploit disorder to discredit the opposition and rally supporters.

Table 2 displays our data set, disclosing the answers to all 12 questions for each election year.

Answers to several of the questions, notably nos. 8–12, depend on judgments about historical circumstance that are frequently put forth by scholars but are not reduced to precise numerical criteria. By offering our full set of data, we enable readers to make judgments of their own about how to answer these questions and to compare the results obtained from the use of alternative responses with the findings we present.

ALGORITHM

The data in Table 2 constitute our “learning material.” As always in pattern recognition, our problem is to derive from this material a “rule of recognition” that can assign an election to class *I* or *C*, given the answers to the questionnaire for that particular election. If found, such a rule would disclose the “distinctive traits” that distinguish elections won by the incumbent party from those won by the challenging party.

Because the learning material is limited (18 elections of class *I* and 13 elections of class *C*), we sought to choose as simple an algorithm as possible for the derivation of a rule of recognition. We have chosen the algorithm “Cherning’s distance” suggested in ref. 4; a brief description of it follows.

Each election year is described by the binary vector $Y_i(X_1, X_2, \dots, X_n)$, where $X_i = 1$ or $X_i = 0$ represents the answer to the

Table 1. Questionnaire

| | |
|--|-------|
| 1. Has the incumbent party been in office more than a single term? | (no) |
| 2. Did the incumbent party gain more than 50% of the vote cast in the previous election?* | (yes) |
| 3. Was there major third party activity during the election year? | (no) |
| 4. Was there a serious contest for the nomination of the incumbent party candidate? | (no) |
| 5. Was the incumbent party candidate the sitting president? | (yes) |
| 6. Was the election year a time of recession or depression? | (no) |
| 7. Was the yearly mean per capita rate of growth in real gross national product during the incumbent administration equal to or greater than the mean rate in the previous 8 years and equal to or greater than 1%?† | (yes) |
| 8. Did the incumbent president initiate major changes in national policy? | (yes) |
| 9. Was there major social unrest in the nation during the incumbent administration? | (no) |
| 10. Was the incumbent administration tainted by major scandal? | (no) |
| 11. Is the incumbent party candidate charismatic or a national hero? | (yes) |
| 12. Is the challenging party candidate charismatic or a national hero? | (no) |

The answers in parentheses favor the victory of the incumbent party according to analysis of the whole data set in Table 2 (last kernel in Table 3)

* Rounded to the nearest percent.

† Prior to the 1890s, the available statistics are approximate.

*i*th question in the questionnaire. For each question, two numbers are computed that indicate the predominant values of X_i in class *I* and class *C*:

$$P(i/I) = n(i,I)/n(I), \text{ and } P(i/C) = n(i,C)/n(C).$$

Here, $n(i,I)$ is the number of elections in which $X_i = 1$ for class *I*, $n(i,C)$ is the number of elections in which $X_i = 1$ for class *C*, and $n(I)$ and $n(C)$ show how many elections *I* and *C* are included in the learning material.

These ratios are then used to form a "kernel" representing the distinctive traits (i.e., the set of preferential answers for victory by an incumbent candidate). The kernel is a binary vector (K_1, K_2, \dots, K_n) . $K_i = 1$ if $P(i/I) - P(i/C) \geq k$; $K_i = 0$ if $P(i/C) - P(i/I) \geq k$; otherwise, X_i is not used in the kernel.

The distance *D* between the kernel and a given election is defined, as

$$D \equiv \sum_i W_i \delta(K_i, X_i).$$

Here $\delta(K_i, X_i) = 1$ if $K_i \neq X_i$ (showing that the value of X_i for that election year differs from the value associated with victory for the incumbent) and $\delta(K_i, X_i) = 0$ when $K_i = X_i$. We assume here that $W_i = 1$, assigning equal weights to the answers to each question. Then *D* becomes the number of answers preferential for class *C*; it is Cheming's distance, and hence the name of the algorithm. Let us denote \overline{DI} , the maximal value of *D* for all preceding elections *I*, and \underline{DC} , the minimal value of *D* for all preceding elections *C*. We will recognize an election as

$$I \text{ if } D < \underline{DC} \text{ and } D \leq \overline{DI} \text{ and as}$$

$$C \text{ if } D > \overline{DI} \text{ and } D \geq \underline{DC}.$$

If neither of these conditions is satisfied, the recognition is indefinite.‡ We also assumed $k = 0.1$.

