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Supplementary Information for

Aversion to Ambiguity and Model Misspecification in Dynamic Stochastic Environments

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Supplementary text

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Supporting Information Text

A. Problem 2.1. As noted in section 1, there is an equivalent way to represent the relative entropy Eq. (1) in terms of posteriors and predictive densities:

$$D\left(\mathbf{p} \mid \widehat{\mathbf{p}}\right) = \int_{\mathcal{Y}} \int_{\Theta} \log\left[\frac{d\pi^{+}}{d\hat{\pi}^{+}}(\theta \mid y)\right] \pi^{+}(d\theta \mid y)\phi(y)\tau(dy) + \int_{\mathcal{Y}} \log\left[\frac{\phi(y)}{\hat{\phi}(y)}\right] \phi(y)\tau(dy).$$

Provided that priors π and $\hat{\pi}$ are absolutely continuous, the same can be said of posteriors. Since the objective of Problem 2.1 can be expressed in terms of the predictive density, it follows from this entropy decomposition that minimization will distort only this density and not the posterior distribution.

B. Section 4 derivations. In section 4 we asserted that

$$\log B_t = \log Y_t + (1-t)m_t + \mathsf{b}_t,$$

where

$$\mathsf{b}_{t+\epsilon} - \mathsf{b}_t = \frac{\epsilon^2}{2\kappa(\epsilon)} q_t \left(\frac{(1-t)q_t + |\varsigma|^2}{|\varsigma|^2 + \epsilon q_t}\right)^2 + \frac{\epsilon|\varsigma|^2}{2}$$

and where $\mathsf{b}_1 = 0$. To derive this recursive formula, note that

$$E\left(\log B_{t+\epsilon} \mid \mathcal{F}_t^{\epsilon}\right) = \log Y_t + (1-t)m_t + \epsilon\left[\frac{(1-t)q_t + |\varsigma|^2}{|\varsigma|^2 + \epsilon q_t}\right](\theta - m_t) + \mathsf{b}_{t+\epsilon} - \frac{\epsilon|\varsigma|^2}{2}.$$

Now apply

$$\mathbb{B}_{\epsilon,t}\left[E\left(\log B_{t+\epsilon} \mid \mathcal{F}_t^{\epsilon}\right)\right] = \log Y_t + (1-t)m_t + \mathsf{b}_{t+\epsilon} - \frac{\epsilon|\varsigma|^2}{2} - \frac{\epsilon^2}{2\kappa(\epsilon)} q_t \left(\frac{(1-t)q_t + |\varsigma|^2}{|\varsigma|^2 + \epsilon q_t}\right)^2.$$

The recursion follows from our formula for $\log B_t$.

In section 4 we also claimed that

$$\log A_t = \log Y_t + (1-t)m_t + \mathsf{a}_t$$

where

$$\mathsf{a}_{t+\epsilon} - \mathsf{a}_t = \frac{1}{2}\left(\frac{|\varsigma|^2 + (1-t)q_t}{|\varsigma|^2 + \epsilon q_t}\right)^2 \left[\epsilon(\gamma-1)|\varsigma|^2 + \alpha(\epsilon)\epsilon^2 q_t\right] + \frac{\epsilon|\varsigma|^2}{2}$$

and $\mathsf{a}_1 = 0$. To deduce this recursion, we first compute

$$\begin{aligned} \log \mathbb{R}_{\epsilon,t}\left(A_{t+\epsilon}\right) &= \frac{1}{1-\gamma}\log E\left[\left(A_{t+\epsilon}\right)^{1-\gamma} | \mathcal{F}_t^{\epsilon}\right] \\ &= \log Y_t + (1-t)m_t + \epsilon\left[\frac{|\varsigma|^2 + (1-t)q_t}{|\varsigma|^2 + \epsilon q_t}\right](\theta - m_t) \\ &\quad + \frac{\epsilon(1-\gamma)}{2}\left(\frac{|\varsigma|^2 + (1-t)q_t}{|\varsigma|^2 + \epsilon q_t}\right)^2 |\varsigma|^2 + \mathsf{a}_{t+\epsilon} - \frac{\epsilon|\varsigma|^2}{2}. \end{aligned}$$

Applying the operator $\mathbb{A}_{\epsilon,t}$ and the recursive equation Eq. (9), the recursion follows.