

Model choice versus model criticism

Ratmann et al. (1) radically modifies the perception of the Approximate Bayesian Computation (ABC) error into a genuine parameter, whereas the unification of ABC representations in Section 2 is immensely valuable. However, it requires that the data are informative about ε , which is not necessarily true. For instance, when $x_0 \sim \mathcal{B}(n, \theta)$, $\theta \sim \mathcal{U}(0, 1)$, $\varepsilon = x - x_0 \sim \mathcal{U}(\{-n, \dots, n\})$, then $\pi_\varepsilon(\varepsilon|x_0) = \pi_\varepsilon(\varepsilon)$.

Model Assessment

ABC _{μ} validates a model based on the marginal $m(x)$, not the predictive $p(x|x_0)$. Because it is strongly impacted by prior modeling, ABC _{μ} fails to condition on the observed x_0 . Checking prior adequacy versus model adequacy is valuable but challenging, as in location families where the difference ($\varepsilon - \theta$) is unidentifiable.

In ABC, using $p(x|x_0)$ requires the same computing times, whereas estimating Bayes factors as acceptance rates is faster. For instance, when $x_0 \sim \text{Poi}(\theta)$, $\theta \sim \mathcal{E}(1)$, the evidence is

$$\int \pi_\theta(\theta) f(x_0|\theta) d\theta = 2^{-x_0-1},$$

whereas the ABC _{μ} assessment is the puzzling nonmonotonous

$$\sum_{k \geq -x_0} 2^{-k-x_0} / (1+k^2) \mathbb{I}_{\pi_\varepsilon(k|x_0) \leq \pi_\varepsilon(0|x_0)}.$$

Model Criticism

The Bayesian foundations of ABC _{μ} are questionable. The consequences of rejecting a model are ignored by ABC _{μ} , but they are included in constructing another model (see figure 1 in ref. 1).

The nonparametric evaluation in ABC _{μ} can approximate the marginal density $m(x)$, but the smooth version of [S8] is a poor density estimate of $\xi_{x_0, \theta}(\varepsilon)$ using a single realization; this suggests

integrating over h . Thus, the estimator [S9] cannot be used as a practical device, because B is necessarily small, although the quality of the approximation $f_\rho(\theta, \varepsilon|x_0)$ may be irrelevant.

Error or Parameter

Being endowed with a prior, ε is part of the model. The product $\xi_{x_0, \theta}(\varepsilon)\pi(\varepsilon)$ is probabilistically incoherent, even when labeling $\xi_{x_0, \theta}(\varepsilon)$ as a likelihood. Simulation is feasible when it integrates, but it varies under reparameterisation. Whereas ABC strategies use a formal prior + likelihood representation of ε , this does not turn ε into a true parameter. When $x_0 \sim \text{Poi}(\theta)$, $\xi_{x_0, \theta}(\varepsilon)$ is the translated Poisson distribution truncated to positive values, which cannot be used as the original distribution because of the unidentifiability of ε .

Comparing models through the resulting distributions of ε is missing the model complexity penalization from Bayesian model comparison. First, more complex models should lead to more dispersed error distributions. Second, the choices of both $\pi(\varepsilon)$ and ε are model-dependent, and the comparison reflects prior modeling, not data assessment. Using one rejection bound over all models (figure 1 in ref. 1) does not seem recommendable on a general basis.

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1. Ratmann O, Andrieu C, Wiuf C, Richardson S (2009) Model criticism based on likelihood-free inference, with an application to protein network evolution. *Proc Natl Acad Sci USA* 106:1–6.

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