Appendix B. Derivation of Endothelial Surface Layer (ESL) Compression Due to Red Cell Arrest

1. Governing Equations

The draining of fluid from the ESL due to red cell arrest is modeled in the following figure.

In the above figure, $v_y(y)$ is the local velocity of fluid in the ESL; $W$ is the width of red cell; $P_c$ is compression pressure of the red cell membrane on the ESL; and $L_f$ is the height of the ESL. For a flexible membrane, $L_f$ is a function of both $y$ and $t$ [see Wu and Weinbaum (1)]. However, for present purposes, where we are primarily interested in the characteristic time for the fluid drainage, we treat $L_f$ as a rigid planar surface. In the absence of membrane curvature, $P_c$ is equal to the internal cell pressure. As described in the text, the process is governed by Darcy’s law,

$$\frac{\partial p}{\partial y} = -\frac{\mu}{K_p} v_y, \quad [B1]$$

and continuity,
\[
y \frac{dL_f}{dt} = -v_y L_f. \tag{B2}
\]

Combining the above two equations, one obtains the pressure distribution beneath the red cell membrane,

\[
p - p_0 = \frac{\mu}{8K_p} \left( -\frac{dL_f}{dt} \right) \left( \frac{W^2 - 4y^2}{L_f} \right), \tag{B3}
\]

where \( p_0 \) is the ambient pressure at the edge of the compression zone. The average compression pressure on the ESL must equal the cell pressure. Thus,

\[
P_c = \frac{2}{W} \int_{\frac{W}{2}}^{\frac{W}{2}} (p - p_0) dy = \frac{\mu W^2}{12K_p L_f} \left( -\frac{dL_f}{dt} \right). \tag{B4}
\]

Assuming \( P_c \) and \( W \) are constant, one obtains

\[
t = \frac{\mu W^2}{12P_c} \int_{\frac{d}{2}}^{\frac{d}{2}} \frac{dL_f}{K_p L_f}, \tag{B5}
\]

where \( L_{f0} \) is the initial ESL thickness. If \( K_p \) is constant, then

\[
L_f / L_{f0} = \exp(-t/\tau), \tag{B6a}
\]

with

\[
\tau = \frac{\mu W^2}{12P_c K_p}. \tag{B6b}
\]
However, for the large compressions of the ESL considered herein, $K_p$ is a function of the instantaneous solid fraction $c$.

2. Estimation of $K_p$

In ref. 2, the expression for the drag force on a single spherical scattering center along the core protein is

$$F = 6\pi K \mu r \nu,$$ 

[B7]

where $r$ is the radius of the scattering center and $K$ is the drag force coefficient. Sangani and Acrivos (2) showed that for a face-centered cubic array of spheres,

$$K = \sum_{s=0}^{30} q_s \left( \frac{c}{c_{\text{max}}} \right)^{1/3},$$ 

[B8]

where $c_{\text{max}} = 0.74$ is the maximum solid fraction for the face-centered array, and the $q_s$ are coefficients given in ref. 2. From Darcy’s law, one can show that $K_p$ is related to $K$ by

$$K_p = \frac{cK^{-1}}{6\pi r},$$ 

[B9]

where $\varepsilon = \frac{4\pi r^3}{3c}$.

3. Estimation of other parameters

To solve the above equations, we have to estimate two other parameters: the solid fraction $c$ and the width $W$ of the red cell compression along the ESL. Mass conservation within the solid phase requires that
where \( c_0 = 0.13 \) for the initial array of spherical scattering centers depicted in Fig.1. For a red cell volume of 90 \( \mu m^3 \), a capillary diameter of 5 \( \mu m \) and \( L_f = 0.4 \mu m \), \( W = 4.6 \mu m \) initially and increases to 6.5 \( \mu m \) after maximum crushing of the ESL if the assumed shape is that of a circular cylindrical pellet. In our calculation, we let \( W \) be the mean of these two values.

4. Calculation

a. Constant \( K_p \)

Substituting \( t = 0.5 \) s into Eq. B6a,

\[
\exp(-t/\tau) = \frac{L_{f,\min}}{L_{f,0}} = \frac{c_0}{c_{\text{max}}} = 0.176.
\]

Therefore,

\[ \tau = 0.29 \text{ s}. \]

Substituting \( \tau \) into Eq. B6b and rearranging, one obtains

\[ P_c = 2,421 \text{ dyn/cm}^2. \]

b. Variable \( K_p \)

Combining Eqs. B7, B8, and B9, one finds \( K_p \) is only a function of \( L_f \). Thus, the integrand of the integral in Eq. B5 is only a function of \( L_f \). The integral in Eq. B5 has been
evaluated numerically. For $P_c = 2,421 \text{ dyn/cm}^2$, the relationship between $t$ and $L_f$ is given in the following table and plotted in Fig. 6.

Table 1. Time-dependent compaction of ESL for variable $K_p$

<table>
<thead>
<tr>
<th>$t$(s)</th>
<th>$L_f/L_f0$</th>
<th>$t$(s)</th>
<th>$L_f/L_f0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.28</td>
<td>0.55</td>
</tr>
<tr>
<td>0.015</td>
<td>0.95</td>
<td>0.36</td>
<td>0.50</td>
</tr>
<tr>
<td>0.033</td>
<td>0.90</td>
<td>0.47</td>
<td>0.45</td>
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<tr>
<td>0.053</td>
<td>0.85</td>
<td>0.62</td>
<td>0.40</td>
</tr>
<tr>
<td>0.076</td>
<td>0.80</td>
<td>0.86</td>
<td>0.35</td>
</tr>
<tr>
<td>0.10</td>
<td>0.75</td>
<td>1.29</td>
<td>0.30</td>
</tr>
<tr>
<td>0.14</td>
<td>0.70</td>
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</tr>
<tr>
<td>0.17</td>
<td>0.65</td>
<td>7.03</td>
<td>0.20</td>
</tr>
<tr>
<td>0.22</td>
<td>0.60</td>
<td>25.5</td>
<td>0.176</td>
</tr>
</tbody>
</table>

Note that the time for maximum compaction has been extended by a factor of 50 from 0.5 s to 25 s.