Anomalous heat transport and condensation in convection of cryogenic helium

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When a hot body A is thermally connected to a cold body B, the textbook knowledge is that heat flows from A to B. Here, we describe the opposite case in which heat flows from a colder but constantly heated body B to a hotter but constantly cooled body A through a two-phase liquid–vapor system. Specifically, we provide experimental evidence that heat flows through liquid and vapor phases of cryogenic helium from the constantly heated, but cooler, bottom plate of a Rayleigh–Bénard convection cell to its hotter, but constantly cooled, top plate. The bottom plate is heated uniformly, and the top plate is cooled by heat exchange with liquid helium maintained at 4.2 K. Additionally, for certain experimental conditions, a rain of helium droplets is detected by small sensors placed in the cell at about one-half of its height.

O ur cryogenic experiment takes place in a cryostat containing a Rayleigh–Bénard (RB) convection cell shown in Fig. 1. The cylindrical RB cell, 300 mm in both diameter and height, is designed to minimize the influence of its structure on the convective flow (1) and is capable of running at very high Rayleigh numbers up to $10^{15}$ (2, 3). These studies used cryogenic helium gas as the working fluid and have been performed under nearly Oberbeck–Boussinesq conditions (all physical properties of working fluid assumed constant except its density varying linearly with temperature) (2), as well as for the case when non–Oberbeck–Boussinesq conditions cause asymmetry between the top and bottom boundary layers (3). Here, we report results of experiments on two-phase heat transport, using cryogenic helium vapor and normal liquid $^4$He as working fluids with remarkable, well-known and in situ tunable properties (4, 5). We stress that the heat conductivity of the thin stainless-steel cylindrical wall and any possible parasitic heat input to the RB cell are negligibly small in this work.

We start very near equilibrium conditions, with the RB cell filled one-half with normal liquid helium and one-half with helium vapor. The temperature of the cell is approximately that of the thermally connected liquid helium vessel (LHeV), as shown for time $t<0$ in Fig. 2A, before the homogeneously distributed resistive heater in the bottom plate is turned on at $t=0$. This condition closely corresponds to a point on the equilibrium saturated vapor curve (SVC), calculated based on the continuously monitored value of pressure, $P$, in the cell. Then we start heating the bottom plate with a constant input power into the resistive heater, and, using built-in germanium thermometers (1, 2), continuously monitor the temperatures $T_B$ and $T_T$, of the highly conductive bottom and top copper plates, respectively, as well as the temperature readings $T_1$ ... $T_4$ of four small Ge sensors (6) installed within the cell as shown in Fig. 1. The temperature of the upper plate is not controlled; it is only affected by heat exchange with the helium inside the cell and by the weak thermal link to the LHeV. For an isolated system consisting of liquid and vapor in equilibrium, one would expect that slow heating would result in partial evaporation of the liquid and uniformly rising temperature of the entire system along SVC.

The experimental data in Fig. 2A show, however, that this is not the case. Upon switching on the bottom plate heater (the heat flux $Q_B=2$ W in this particular case), the bottom plate temperature $T_B$ rises quickly with respect to the temperature of the liquid, $T_L$, as determined by $T_2$ and $T_4$ (sensors immersed in the liquid), by $\Delta T_{BL} = T_B - T_L \approx 60$ mK. This overheating of the bottom plate relative to the liquid, which would take much longer to heat up, results in nucleate boiling on the surface of the plate. From this point on, all subsystems in the cell heat up gradually.

Although the submerged Ge sensors $T_2$ and $T_4$ seem to follow the equilibrium saturation temperature $T_{sat}$ calculated from measurements of pressure inside the cell using the data in refs. 4 and 5 closely, the overheat of the bottom plate $\Delta T_{BL}$ slowly decreases to its final value $\approx 50$ mK, before the heater is switched off. Contrary to expectations based on liquid–vapor equilibrium, $T_1$, $T_3$, and $T_7$ do not stay close to, or remain below, $T_2$ and $T_4$, but, in fact, rise faster than $T_B$. With $T_1$ and $T_7$ crossing $T_B$, the vapor becomes the hottest subsystem and, soon, even the cooled top plate becomes hotter than the heated bottom one, resulting in an apparent inversion of the heat flow. The behavior that occurs after the heater is switched off at the end of the yellow shaded area in Fig. 2 will be discussed subsequently.