ANALYSIS

The last two lines of Table 2 show the values of $P(i/I)$ and $P(i/C)$ for all the learning material. The corresponding kernel is displayed in the last line of Table 3. Table 4 gives the value of *D* for each election. Several aspects of these results merit discussion.

(i) Taken individually, the answers to none of the questions can accurately divide elections into those won by incumbent and by challenging parties. This can be seen from Table 2. One question (Was there a serious contest for the nomination of the incumbent party) emerges as by far the most powerful discriminator. The answer to this question identifies incorrectly only one of the incumbent victories and three of the challenger victories. The special relevance of this question suggests that nomination contests within the incumbent party are both sympto-

‡ We also used the algorithm CORA-3 (1-3) to generate, from the learning material in Table 2, multiple distinctive traits, representing combinations of answers to several questions. Such traits can provide additional insight into the nature of the difference between elections *I* and *C*, but with limited learning material the multiple traits are especially susceptible to the influence of random, spurious combinations of the answers. Multiple traits, however, showed no significant additions to the results generated by the algorithm "Cheming's distance." With a broader set of questions, multiple distinctive traits may be more revealing.

Table 2. Answers to questions in Table 1 (1 = yes, 0 = no)

| Election year | Question number in Table 1 | | | | | | | | | | | |
|----------------------|----------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Incumbent victory | | | | | | | | | | | | |
| 1864 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1868 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1872 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1880 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1888 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1900 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1904 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1908 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1916 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1924 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1928 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1936 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1940 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1944 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1948 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1956 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1964 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1972 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| Challenger victories | | | | | | | | | | | | |
| 1860 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1876 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1884 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 1892 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1896 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1912 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1920 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1932 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1952 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 1960 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1968 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1976 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1980 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| $P(i/C)$ | 0.769 | 0.462 | 0.385 | 0.769 | 0.385 | 0.462 | 0.385 | 0.308 | 0.538 | 0.308 | 0.154 | 0.385 |
| $P(i/I)$ | 0.500 | 0.667 | 0.111 | 0.056 | 0.778 | 0.167 | 0.667 | 0.611 | 0.167 | 0.056 | 0.389 | 0.056 |

Table 3. Kernels based on information accumulated through consecutive election from 1860 to T

| Year T | Question number | | | | | | | | | | | |
|-----------|-----------------|---|---|---|---|---|---|---|---|----|----|----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1864 | 0 | * | 0 | 0 | 1 | * | 0 | 1 | * | * | * | * |
| 1868 | 0 | 1 | 0 | 0 | 1 | * | 0 | 1 | * | * | 1 | * |
| 1872 | 0 | 1 | 0 | 0 | 1 | * | 0 | 1 | 0 | * | 1 | * |
| 1876 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | * |
| 1880 | 0 | * | 0 | 0 | 1 | 0 | 1 | 1 | * | 0 | 1 | * |
| 1884 | 0 | 1 | 0 | 0 | 1 | 0 | * | 1 | 0 | 0 | 1 | * |
| 1888 | 0 | * | 0 | 0 | 1 | 0 | * | 1 | 0 | 0 | * | * |
| 1892 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1896 | * | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | * | 0 |
| 1900 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | * | 0 | 0 | * | * |
| 1904 | * | 1 | 0 | 0 | 1 | 0 | 1 | * | 0 | 0 | * | * |
| 1908 | * | 1 | 0 | 0 | 1 | 0 | 1 | * | 0 | 0 | * | * |
| 1912 | * | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | * | * |
| 1916 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | * | * |
| 1920 | 0 | 1 | 0 | 0 | 1 | 0 | * | 1 | 0 | 0 | * | * |
| 1924 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | * | * | * |
| 1928 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | * | * | * |
| 1932 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | * | * | * |
| 1936 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | * | * | 0 |
| 1940 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | * | 1 | 0 |
| 1944 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | * | 1 | 0 |
| 1948 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | * | 1 | 0 |
| 1952 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1956 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1960 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1964 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1968 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1972 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1976 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1980* | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |

$\hat{K}i = 1$ if $P(i/D) - P(i/C) \geq 0.1$; $\hat{K}i = 0$ if $P(i/C) - P(i/D) \geq 0.1$; $\hat{K}i = *$ otherwise.

