Self-organization of tidal deltas

Sergio Fagherazzi[1]

Department of Earth Sciences and Center for Computational Science, Boston University, Boston, MA 02122

Edited by Paolo D'odorico, University of Virginia, Charlottesville, VA, and accepted by the Editorial Board October 16, 2008 (received for review July 10, 2008)

Tidal deltas are characterized by a dendritic network of distributaries that transport water and sediments to the ocean. Here, I show that the distributaries self-organize to uniformly redistribute the tidal prism across the entire delta system. The 2 opposite mechanisms of channel formation by avulsion and channel abandonment drive the entire delta toward a critical state at which every channel is close to the silting threshold. Under these conditions the delta reaches self-organized criticality, with changes of its planimetric channel distribution occurring across several spatial scales.

distributary | tide | tidal prism | avulsion | discharge

Of the 3 main classes of deltas (river dominated, wave dominated, and tidal dominated), the morphology of tidal-dominated deltas presents the most dendritic structure (1–4). I hypothesize in this article that tidal delta evolution is driven by channel avulsion (defined herein as the abrupt change in the course of a channel caused by floods, storm surges, or variations in tidal regime) that creates new distributaries, and by the silting of old branches when the discharge is not large enough to transport its entire sediment load. In fluvial-and wave-dominated deltas old distributaries are rapidly abandoned once the river flow is diverted along a new path, so that only a few distributaries are active at any given time (5, 6).

On the contrary, in macrotidal environments the fluvial discharge can be magnified by tidal fluxes, so that more distributaries are maintained flushed despite a limited freshwater input, thus creating a complex dendritic network of hundreds of channels. An example of a tidal network is the Ganges delta, which has migrated eastward forming 3 prograding deltaic systems in the past 5,000 years (7). The seaward portion of the oldest distributaries to the west, forming the Sunderbans, became in time tidally dominated, and, nowadays, they receive a limited fluvial input. Only tidal fluxes keep the dendritic network of the Sunderbans hydrodynamically active (Fig. 1). In tidal deltas the formation of new channels by avulsion (positive feedback) and elimination of channels with low discharge (negative feedback) gives rise to a channel selection that spontaneously increases the organization and complexity of the delta, with more and more branches selectively added to the system, in a self-organized process.

Tidal fluxes are inherently linked to the tidal prism (i.e., the volume of water that enters the delta during one tidal cycle), which, to a first approximation, can be simply computed by multiplying the planimetric submerged area of the delta by the local tidal amplitude (8). Therefore, critical for delta dynamics is the partitioning of upstream submerged area among different distributaries, in a way similar to the relationship between discharge and drainage area in fluvial watersheds (9). However, contrary to rivers, the bottom slope of tidal channels plays a secondary role on tidal fluxes, so that loops are common in the network (Fig. 1 *A* and *B*).

By using a simple yet physically based method, I relate every location of the tidal network to a corresponding submerged delta area flooded and drained during a tidal oscillation (specific tidal discharge). By assuming, to a first approximation, a uniform tidal oscillation within the delta, the specific tidal discharge becomes a proxy for tidal prism and can then be used to test whether each branch is hydrodynamically stable or will be silted in time.

Tidal Delta Model

Tidal fluxes are directly linked to the tidal prism, defined as the total volume of water entering and exiting an embayment during a tidal cycle. In a small tidal embayment the tidal prism can be simply expressed as the product of the embayment area times the tidal excursion, so that the tidal prism, to a first approximation, is directly proportional to the area flooded by the tide (8). If we assume that the volume of water flooding the emerged area between the channels is negligible with respect to the water stored within the channels, we can then assume that the tidal prism is proportional to the total area of the channel network.

This hypothesis will prevent the formation of headless channels in the model simulations. In reality headless channels are present in tidal deltas, particularly in low lying areas subject to flooding and in the prograding foreset, where the tide has the opportunity to channelize the surface during aggradation (10, 11). However, headless channels in the Ganges and Kikori deltas are much smaller than the main delta distributaries forming the network, which are either connected to a terrestrial stream in the upland area (Kikori delta) or display signs of such a connection in the geological past (Ganges delta). We therefore assume that headless channels formed only by tidal flooding are an order of magnitude smaller than delta distributaries, and to a first approximation, we do not include them in the modeling framework.