Because there is no other external source of heating in the system except the bottom plate heater, this situation at first sight appears to contradict the second law of thermodynamics, particularly Clausius’ formulation (7): “No process is possible whose sole result is the absorption of heat from a body of lower temperature to a body of higher temperature.” Indeed, the heater supplies heat to the bottom plate that in turn heats the liquid in the lower part of the cell—partly raising its temperature, partly evaporating it—and subsequently heating the vapor in the upper part of the cell. This complex process is the key to understanding why heat flows from the bottom plate at temperature $T_B$ to the top plate even when $T_T > T_B$.

Before demonstrating that the described heat transfer does not contradict the second law of thermodynamics, and that it is possible to simulate the observed processes numerically (see Fig. 2B and C, which will be explained subsequently), let us consider nominally the same experiment (Fig. 3), but starting with a smaller amount of liquid in the cell. Upon switching on the bottom plate heater ($Q_B=1$ W), $T_B$ quickly jumps up, this time by about $\Delta T_{BL} \approx 50$ mK, and then keeps rising slowly, more or less parallel with the calculated SVC. The temperatures $T_1$ ... $T_4$ grow faster than $T_B$, but all of the small sensors are now contained within the vapor and their readings are thus almost identical. Before long, $T_1$ ... $T_4$ cross $T_B$ (in the vicinity of point C in Fig. 3A), and, again, the vapor displays the highest detected temperature in the system. $T_T$ also grows, albeit more slowly than $T_B$.

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Fig. 1. (Left) Three-dimensional view of the cryogenic insert of our RB convection apparatus (1). The cylindrical RB cell (1), thermally attached via a heat exchange chamber (2) to the liquid helium vessel (LHeV) (3) above it, is placed in high cryogenic vacuum, inside a radiation shield thermally anchored to LHeV. (Right) A schematic of the RB cell heated from below and cooled from above, showing the placement of four small Ge sensors in its interior.

The temperature profile of the top plate at temperature \( T_B \) is to the bottom plate at temperature \( T_T \) and to the top plate at \( T_T > T_B \). This behavior holds up to point E in Fig. 3A when all of the liquid evaporates, resulting in a very steep increase in \( T_B \) that quickly crosses both \( T_T \) and \( T_1 \ldots T_4 \), restoring the “normal” temperature profile of the high-Rayleigh number single-phase turbulent convection: the temperature decreases monotonically with height from the highest temperature \( T_B \) of the bottom plate to the smallest temperature \( T_T \) of the top plate.

Upon switching the bottom plate heater off (Fig. 3A, point O), the bottom plate quickly acquires the temperature of the helium vapor in the cell interior, and \( T_B \) as well as \( T_1 \ldots T_4 \) decrease, more or less similarly to the top plate that is cooled via the thermal link to the LHeV. This cooling continues monotonically until \( T_T \) reaches the SVC and closely follows it. As shown in detail in Fig. 3A, Inset, soon after this point, the character of \( T_1 \ldots T_4 \) changes significantly: the record starts to display short downward spikes in temperature. Importantly, none of these spikes reach below \( T_T \) so that the top plate stays the coldest part of the system. These uncorrelated spikes of temperature detected by small sensors serve as the evidence of rain of helium in the cell: helium vapor gets cooled by the top plate, condenses into droplets that fall downward and hit the sensors and/or electrical leads in their vicinity. The condensation of helium at the top plate prevents \( T_T \) from dropping appreciably below the SVC. Helium droplets fall down to the bottom plate, which quickly acquires the temperature close to \( T_T \) and thus close to SVC.

As it is evident from Fig. 3B, the rain in the cell can occur in both the heating and cooling phases. In this particular run, less power \( (Q_B = 0.5 \text{ W}) \) was applied to the bottom heater, so that the cooling link to LHeV was strong enough to sustain \( T_T \) at \( T_{sat} \) for a longer period. The temperature inversion occurs only for a short time, and soon, even upon heating, we have \( T_B > T_2 \approx T_2 > T_3 > T_4 > T_T \approx T_{sat} \). Note that there is a clear evidence of rain

![Fig. 2](image-url)

