$$\lim_{n=+\infty} \int_\alpha^b f(x)\,\varphi(n, x-\alpha)\,dx = \tfrac{1}{2} f(\alpha+0)$$

$$\text{''} \left(\lim_{n=+\infty} \int_a^\alpha f(x)\,\varphi(n, x-\alpha)\,dx = \tfrac{1}{2} f(\alpha-0)\right)$$

Theorem IV is a corollary of Theorem III.

While the forms of representation for an arbitrary function $f(x)$ afforded by the preceding theorems do not, strictly speaking, represent the function in terms of definite integrals, but rather in terms of the limits of such integrals as the parameter n increases to $+\infty$, it is to be observed that the first member of (2) may always be expressed as a convergent series, viz:

$$I_0(\alpha) + \sum_{n=0}^{\infty} [I_{n+1}(\alpha) - I_n(\alpha)]$$

and thus it appears that to every integral (2) obtained by any one of the preceding theorems there corresponds an actual representation of the arbitrary function in series of definite integrals.

THE LYMPHOCYTE AS A FACTOR IN NATURAL AND INDUCED RESISTANCE TO TRANSPLANTED CANCER

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Histologically there is a striking resemblance between the series of phenomena which take place about a failing tissue graft in a host of a foreign species, and an homologous cancer graft in an animal with a natural or induced immunity to transplanted cancer. A constant finding in both cases is a local lymphoid reaction which appears early in the process, and lasts till the destruction of the tissue or cancer graft is complete. We have shown in previous communications that the lymphoid tissue is apparently the important factor in the destruction of a tissue graft in an animal of a foreign species. The facts which lead up to this conclusion are, that an organism like the chick embryo, which normally has no defensive agents against the cells of a foreign species, if supplied with adult lymphoid tissue becomes as resistant as the adult animal in this respect. Furthermore when the adult animal is deprived of the major portion of its lymphoid tissue by repeated small doses of X-ray, it loses the ability to destroy the cells of a foreign species and these will live and grow as well as they would in a native host. It seemed of importance from these observations to study the lymphocytes in animals immune to transplanted cancers.

There are two types of this so-called immunity; the natural, possessed normally by a certain variable proportion of animals inoculated; and the induced variety, obtained by a previous treatment of the animal with an injection of homologous living tissues. We have studied the circulating lymphocytes in these two types of immunity and for comparison in susceptible mice with growing tumors.

In the mice with induced immunity the circulating lymphocytes show no change either in actual numbers or relative proportion to the other white cells during the ten days which must elapse between the immunizing tissue injection and the cancer inoculation. Within twenty-four hours after the cancer graft is introduced, however, there is a sharp rise in the number of these cells amounting to an average increase for the group of 100% above the former level. This increase continues with slight variations for something over 50 days, with a maximum average for the series of between 200 and 300% above the normal. The other white cells of the blood retain their normal level.

The natural immune animals, those which without treatment were able to overcome the cancer graft, showed a similar response on the part of the lymphocyte to that seen in the preceding group. The period of increase, however, is not evident for several days or a week after inoculation and the average maximum is not so high, being between 100 and 200% above the normal. Like the first group, there is slight, if any change in the actual numbers of the other white cells of the blood.

Animals in which the cancer graft resulted in a take, showed no such lymphoid response as did the immune animals. A composit curve of the white cells plotted for a number of such animals showed a slight tendency on the part of the lymphocyte to increase during the first two weeks, but this was followed by a gradual decline in number as the cancer increased in size. The polymorphonuclear cells during this period showed an actual increase in numbers.

In order to ascertain the importance of the lymphoid reaction in the immunity process, we have destroyed the major portion of the lymphoid tissue in mice having one of the two types of immunity. This was done by giving the animal several small exposures to X-ray, previously estimated to be sufficient for the purpose. In the induced immunity the X-ray was given between the time of the immunizing tissue injection and the cancer inoculation. This treatment resulted in the complete destruction of the immunity and the inoculated cancers grew more readily than in the normal animals.

For testing the importance of the lymphoid tissue in natural immunity we can only compare the percentage of takes in X-rayed and normal animals inoculated with the same cancer. This has been done with a variety of different cancers and a large series of animals. The average number of takes in the X-rayed animals was 94%, while in the untreated animals only 32% of those inoculated grew the cancer. This shows a very considerable destruction of the natural immunity accompanying a destruction of the lymphocytes.

To summarize, we have shown that a marked increase in the circulating lymphocytes occurs after cancer inoculation in mice with either a natural or induced immunity. When this lymphoid reaction is prevented by a previous destruction of the lymphoid tissue with X-ray the immune states are destroyed. Hence it would seem fair to conclude that the lymphocyte is a necessary factor in cancer immunity.

SOME THEOREMS CONNECTED WITH IRRATIONAL NUMBERS

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As is well known to those who have investigated the fields of celestial mechanics, the series which arise there from the integration of the equations of motion involve factors of the form $(i-j\gamma)$ in the denominators of the coefficients, where i and j run over the entire series of positive integers and γ is a positive number which may be rational or irrational. Previous to the time when Poincaré had shown the existence and construction of periodic solutions (in which γ is always rational) it had been the custom for the astronomers to regard γ as irrational since with this hypothesis the factors $(i-j\gamma)$ never vanish and consequently nonperiodic terms did not arise in the solutions. The presence of these factors in the denominators naturally led to very grave doubts as to the convergence of these series since there are infinitely many such factors which are smaller than any assigned limit, and the convergence has never been proved.

Considerations of this nature have led me recently to examine the convergence of simple types of power series in which this phenomenon occurs, and it has been found that the series $\Sigma\Sigma\, a_{ij}/(i-j\gamma)\, x_1^i\, x_2^j$ has precisely the same domain of convergence as the series $\Sigma\Sigma a_{ij}\, x_1^i\, x_2^j$, provided γ is a positive irrational number which satisfies a rather mild condition, which is stated below.