To partition the delta area among the different distributaries we use the potential discharge Φ defined as (12):

$$q_x = \frac{\partial \Phi}{\partial x}, q_y = \frac{\partial \Phi}{\partial y} \qquad [1]$$

Where q_x and q_y are the discharges per unit width (average velocity times water depth) in the x and y directions, respectively.

The substitution of Eq. **1** in the continuity equation

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_x}{\partial x} + \frac{\partial \eta}{\partial t} = 0 \qquad [2]$$

leads to the Poisson equation:

$$\nabla^2 \Phi = -\frac{\partial \eta}{\partial t} \qquad [3]$$

where η is the elevation of the water surface. If we assume that the spatial differences in water elevations are small with respect to the tidal oscillation, the term on the right-hand side can be

---

Author contributions: S.F. designed research, performed research, contributed new reagents/analytic tools, analyzed data, and wrote the paper.

The author declares no conflict of interest.

This article is a PNAS Direct Submission. P.D. is a guest editor invited by the Editorial Board.

[1] To whom correspondence should be addressed. E-mail: sergio@bu.edu.

This article contains supporting information online at www.pnas.org/cgi/content/full/0806668105/DCSupplemental.

www.pnas.org/cgi/doi/10.1073/pnas.0806668105

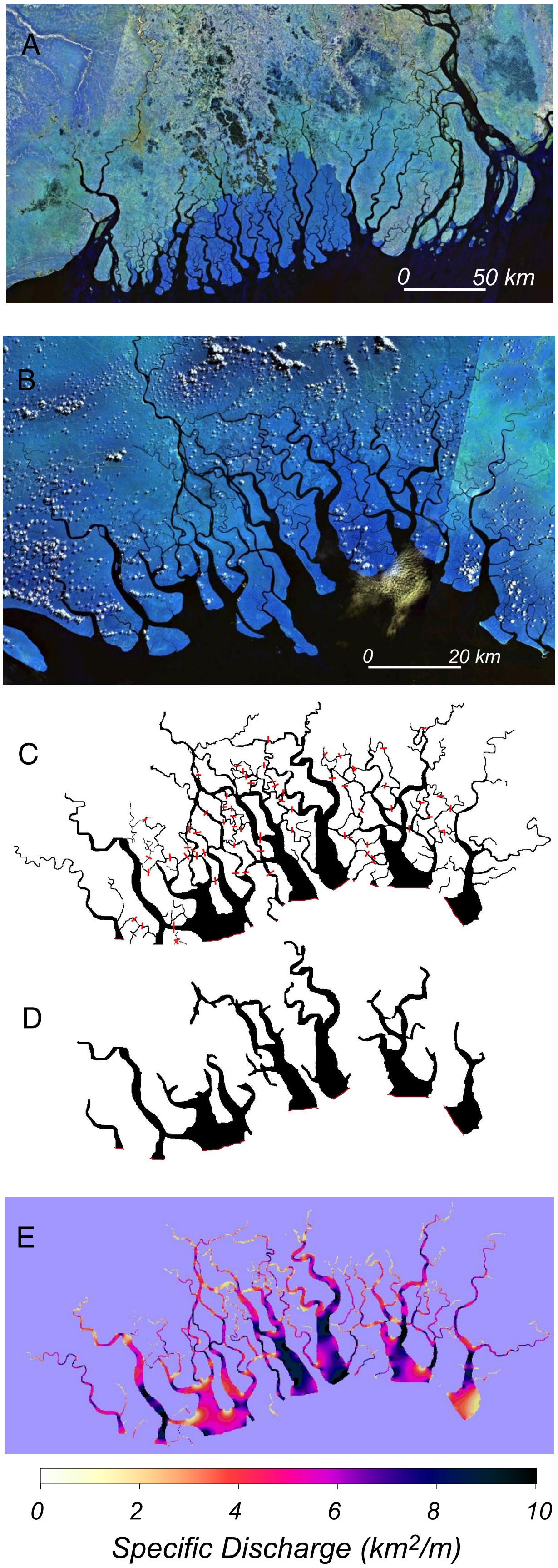

Fig. 1. Morphological analysis of tidal deltas. (*A*) LANDSAT image of the Sunderbans, in the Ganges Delta, Bangladesh (courtesy NASA World Wind). (*B*) LANDSAT image of the Kikori delta, Papua New Guinea. (*C*) Tidal channel network in the Kikori delta extracted from satellite images; the red segments are the locations at which the tidal loops were disconnected in the tidal area analysis. (*D*) Major tidal estuaries in the Kikori delta. (*E*) Computation of the specific tidal discharge at each location within the Kikori tidal delta (width > 500 m).

