Supporting Information

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SI Text

Calculation of the Time Delays at the Lens–Host Medium Interface. The position of the focused acoustic signal in the host medium is a function of the static precompression applied to the lens. To achieve focusing, we applied a different static precompression on each chain of particles that composes the lens. To calculate the necessary time delay distribution for the signals emanating from the individual chains, we assumed that the chains of spheres transmit mechanical disturbances to the host medium as radiative point sources. This is possible thanks to the small contact area between a sphere and an adjacent planar surface. Such point sources produce spherical acoustic waves that propagate unobstructedly in the host medium, which we assume to be linear and isotropic. Geometric or ray acoustics (1) can thus be used to estimate the delay Δt distribution necessary to focus energy at a desired location (x_f, y_f). The delays satisfy

$$c^2(t_0 - \Delta t_n)^2 = (x_f - x_n)^2 + (y_f - y_n)^2, \qquad \textbf{[S1]}$$

where (x_n, y_n) is the location of the nth source, c is the speed of sound of the linear medium, and t_0 is the travel time between the farthest source and the focal point.

The time delay Δt_n can be converted to a precompression distribution on the chains of particles based on the dependence of the solitary wave velocity V_s on the ratio $F_r = F_m/F_0$ of the dynamic force F_m to the static force F_0 (2),

$$V_s = \frac{c_0}{(F_r - 1)}\left\{\frac{4}{15}\left[3 + 2F_r^{5/3} - 5F_r^{2/3}\right]\right\}^{1/2}. \qquad \textbf{[S2]}$$

Here, the characteristic "speed of sound" c_0 in each chain of particles is (3)

$$c_0 = \sqrt{\frac{E}{\rho_p}}\left[\sqrt{\frac{81F_0}{\pi E}}\frac{1}{\pi D(1-\nu^2)}\right]^{1/3}, \qquad \textbf{[S3]}$$

where ν denotes Poisson's ratio of the material, E is Young's modulus, D is the sphere diameter, and ρ_p is the particles' material density. Note that c_0 is always lower than the speed of sound of the material ($\sqrt{E/\rho_p}$), unless $F_0 \sim D^2E$.

Pressure Field in the Host Medium. At the interface between the nth chain of the lens and the host medium, the solitary waves traveling along the chain are expected to generate displacements of the form (3)

$$u_n(t) = \begin{cases} A_n \cos^4 \alpha_n & \alpha_n \in [-\pi/2, \pi/2], \\ 0 & \text{otherwise}, \end{cases} \qquad \textbf{[S4]}$$

with phase

$$\alpha_n = k_s[V_{s,n}(t - \Delta t_n)]. \qquad \textbf{[S5]}$$

Here, $u_n(t)$ is the normal component of the interface displacement with amplitude A_n. The wavenumber $k_s = \sqrt{10}/(5D)$ only depends on the spheres' diameter D, which is the same in each chain, whereas the solitary wave velocity $V_{s,n}$ depends on the precompression and thus is specific to chain n.

In a fluid host medium, the linear pressure field p_n resulting from traveling sound waves is determined by the nth source at the boundary. The pressure and velocity fields are expressed in terms of the potential $\psi(r,t)$ as $\vec{v} = \vec{\nabla}\psi$ and $p = -\rho\partial\psi/\partial t$, resulting in the wave equation (4)

$$c^2\frac{\partial^2 r_n\psi}{\partial r_n^2} - \frac{\partial^2 r_n\psi}{\partial t^2} = 0, \qquad \textbf{[S6]}$$

with ρ being the density of the fluid medium. We restrict ourselves to the solution in the (x,y) plane, so $r_n^2 = (x - x_n)^2 + (y - y_n)^2$. Solutions of Eq. **6** for outgoing waves only have the form (4)

$$\psi(r_n, t) = \frac{f(t - r_n/c)}{r_n}, \qquad \textbf{[S7]}$$

with

$$f(t) = -ca\int_{-\infty}^{t} e^{-\frac{c}{a}(t-\tau)} v_s(\tau)d\tau, \qquad \textbf{[S8]}$$

where a is a shift from the center of the coordinate system to avoid singularities, and $v_s(t) = \partial u_n(t)/\partial t$ is the interface velocity. Given the relation between pressure and the potential ψ, the assumed interface displacement u_n from Eq. **4**, and considering the contribution of all N sources, the pressure in the far field becomes

$$p(x,y,t) = \frac{\rho c k_s a}{2}\sum_{n=1}^{N}\frac{A_n V_{s,n}[2\sin(2k_s\phi_n) + \sin(4k_s\phi_n)]}{r_n}, \qquad \textbf{[S9]}$$

where $\phi_n = V_{s,n}(r_n/c - a/c - t + \Delta t_n)$. The pressure field is understood to be nonzero only where $k_s\phi_n \in [-\pi/2, \pi/2]$; it is zero otherwise. The interface of nonlinear chains with the host medium through an elastic interface is not amenable to simple analytical expressions. As a result, the amplitude aA_n is chosen (same for all n) to fit the results of numerical models.