**Fig. 2.** (A) Recorded time traces of pressure, \( P \), in the cell and temperatures of the bottom plate, \( T_B \), top plate, \( T_T \), and of four small sensors (color-coded lines as indicated); \( T_1 \) and \( T_2 \) are in the liquid, \( T_3 \) and \( T_4 \) are in the vapor. Except for the time interval marked as “Rain,” the readings of each pair overlap within the resolution used in this plot. The thick broken line is the saturated temperature \( T_{sat} \) calculated from the measured pressure in the cell. The shaded area indicates the time interval during which the heater was turned on with a power of 2 W. The average density of helium in the cell was 72 kg/m\(^3\). (B) Schematic illustration of heat flows (red arrows) and mass flows (blue arrows) as considered in the text. The heat supplied to the bottom plate must be either stored in the system or leave through the top plate into LHeV. Inside the cell, heat is exchanged between the two plates, liquid and vapor, either by means of direct contact at interfaces, or via phase transitions—nucleate boiling at the lower plate, evaporation/condensation at the liquid level, or condensation at the top plate. These phase transitions are indicated by pairs of red and blue arrows, as they include both heat and mass currents, respectively. The arrow thickness is indicative of the relative magnitudes of heat flows. (C) Computer simulation of this experiment using the outlined model; the meaning of the lines and shaded area is the same as in A. There is a remarkable conformity with experiments despite the shortcomings of the model discussed and explained in the text. Condensation on the upper plate is indeed reproduced in the simulation after \( T_T \) relaxes to \( T_{sat} \), thus confirming our interpretation of the downward spikes in temperature traces as an effect of rain droplets hitting the thermometers.
even past the steep increase in $T_B$, marking the time when the liquid layer on the bottom plate disappears, but $T_T \leq T_{\text{sat}}$ for that period.

To understand the underlying thermodynamical processes in our RB cell, it is useful to consider recent experimental study by Zhong et al. (8), who observed an enhanced heat transport by two-phase turbulent thermal convection when the applied temperature difference between top and bottom plates in their RB cell straddled the (ethane) liquid–vapor coexistence curve. The authors state that enhanced heat transport efficiency takes place thanks to droplet condensation at the top of the RB cell, and, indeed, such a droplet formation was observed in their shadow graphs. This explanation applies to our experiment when there is no liquid layer on the bottom plate (Fig. 3B) and the small sensors clearly indicate rain in the cell. Liquid helium droplets are formed at the top of the RB cell, but they either do not reach the bottom plate or evaporate upon reaching it; indeed, we have ample experimental evidence that the heat transport efficiency is enhanced, in qualitative agreement with ref. 8.

Even more intriguing is the inversion of the plate’s temperatures, which formally corresponds to negative Rayleigh numbers. How could one understand that heat is transferred from the (heated) bottom plate at temperature $T_B$ to the (cooled) top plate at temperature $T_T$? It is clear that we are dealing with a nonequilibrium and unsteady system in which not all of the heat supplied, $Q_B$, passes the RB cell and comes out as $Q_T$: it is partly spent as latent heat to evaporate the liquid and to heat up and pressurize the cell interior. As illustrated by Fig. 3, we are dealing with a “one-shot” regime here—the observed anomalous heat transfer lasts only until the liquid layer in the cell disappears: this could happen, depending on the starting mean density of helium in the RB cell, either by evaporation or upon exceeding the critical temperature of about 5.195 K.

The observed behavior can be broadly explained by means of a phenomenological model that assumes that heat from the bottom plate is partly absorbed by the liquid layer above it and partly carried to the vapor phase directly, increasing its density and pressure. The latter is accomplished by means of vapor bubbles most likely created at surface defects of the bottom plate (1). According to the literature (9, 10), nucleate boiling should indeed occur under our experimental conditions at the heated bottom plate. An empirical relation between $\Delta T_{\text{BL}}$ and the heat flux due to boiling $Q_{\text{boil}}$, based on refs. 9 and 10, can be used to calculate $Q_{\text{boil}}$ and from it also the boiling rate. This relation is taken in the form $Q_{\text{boil}} = c \Delta T_{\text{BL}}^n$, where $c$ is an adjustable coefficient and $n$ is taken typically between 1 and 3 (in our case $n = 2.5$ was used, to fit the experimental data of refs. 9 and 10).

To describe the relevant processes occurring in the RB cell in the simplest possible manner, we further assume mechanical equilibrium in the entire RB cell, i.e., uniform pressure, neglecting its hydrostatic changes due to gravity. Thermal equilibria (well-defined uniform temperatures) are assumed for each subsystem (bottom and top plates, liquid and vapor), requiring separation of timescales of heat transfer within each subsystem compared with across their boundaries. Thus, in the model, temperature gradients occur exclusively at the boundaries of subsystems. Heat transfer by conduction/convection across each boundary XY, including heat lost from the top plate to LHeV, $Q_T$, is modeled simply as $Q_{XY} = \rho h_{XY} S(T_Y - T_X)$, where $S$ denotes the boundary surface area, $T_X$ and $T_Y$ stand for the respective temperatures on either side of the XY boundary, and $h_{XY}$ are parameters of the model.