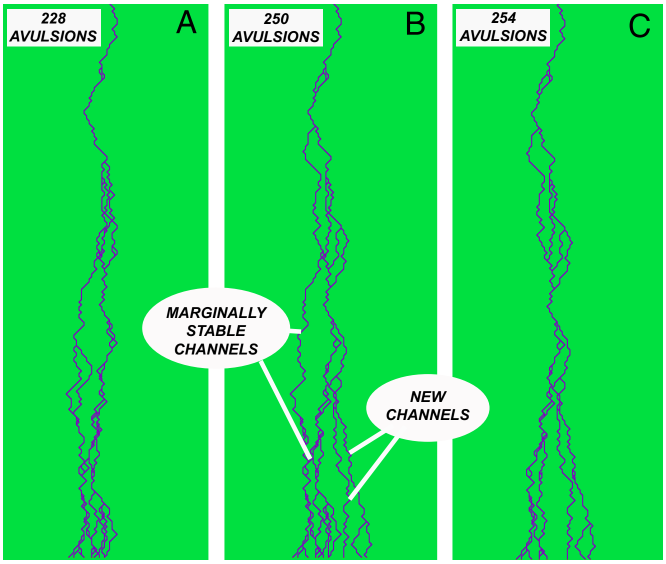

Fig. 2. Simulation of tidal delta evolution. At each time step a new avulsion is created within the delta. All of the channels with a specific tidal discharge below the threshold value are abandoned and removed from the delta. Delta after 228 avulsions (*A*), after 250 avulsions (*B*), and after 254 avulsions (*C*). The delta extends in the lower right area between avulsion 228 and 250, but this extension destabilizes the upper left part of the delta that collapses at avulsion 254.

assumed identical, to a first approximation, across the entire delta. Dividing both the discharge per unit width and the potential discharge by $\frac{\partial\eta}{\partial t}$, we obtain the following equations:

$$a_x = \frac{\partial\varphi}{\partial x},\ a_y = \frac{\partial\varphi}{\partial y} \qquad [4]$$

$$\nabla^2\varphi = -1 \qquad [5]$$

$$a = \sqrt{\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2} \qquad [6]$$

In which $\varphi = \Phi/\frac{\partial\eta}{\partial t}$ is the potential discharge per unit of tidal oscillation, $a_x = q_x/\frac{\partial\eta}{\partial t}$ and $a_y = q_y/\frac{\partial\eta}{\partial t}$ are the specific discharges (discharge per unit increase/decrease of tidal oscillation), and $a = \sqrt{a_x^2 + a_y^2}$ is the module of the specific discharge.

The specific discharge (Eq. **6**) is independent of tidal oscillations, and has units of area per unit width (m^2/m). The specific discharge thus represents a physically based redistribution of intertidal area among all tidal channels in the network. Moreover, if integrated along each channel cross-section, the specific discharge represents the upstream delta area that is drained or flooded by the tide through that channel, and therefore it is equivalent to the drainage area in terrestrial watersheds. Finally, it is possible to prove that the specific discharge (Eq. **6**) is proportional to the tidal discharge in a tidal embayment whose dimensions are small with respect to the tidal wavelength and