Material Properties of Spheres and Neighboring Media. The list of materials and geometric properties used in our nonlinear lens is summarized in Table S2. The considered host media for acoustic focusing are air and polycarbonate (5). The material properties for each are reported in Table S2. For an impact of a striker with mass equal to that of 21 steel spheres with properties listed in Table S2 and an initial velocity $v_0 = 1\ \mathrm{ms}^{-1}$, the phase velocity V_s of the solitary waves within the lens is approximately 658 ms^{-1} and $k_s = 66.4\ \mathrm{rad\,m}^{-1}$.

Combined Discrete-Particle (DP) and Finite-Element (FE) Model of the Acoustic Lens. The numerical analysis serves as tool to quantify the pressure field produced by the transmission of waves within arrays of spheres into the host medium. The fluid or solid space adjacent to the acoustic lens is considered linear, isotropic, and modeled as 2D. A fluid-structure-interaction (FSI) model is established to analyze the interaction of a flexible baffle with the neighboring fluid. The baffle is discretized via Timoshenko beam elements, while coupling fluid-structure and fluid-only elements are plane, four-node, isoparametric elements (6). The acoustic lens is instead considered to be directly in contact with a solid half space, which is discretized via plane, foru-node, isoparametric elements as well. The choice if isoparametric elements is motivated by the fact that significant resolution is needed at the boundary between spheres and the host medium, whereas a coarser discretization may be reasonable in other areas. This scenario is most easily treated with an unstructured or free mesh. If the neighboring medium is a fluid (Fig. S1*A*), arrays of spheres interact with a baffle (end plates shown as orange lines in Fig. S1*A*), which in turn interacts with the adjacent fluid. The FSI model (6) allows to simulate the coupled dynamics of the baffle and fluid. The interaction of the spheres with the baffle itself is modeled by imposing equilibrium of forces as shown in Fig. S1*B*. Spheres are considered point masses and their interaction is modeled by nonlinear springs (3, 7) whose stiffness constant is provided by Hertz contact theory (7). If the neighboring medium is a solid, there is no interface baffle and the spheres contact the solid directly.

The mesh employed for the fluid case in shown in Fig. S2. In the fluid case, beam elements simulating the baffle are superposed to the plane elements. In the solid case, the same discretization is employed, but the solid domain has dimensions of the polycarbonate plate reported below.

To compare numerical and experimental models, the numerical stress field was converted to visible photoelastic fringes by the stress optic law (5):

$$n = \frac{h}{f_\sigma}(\sigma_1 - \sigma_2), \qquad \textbf{[S10]}$$

where h is the polycarbonate plate thickness and $f_\sigma = 7$ KN/m is the material fringe constant. The number of visible fringes n is proportional to the difference of principal stresses σ_1 and σ_2.

Experimental Setup. Figure S3*A* shows the experimental setup. A snapshot of the camera's field of view at rest is provided in Fig. S3*B*. The time delay distribution used for the acoustic focusing (calculated using Eqs. **S1–S3**) is shown in Fig. S3*C*, and the amount of static precompression applied on each chain is reported in Fig. S3*D*.

A movie of the pressure field obtained from the DP/FE model is also provided.

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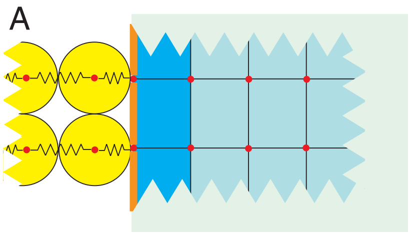


Fig. S1. Combined DP/FE model. (*A*) Arrays of spheres interact with a baffle (orange lines) which in turn interacts with the adjacent fluid. (*B*) The interaction of granular media and baffle.