* This kernel is repeated in Table 1, as the answers given in parenthesis.

matic of a lack of confidence in the incumbent administration (especially when an incumbent president is seeking renomination) and productive of potentially damaging disunity within the ruling party. Since 1860, only one incumbent party candidate—James A. Garfield in 1880—survived a serious contest for the presidential nomination.

Taken together, the answers to our questionnaire divide the elections into classes I and C as shown in Table 4. Elections of class I have distance $D \leq 5$ and elections of class C have distance $D \geq 5$. Three elections attain the indeterminate distance of $D = 5$; two of them belong to class I and one, the election of 1912, belongs to class C. This overlap has a clear explanation which is offered not to disspell an inconvenient result but to show that certain factors may be of sufficient weight in particular elections to override other considerations. The presidential election of 1912 is considered to be one of the most unusual in the nation's history. Dissatisfied with the policies of his hand-picked successor William Howard Taft, former Republican president Theodore Roosevelt sought to wrest the 1912 nomination from Taft and again become the Republican contender. Failing in this effort, he organized a third-party movement that split the Republican vote and handed the election to Democratic nominee

Table 4. Distances for presidential elections computed from the kernel for 1860–1980 (last line in Table 3)

| | | 1964I | 1972I | | | 1976C | | | 1980C | |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----|
| | | 1924I | 1948I | | | 1968C | | | 1920C | |
| 1956I | 1940I | 1916I | 1928I | 1912C | 1896C | | | 1920C | | |
| 1944I | 1904I | 1900I | 1888I | 1908I | 1892C | 1960C | 1876C | | | |
| 1936I | 1872I | 1868I | 1864I | 1880I | 1884C | 1932C | 1860C | 1952C | | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Distance D | | | | | | | | | | |

Table 5. Test on mutually independent data

| Kernel and DC, DI for | Elections of | Identifications | | |
|-----------------------------|-----------------|----------------------------------|-------|---------|
| | | Indeterminate | Wrong | Correct |
| 1860–1920 DC = 3; DI = 4 | 1924–1980 | 4(1928I, 1932C, 1960C, 1976C) | 0 | 12 |
| 1924–1980 DC = 6; DI = 4 | 1860–1920 | 3(1908I, 1880I, 1912C) | 0 | 12 |

First kernel (1860–1920) is given in the line 1920 of Table 3; second kernel (1860–1920) is the same as in the last line of Table 3. The only difference between these kernels is that in the first kernel, three traits (nos. 7, 11, and 12) are eliminated.

Woodrow Wilson. Together, Taft and Roosevelt polled 51% of the vote, compared with 42% for Wilson.[§]

(ii) The definition of the kernel may predetermine and enhance the separation of elections I and C on the axis D (bootstrap effect). The simplest hypothesis, which may compete with our conclusions, is that our recognition is irrelevant to the outcome of elections. To test this hypothesis, we included in the learning material only the 16 elections from 1860 to 1920 and determined the corresponding kernel. Then we used this kernel to identify the 15 elections from 1924 to 1980. Also we reversed the procedure: determined the kernel for the elections of 1924–1980 and used this kernel to identify the elections of 1860–1920. The results of this experiment are shown in Table 5. They are satisfactory: the competing hypothesis can be rejected at the level above 99.9%. A more refined competing hypothesis would be to assume that the probabilities of elections I and C are related, as $n(I):n(C)$ (9:7 in 1860–1920; 9:6 in 1924–1980). This hypothesis is also rejected at the level above 99.9%.