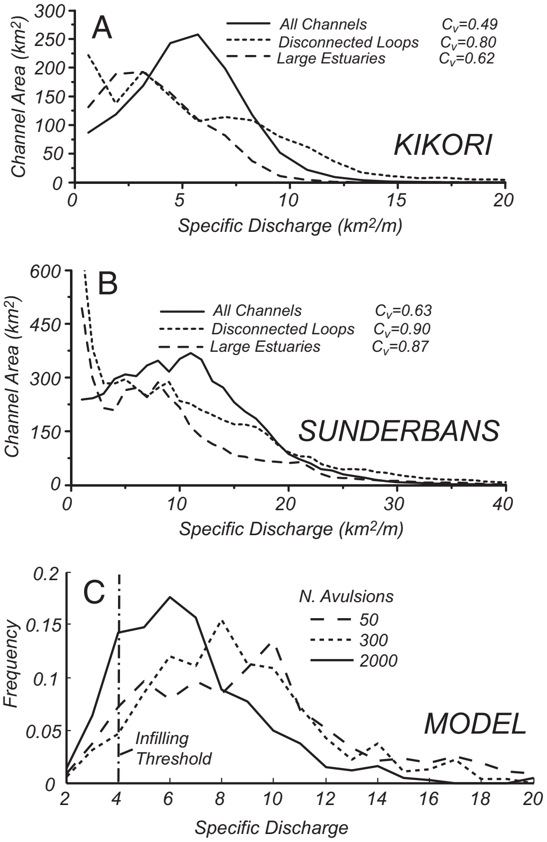

Fig. 3. Distribution of specific tidal area in a delta. (*A*) Kikori delta, Papua, New Guinea, the distribution of specific tidal area for the entire delta (solid line) is compared with the distribution of specific tidal area for the large estuaries (dashed line) and to the distribution after cutting the channel loops at the location with minimum width (dotted line). The coefficient of variation C_V indicates that the natural configuration efficiently redistributes the tidal prism among all tidal channels. (*B*) Sunderbans, Ganges Delta. (*C*) Distribution of specific tidal area during the evolution of a simulated tidal delta, repeated channel avulsions and infilling select a small range of specific tidal area, with the network redistributing the tidal prism to all channels.

with negligible bottom friction (12). Eq. **5** is solved with a no flux boundary condition between the channels and the coastal plain and an elevation equal to mean sea level at the channels mouth (supporting information (SI) Fig. S1).

The tidal delta model simulates the 2 competing processes that regulate the development of the tidal network in time. New channels are formed by avulsion, whereas channels with a tidal discharge below a threshold value are abandoned and eliminated from the network. These 2 opposed processes select the delta channels having a specific range of tidal discharges, thus producing an emergent complexity in the delta by redundancy of distributaries. The model does not represent the avulsion process in detail; rather, it captures the consequences of the addition of a new channel on the hydrodynamic stability of the entire network. At every time step a new avulsion is implemented by randomly choosing a point of the network. Starting from this location, a new channel is then created as a random walk toward the ocean until either the channel reaches the ocean or encounters another channel (Fig. 2). Once the new channel is formed, the specific tidal discharge is recalculated for the entire network by using Eqs. **5** and **6**. If a point of the network has a specific tidal discharge below the threshold value, the point is removed from the network together with all of the other points belonging to the same channel branch, both upstream and downstream.

