Model choice versus model criticism

Ratmann et al. (1) radically modifies the perception of the Approximate Bayesian Computation (ABC) error into a genuine parameter, whereas the unification of ABC representations in Section 2 is immensely valuable. However, it requires that the data are informative about ε, which is not necessarily true. For instance, when $x_0 \sim \mathcal{B}(n,\theta)$, $\theta \sim \mathcal{U}(0,1)$, $\varepsilon = x - x_0 \sim \mathcal{U}(\{-n,\ldots,n\})$, then $\pi_\varepsilon(\varepsilon|x_0) = \pi_\varepsilon(\varepsilon)$.

Model Assessment

ABC_μ validates a model based on the marginal $m(x)$, not the predictive $p(x|x_0)$. Because it is strongly impacted by prior modeling, ABC_μ fails to condition on the observed x_0. Checking prior adequacy versus model adequacy is valuable but challenging, as in location families where the difference $(\varepsilon - \theta)$ is unidentifiable.

In ABC, using $p(x|x_0)$ requires the same computing times, whereas estimating Bayes factors as acceptance rates is faster. For instance, when $x_0 \sim \mathcal{P}oi(\theta)$, $\theta \sim \mathcal{E}(1)$, the evidence is

$$\int \pi_\theta(\theta) f(x_0|\theta)\mathrm{d}\theta = 2^{-x_0-1},$$

whereas the ABC_μ assessment is the puzzling nonmonotonous

$$\sum_{k\geq -x_0} 2^{-k-x_0} / (1+k^2)\,\mathbb{I}_{\pi_\epsilon(k|x_0)\leq\pi_\epsilon(0|x_0)}.$$

Model Criticism

The Bayesian foundations of ABC_μ are questionable. The consequences of rejecting a model are ignored by ABC_μ, but they are included in constructing another model (see figure 1 in ref. 1).

The nonparametric evaluation in ABC_μ can approximate the marginal density $m(x)$, but the smooth version of [S8] is a poor density estimate of $\xi_{x_0,\theta}(\varepsilon)$ using a single realization; this suggests integrating over h. Thus, the estimator [S9] cannot be used as a practical device, because B is necessarily small, although the quality of the approximation $\widehat{f}_\rho(\theta,\varepsilon|x_0)$ may be irrelevant.

Error or Parameter

Being endowed with a prior, ε is part of the model. The product $\xi_{x_0,\theta}(\varepsilon)\pi(\varepsilon)$ is probabilistically incoherent, even when labeling $\xi_{x_0,\theta}(\varepsilon)$ as a likelihood. Simulation is feasible when it integrates, but it varies under reparameterisation. Whereas ABC strategies use a formal prior + likelihood representation of ε, this does not turn ε into a true parameter. When $x_0 \sim \mathcal{P}oi(\theta)$, $\xi_{x_0,\theta}(\varepsilon)$ is the translated Poisson distribution truncated to positive values, which cannot be used as the original distribution because of the unidentifiability of ε.

Comparing models through the resulting distributions of ε is missing the model complexity penalization from Bayesian model comparison. First, more complex models should lead to more dispersed error distributions. Second, the choices of both $\pi(\varepsilon)$ and ε are model-dependent, and the comparison reflects prior modeling, not data assessment. Using one rejection bound over all models (figure 1 in ref. 1) does not seem recommendable on a general basis.

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