As an additional test, we compared the differences $P(i/I) - P(i/C)$ for the two halves of our time period (1860–1920 and 1924–1980). For all items in the questionnaire, the sign of this difference for 1860–1920 is the same as for 1924–1980. Accordingly, the competing hypothesis—that the learning material is a realization of a random binomial process, independent of the outcome of election—is rejected at the level above 99.9%.

(iii) The kernel for discriminating elections of classes I and C proved to be highly stable throughout the time period covered by our analysis. This is clear from Table 3 which displays the consecutive kernels created first by using learning material only from 1860 and 1864 and then adding learning material from each succeeding election, one by one, until 1980 is reached. Table 3 reveals not a single change after 1880 in the values of the distinctive traits. The only change from year to year is that different questions are eliminated. No question, however, is eliminated after 1952.

The final values of the kernel confirm what might be expected from a knowledge of American politics. *The full set of favorable circumstances for continuity in party control of the White House has not changed significantly in the past 100 years.*

(iv) To test further the stability of our recognition rule, we performed an experiment similar to the experiment “earthquake’s history” described in ref. 3. Beginning with 1896, we successively used each of the kernels displayed in Table 3 to compute the distribution of D for all elections included into the learning material for the formation of the kernel. We then used the same kernel to determine the value of D for the upcoming election (not included in learning material) and to project its outcome. This experiment simulates the situation of an observer

[§] Lichtman, A. J. and Lord, J. B., III (1979) *Party Loyalty and Progressive Politics: Quantitative Analysis of the Vote for President in 1912*, Annual Convention, Organization of American Historians, New York, April 1979.

Table 6. Separation of elections *I* and *C* and projections of upcoming elections (based on data from 1860 to *T*, inclusive)

| T | Actual outcome* | | | | | | | | | | | | | | | | | | | | Projection for next | | | |
|------|-----------------|---|---|---|---|---|---|---|---|---|-----------------|---|---|---|---|---|---|---|---|---|---------------------|---|--------------------|--------------------|
| | Incumbent wins | | | | | | | | | | Challenger wins | | | | | | | | | | | | | |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | 9 | 10 | |
| | 1900 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1896 | - | - | - | - | - | 2 | 3 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | C | | |
| | 1904 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1900 | 1 | 1 | 1 | 2 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | <i>I</i> (correct) | |
| | 1908 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1904 | 3 | - | 2 | 2 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | <i>I</i> (correct) | |
| | 1912 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1908 | | | | | | | | | | | | | | | | | | | | | | | C | |
| | | | | | | | | | | | | | | | | | | | | | | | I | <i>I</i> (wrong) |
| | 1916 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1912 | - | 3 | 2 | 3 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | <i>I</i> (correct) |
| | 1920 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1916 | | | | | | | | | | | | | | | | | | | | | | | C | |
| | | | | | | | | | | | | | | | | | | | | | | | I | <i>C</i> (correct) |
| | 1924 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1920 | - | 2 | 4 | 2 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | <i>I</i> (correct) |
| | 1928 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1924 | - | 2 | 3 | 3 | 2 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | Indeterminate |
| | 1932 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1928 | | | | | | | | | | | | | | | | | | | | | | | C | |
| | | | | | | | | | | | | | | | | | | | | | | | I | <i>C</i> (correct) |
| | 1936 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1932 | - | 2 | 3 | 4 | 2 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | <i>I</i> (correct) |
| | 1940 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1936 | - | 2 | 4 | 5 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | <i>I</i> (correct) |
| | 1944 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1940 | - | 1 | 4 | 3 | 4 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | <i>I</i> (correct) |
| | 1948 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1944 | - | 2 | 4 | 3 | 3 | 2 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | <i>I</i> (correct) |
| | 1952 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1948 | | | | | | | | | | | | | | | | | | | | | | | C | |
| | | | | | | | | | | | | | | | | | | | | | | | I | <i>C</i> (correct) |
| | 1956 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1952 | - | 2 | 3 | 4 | 4 | 2 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | I | <i>I</i> (correct) |
| | 1960 | | | | | | | | | | | | | | | | | | | | | | | |
| | ↓ | | | | | | | | | | | | | | | | | | | | | | | |
| 1956 | | | | | | | | | | | | | | | | | | | | | | | C | |
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