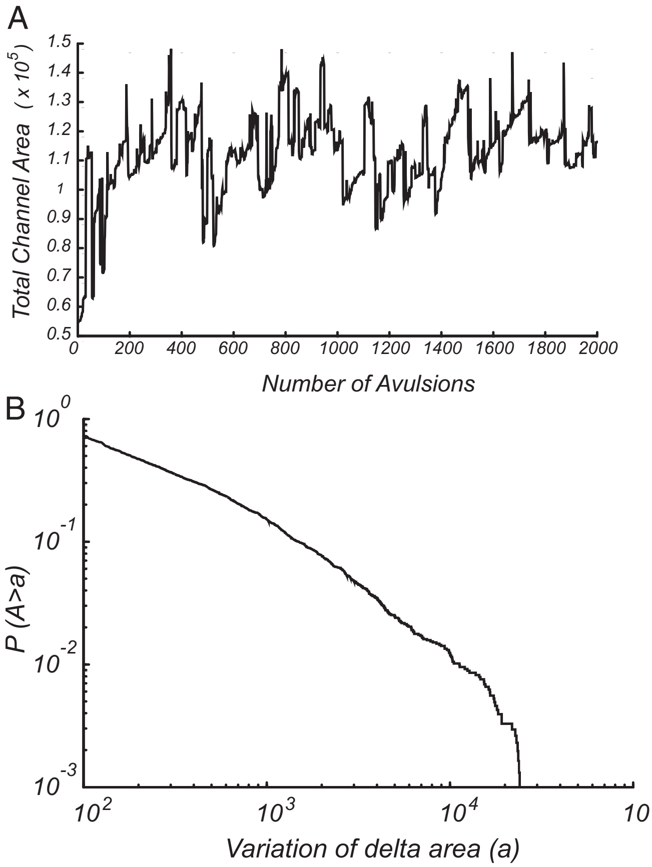

Fig. 4. Numerical simulations of tidal delta evolution. (*A*) The total channel area grows during delta formation but then stabilizes around a critical state when the delta is mature. At the critical state the addition of new distributaries can trigger catastrophic failures of large parts of the network, producing wide oscillations in channel area. (*B*) Distribution of variations of total delta area at criticality.

It is important to note that in tidal deltas the creation of new channels increases the intertidal area and therefore tidal discharges, thus favoring the formation of new distributaries (positive feedback). Similarly, the abandonment of a distributary reduces the intertidal area and therefore tidal discharges, promoting the abandonment of new channels (negative feedback).

Self-Organization of Tidal Deltas

Here, I hypothesize that the system tends to uniformly redistribute the tidal prism within all tidal branches. In fact, if we assume that avulsion is frequent in the delta at the geological timescale, sooner or later a distributary with high tidal discharge will be divided in 2 branches, thus partitioning and reducing the tidal fluxes. However, branches with discharge below a critical threshold will not be able to maintain the channel in a flushed condition, so that they will be abandoned in time. These 2 opposite mechanisms are selecting a narrow range of possible discharges, producing a redistribution of tidal prism across the entire network. A complex network of dendritic channels emerges from the repetition of the 2 simple processes of channel avulsion and abandonment, thus spontaneously increasing the redundancy of the system in a self-organized process.

The specific tidal discharge model described herein is applied to both the Sunderbans in India and Bangladesh and the Kikori delta in the Gulf of Papua after extracting the channel network from satellite images (Fig. 1 *A* and *B*). For both networks the distribution of specific tidal discharge (a proxy for tidal prism) is clustered around a narrow range of values (solid line in Fig. 3 *A* and *B*). To show that the system is self-organized to redistribute the total tidal prism, I compare the specific tidal discharge distribution of 2 artificially modified delta network geometries. In the first test case, I cut every tidal loop at the narrowest channel location (Fig. 1*C*). The corresponding distribution of specific tidal discharge becomes wider (dotted line in Fig. 3 *A* and *B*) with a higher coefficient of variation, proving that indeed the loops are critical for the redistribution of tidal prism within the delta. In the second test, I eliminate the fine structure of the network, maintaining only the large delta estuaries (Fig. 1*D*). Again the distribution of specific tidal discharge is wider, further proving that the small channels equilibrate the tidal fluxes among large estuaries.

The tidal delta model well matches the principle of redistribution of tidal prism derived from the geometry of real deltas. In fact, the distribution of tidal discharge in the network becomes narrower during delta formation, as a result of the 2 counteracting processes of channel avulsion and abandonment (Fig. 3*C*). Despite the agreement between model results and the analysis performed on the Kikori and Ganges deltas, more research is needed to determine the existence of a threshold for infilling, as well as its relationship to sediment discharge and tidal processes.

The selective mechanisms of avulsion and abandonment drive the system toward a configuration in which every channel is close to the threshold discharge for infilling (Fig. 3*C*). At this critical state a perturbation of the system (i.e., the addition or elimination of a new tidal branch) can cause the catastrophic collapse of large areas of the network, with the infilling of an upstream network location and the subsequent removal of the entire downstream branches. The critical state is thus characterized by wide oscillations in total channel area and, therefore, delta dimensions (Fig. 4*A*). The critical threshold for infilling regulates the dimensions of the entire delta, with a larger number of channels that form for a small discharge threshold. The discharge threshold also influences the stability of the delta, with new channels that are more stable when the threshold is low.

The cumulative distribution of variations of total delta area shows that the spatial modifications of the delta after each avulsion span several spatial scales, with a power-law decay of changes in channel area versus frequency (Fig. 4*B*). The emergence of a spatially scale-free behavior is a typical clue of self-organized criticality (13).