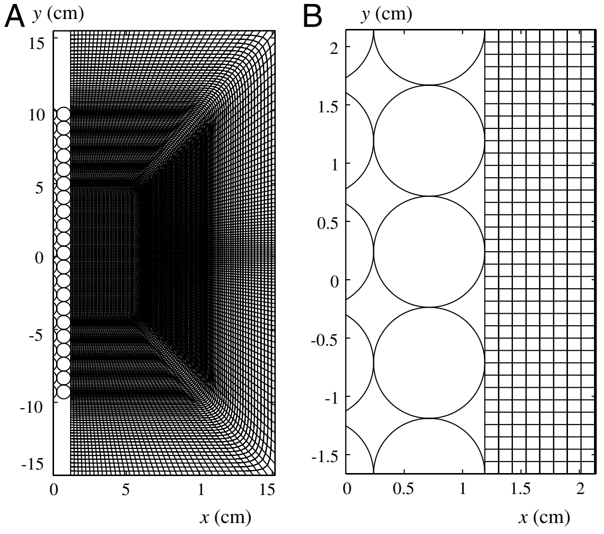


Fig. S2. (*A*) Discretization of host medium with spheres, and (*B*) detail of their interaction.

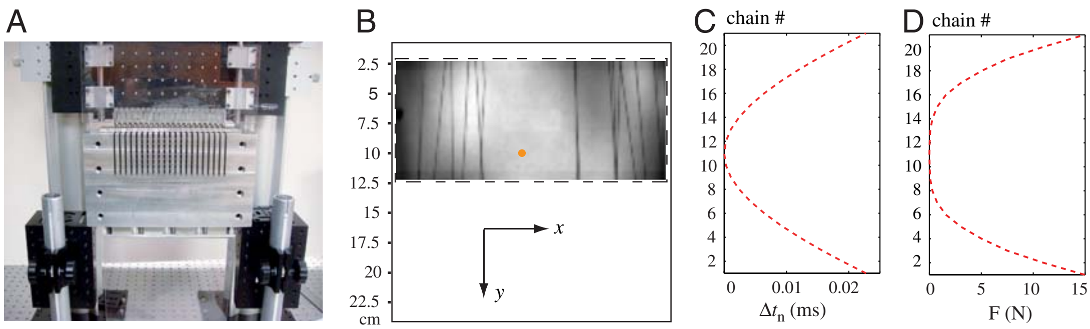
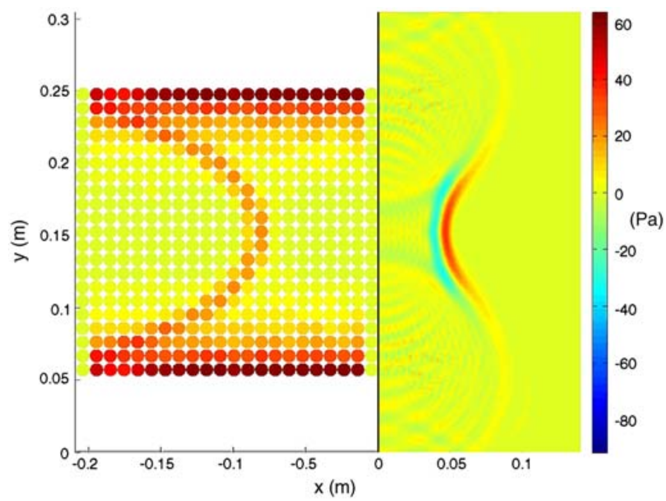


Fig. S3. (*A*) Experimental setup with striker and acoustic lens vertically resting on a polycarbonate plate. Holes filled with resin to properly align the striker edge with the top portion of the lens are visible in the lower portion of the striker. The striker is mounted onto two rails to ensure proper impact. (*B*) The high-speed camera acquired images of a portion of the polycarbonate plate. The nominal focal point is indicated by the orange dot. The dark lines are shadows of the fishing line used to impart static compression to each chain. (*C*) The necessary time delay applied to each chain and (*D*) associated precompression forces applied on each chain are also reported.



Movie S1. Stress waves traveling in a nonlinear acoustic lens and sound bullet formation in the adjacent fluid medium (numerical results). In the lens, the sphere colors are proportional to the contact-force amplitude. Waves emitted by the nonlinear acoustic lens reach the host medium (via an elastic interface), where they generate compact spherical pressure waves. The pressure waves coalesce in the host medium and form a sound bullet. The color bar on the right represents the scale of pressure intensity (Pa) in the host medium.

Movie S1 (AVI)

Table S1. Geometric and material properties (stainless steel 302) from ref. 8 of employed spheres

Diameter	$D = 9.5$ mm
Young's modulus	$E = 195.6$ GPa
Density	$\rho = 8,100$ Kg/m^3
Poisson's ratio	$\nu = 0.33$

Table S2. Material properties of air and polycarbonate (5)

Air	
Density	$\rho = 1.225$ Kg/m^3
Speed of sound	$c = 343$ m/s
Polycarbonate	
Young's modulus	$E = 3.45$ GPa
Density	$\rho = 1230$ Kg/m^3
Poisson's ratio	$\nu = 0.35$
Speed of sound	$c = 1675$ m/s