square. The hardness of the crystal is equal to that of corundum (9 on Mohs' scale), so that specimens are not easily scratched or deformed. The theoretical maximum wave-length of X-rays with which the crystal can be used is 2*d*, or about 22.5 Å, corresponding to an angle of reflection of 90°; the wave-length corresponding to an angle of reflection of 60° is about 19.5 Å.

Previous attempts to find crystals with large grating-constants have been restricted to substances with complicated chemical formulas; such a procedure appears unnecessary in light of the surprising discovery of the unusually large constant possessed by crystals of the simple inorganic substance β-alumina.

[1] Linus Pauling and S. B. Hendricks, *J. Amer. Chem. Soc.*, **47,** 781 (1925).

[2] G. A. Rankine and H. E. Merwin, *J. Amer. Chem. Soc.*, **38,** 568 (1916).

[3] M. Siegbahn, "Spektroskopie der Röntgenstrahlen," Julius Springer, Berlin, 1924, p. 70.

[4] Ref. 3, p. 60.

[5] Albert Björkeson, *Proc. Nat. Acad.* This number, p. 413.

[6] Ref. 3, p. 24.

[7] We wish to thank Dr. A. A. Klein of the Norton Company, Worcester, Mass., for this speciman; also Dr. R. G. Dickinson of this Institute, and Dr. R. W. G. Wyckoff of the Geophysical Laboratory, for supplying us with some crystals, from the same source, used in our preliminary work.

ON QUASI-ANALYTIC FUNCTIONS

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A recent note of G. Julia in the *Comptes Rendes de l'Academie des Sciences in Paris* (**180,** p. 720, March, 1925) may have most remarkable consequences in the coming years, as appears if we pay attention to the general properties of quasi-analytic functions and their mutual relations.

As yet, calculations and special studies in Calculus have, above all, concerned analytic functions. Where does this quite special importance come from? It is clear that it is due to two main circumstances, which are:

(1) That the simplest known functions—beginning with $y=f(x)=x$ itself—those which we want it the most elementary and usual calculations, are analytic:

(2) That they generate each other by the main operations of Calculus: algebraic operation, substitution of functions in functions, differentiation, integration, and so on—i.e., carrying out such operations on analytic functions always leads to functions which are again analytic.

Quasi-analytic functions, or, more exactly, definite classes of quasi-analytic functions, also admit of the latter property. Therefore, if amongst them, there would arise any special one which would prove of importance, the consequence would be that the whole corresponding class would play its role in analysis in a manner analogous to usual analytic functions.

Now, this is precisely what Julia's note shows to be quite possible: for he finds that certain classes of quasi-analytic functions are necessarily to be introduced in the study of some problems concerning iterations.

If it should happen that any special series constituted with iterated functions appears in a concrete problem and, therefore, became an object of calculations, we should have that paradoxial consequence that, in some researches certain classes of quasi-analytic functions would become as usual and important as analytic ones.

SUR LE CALCUL APPROCHÉ DES INTEGRALES DEFINIES

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La méthode la plus simple et celle qui se présente la première a l'esprit pour l'évaluation approchée de l'intégrale définie $\int_a^b f(x)dx$—celle des trapèzes—a été améliorée dans plusieurs sens. La méthode de *Simpson* consiste, comme on sait, à prendre pour le nombre des divisions, supposées égales entre elles, une valeur paire $2n$, ces intervalles étant groupés deux par deux et, dans chaque couple d'intervalles ainsi constitués, à remplacer la fonction f par un polynôme d'interpolation du second degré prenant les mêmes valeurs aux trois points (extrêmes et médian) de division. Si, pour commencer, nous nous bornons à un seul couple d'intervalles, de longueur l chacun, l'origine des abscisses étant prise au point médian, la methode conduit, pour l'intégrale partielle correspondante

$$i = \int_{-l}^{+l} f(x)dx$$

à la valeur approchée

$$j_1 = \frac{l}{3}\left[f(l) + f(-l) + 4f(o)\right], \tag{1}$$

l'intégrale totale

$$I = \int_a^b f(x)dx$$

étant, en conséquence remplacée par l'expression

$$J_1 = \frac{b-a}{3n}(A + 2B) \tag{2}$$