Additional heat transfer and all mass flow within the system are associated with phase transitions—nucleate boiling at the lower plate, condensation/evaporation at the liquid level, condensation of droplets on the upper plate. All of these phase transitions are modeled empirically, bearing in mind conservation of energy. Although the nucleate boiling rate at the lower plate is determined by the empirical relation between $\Delta T_{\text{BL}}$ and $Q_{\text{boil}}$ mentioned above, the condensation/evaporation rates are approximated by a linear dependence on $T_{\text{sat}} - T_S$, where $T_S$ is the temperature of the surface in question. All He properties (temperature-pressure-

![Fig. 3.](image)

Recorded values of $T_1$, $T_2$, $T_3$, $T_4$, and $P$ (color-coded as indicated) plotted versus time in two experimental runs A and B with the same average density of helium in the RB cell equal to 25.7 kg m$^{-3}$. The thick broken lines are the calculated saturated temperatures $T_{\text{sat}}$. The yellow shaded areas indicate the duration over which the bottom heater (power as indicated in the figure) was switched on. The short uncorrelated downward departures in temperature readings $T_1 \ldots T_4$ indicate rain in the RB cell. Insets show the observed rain of helium droplets in greater detail.
dependent densities, enthalpies, heat capacities of He liquid and vapor, latent heat) are derived from refs. 4 and 5 and improved for numerical processing using a specifically devised smoothing algorithm based on 2D polynomials, with weighting in favor of data closer to SVC, which are more relevant to this experiment. Cell geometry and $Q_B$ match the experimental conditions, and a temperature-dependent heat capacity of copper is used, based on the approximative polynomial correlation given in ref. 11. Numerical simulations, in particular those shown in Fig. 2C, closely reproduce the experimental data, justifying the model assumptions.

The most significant shortcomings of the model in terms of comparison with the experiment can be seen in the fact that it does not reproduce the overshoot in $T_B$ due to hysteresis in the onset of nucleate boiling (9), and that it results in $\Delta T_{BL}$ approximately constant throughout the heat-up, whereas in the experiment shown in Fig. 2A, the difference $\Delta T_{BL}$ decreases slowly from $\approx 60$ to $\approx 50$ mK before the heating is switched off. The quantity $\Delta T_{BL}$ is determined mainly by the ratio between the heat supplied by the bottom plate to the liquid directly and the heat used for boiling. In the temperature range between 4.2 and 4.9 K, the proportionality coefficient $h$ in the conductive heat flux between the Cu plate and He liquid will change appreciably due to changes in the thermal (Kapitza) resistance of the boundary, which is not captured by the model ($h = const.$). Similarly, the dependence of the boiling rate on $\Delta T_{BL}$ may change as well, because the overheated layer of He liquid in the immediate vicinity of the plate will have different properties (mainly surface tension) at higher $T_L$. Nevertheless, none of these shortcomings has any appreciable impact on the description of temperature inversion and rain formation in the RB cell, which are reproduced clearly.

As a result of the model calculations, it became evident that, in order for the temperature inversion to occur, it is essential that the coefficients $h_{XY}$ for the interfaces lower plate–liquid and liquid–vapor are comparatively low, resulting in a dominant flow of heat from the lower plate via boiling to vapor and then mainly to the top plate. Recondensation of vapor at the liquid level may represent a significant heat flux as well, but does not impede temperature inversion because of its associated mass flow. The processes that inhibit temperature inversion are all characterized by heat flowing to the liquid without phase transitions occurring. In the region characterized by condensation on the upper plate and rain formation, the model clearly indicates evaporation occurring simultaneously at the liquid level, reminiscent of the Earth’s water cycle, with the caveat that this system consists of helium only, unlike water vapor being one of the many constituents of air.

To conclude, we have provided convincing experimental evidence that heat can flow from a cooler but constantly heated body to a thermally connected and constantly cooled hotter body, and developed a phenomenological model that describes this process quantitatively, without any conflict with the second law of thermodynamics. The observed anomalous heat transfer and the rain formation in our RB cell occur because of evaporation and condensation processes in two-phase working fluid. These observations are not limited to cryogenic helium; however, the conveniently tuneable and well-known properties of cryogenic helium (4, 5) allow performing these experiments (characterized by high Rayleigh numbers) under well-controlled laboratory conditions. Moreover, cryogenic conditions allow the experiments to be performed using extremely clean working fluids with all possible impurities frozen out, and effectively excluding any parasitic heat fluxes. We hope that our work will stimulate further effort toward understanding similar complex and intriguing thermodynamical processes.

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