Discussion and Conclusion

This analysis is valid for tidal deltas with a freshwater input negligible with respect to the tidal fluxes, which display a dendritic network of channels, rather than for major rivers with the characteristic fan shaped tidal delta (2). Moreover, the present framework does not account for the redistribution of sediment load within the delta branches that strongly influences channel siltation and avulsion (3, 6). The simplified model presented herein focuses only on tidal dynamics and is therefore complementary to already existing models of delta formation (14, 15). The results presented herein have important consequences for human settlements and ecosystems in tidal deltas. If avulsion is still an active process in the delta, the formation of a new channel can produce a dramatic modification of the system, with the hydrodynamic abandonment of large parts of the network. Since at criticality the system tends to become scale-free, a catastrophic system change has a probability of occurrence that is not negligible, but comparable to the occurrence of large earthquakes in tectonically active areas (16).

ACKNOWLEDGMENTS. I thank the editor, Doug Jerolmack, and Jon Pelletier for the constructive reviews of this manuscript. This work was supported by the National Science Foundation MARGINS Program Award OCE0505987, The Petroleum Research Fund, Award ACS PRF no. 42633-G8, and the Office of Naval Research Award N0001-07-1-0664.

1. Bhattacharya JP, Walker RG (1992) Deltas. In: *Facies Models—Response to Sea Level Change*, eds Walker RG, James NP (Geological Association of Canada, St. John's, Newfoundland), pp 157–177.
2. Dalrymple RW, Choi K (2007) Morphologic and facies trends through the fluvial-marine transition in tide-dominated depositional systems: A schematic framework for environmental and sequence-stratigraphic interpretation. *Earth-Sci Rev* 81(3–4):135–174.
3. Jerolmack DJ, Paola C (2007) Complexity in a cellular model of river avulsion. *Geomorphology* 91(3–4):259–270.
4. Harris PT, Baker EK, Cole AR, Short SA (1993) A preliminary study of sedimentation in the tidally dominated Fly River Delta, Gulf of Papua. *Continental Shelf Res* 13:441–472.
5. Tornqvist TE (1994) Middle and late Holocene avulsion history of the river Rhine (Rhine-Meuse delta, Netherlands). *Geology* 22(8):711–714.
6. Slingerland R, Smith ND (2004) River avulsions and their deposits. *Annu Rev Earth Planet Sci* 32:257–285.
7. Allison MA, Khan SR, Goodbred SL, Kuehl SA (2003)Stratigraphic evolution of the late Holocene Ganges-Brahmaputra lower delta plain. *Sediment Geol* 155(3–4):317–342.
8. Bruun P (1978) *Stability of Tidal Inlets* (Elsevier, New York).
9. Rodriguez-Iturbe I, Rinaldo A (1997) *Fractal River Basins: Chance and Self-Organization* (Cambridge Univ Press, New York).
10. Fagherazzi S, Sun T (2004) A stochastic model for the formation of channel networks in tidal marshes. *Geophys Res Lett* 31(21):L21503, 10.1029/2004GL020965.
11. D'Alpaos A, Lanzoni S, Marani M, Fagherazzi S, Rinaldo A (2005) Tidal network ontogeny: Channel initiation and early development. *J Geophys Res Earth Surface* 110(F2):F02001, 10.1029/2004JF000182.
12. Fagherazzi S, Wiberg PL, Howard AD (2003) Tidal flow field in a small basin. *J Geophys Res Oceans* 108(C3):3071.
13. Bak P (1996) *How Nature Works: The Science of Self-Organized Criticality* (Copernicus, New York).
14. Syvitski JPM, Hutton EWH (2001) 2D SEDFLUX 1.0C: An advanced process-response numerical model for the fill of marine sedimentary basins. *Comput Geosci* 27(6):731–753.
15. Swenson JB, Paola C, Pratson L, Voller VR, Murray AB (2005) Fluvial and marine controls on combined subaerial and subaqueous delta progradation: Morphodynamic modeling of compound-clinoform development. *J Geophys Res Earth Surface* 110(F2):1–16.
16. Turcotte DL (1997) *Fractals and Chaos in Geology and Geophysics* (Cambridge Univ Press, Cambridge, UK). Vol. 105, No.