Sinuous rivers

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The windings of rivers have long fascinated their human observers. For example, Abor-iginal legend explains the sinuous pattern of the modern Finke River (Fig. 1) as the crea-tion of the immense and powerful Rainbow Serpent as he emerged during the Dreamtime from deep waterholes. Recently in PNAS (1), a new theory for the general origin of such sinuous flow patterns was published, which follows from a long tradition in seeking a sci-entific explanation for the winding patterns of rivers.

Science itself was arguably born when the first explanatory/genetic hypotheses were formulated in the sixth century BCE by the pre-Socratic philosopher Thales (c. 624 BCE–546 BCE) in the Ionian Greek city of Miletus (2). The site of ancient Miletus, now in south-western Turkey, overlooks a river that in an cient times was known as the Maeander. From this name comes the word “meander,” which since classical Greek time has applied to the description of anything winding, in cluding especially, of course, the curving patterns of rivers. We now know that channelized flows of water, lava, and other fluids have incised such patterns into the surfaces of our solar system’s rocky planets and moons (3). Responding to the need for a general theory to explain all these sinuos, threadlike flow patterns, Lazarus and Constantine’s (1) theory employs a numerical model of flow routing to demonstrate that flow resistance \( R \) (involving a kind of landscape roughness) relative to surface slope \( S \) provides the fundamental control on sinuosity, which for rivers is defined as the long, winding path length of the river channel divided by the relatively short and direct path length down the valley or other surface into which that sinuous river has incised.

Although Lazarus and Constantine’s (1) theory is directed at a general explanation for sinuous flow patterns in all types of channelized flows, the long history of scientific interest in those patterns has focused mainly on the flow type exhibited by alluvial rivers, which have beds and banks composed of the same types of sediment that the rivers commonly transport along their channels. Attributes of such rivers are well illustrated in the notebooks of Leonardo da Vinci (1452–1519), who even depicted a meandering river over the right shoulder of the Mona Lisa in his famous painting. Leonardo da Vinci’s notebooks contain numerous sketches showing the swirling movements of secondary currents in water, and there is one dia gram that shows how meanders can migrate in a downstream direction. Moreover, da Vinci’s interest in winding rivers was not confined to the aesthetic and the scientific; he also studied rivers for purposes of practical engineering. Indeed, much of the subsequent progress on understanding river meandering was made by engineers.

As described in the classic 1955 book, An Introduction to Fluidic Hydraulics (4) by Serge Leliavsky (1891–1963), two general schools of practical engineering research developed in regard to alluvial rivers. One school was empirical, using quantitative measures of river properties. For example, an observation that da Vinci had made qualitatively was quantified in the 19th century: there is a regular downstream decrease in the size of sedimentary particles on a streambed that closely follows the downstream decrease in the slope of that stream. This relationship, known as “Sternberg’s Law” (5), was used by Armin Schoklitsch (1888–1969) to infer an explanation for river sinuosity. Presuming from this “law” that a river’s slope must be adjusted to the diameter of sediment transported, Schoklitsch (6) reasoned that, if this slope of transport is less than the average surface slope of the plain into which the river channel is incised [i.e., the slope \( S \) used in Lazarus and Constantine’s (1) theory], then it will be necessary for the river to assume a winding path to make its channel slope equal to the slope that is appropriate for the transported sediment size. Because the channel slope is the ratio of vertical fall to the distance measured along the winding path of the channel, dividing this number into the valley or surface slope, \( S \), which is ratio of the same vertical fall to the direct path down that valley, will yield a ratio of the distance measured along the winding path of the channel to the distance along the more direct path down the valley, which is by definition the sinuosity of the river. Thus, like Lazarus and Constantine’s (1) theory, the Schoklitsch theory (6) places empha-sis on sinuosity in relation to surface slope, but unlike Lazarus and Constantine’s (1) theory, it does so in relation to the sediment size that the forces of the river are transporting instead of the land-surface roughness \( R \) that is opposing those forces.

The Schoklitsch explanation for meander ing accords with the observation that me andering generally takes place in the lower courses of rivers, where sediments are relatively fine-grained and the corresponding slopes are relatively flat. However, as noted by Leliavsky (4), the theory is not very useful to hydraulic engineers who need a rational, mechanical formulation of the problem, which is the motivation for the second school of alluvial river engineering. An early example from this school was the causative explanation for river meandering proposed by the Scottish civil engineer James Thomson (1822–1892), elder brother to William Thomson (later to become Lord Kelvin). Thomson’s paper, “On the origin of windings of rivers in alluvial plains, with remarks on the flow of water round bends in pipes” (7), was communicated to the Royal Society by his brother (who was then a member) on March 14, 1876. The pa per describes the phenomenon that, as wa ter moves around a river bend, a helical
secondary current is set up in the flow. Modern understanding of the mechanics of meander formation largely centers on the effects of this helical secondary current (8). The physical appeal of secondary circulation as a cause of meandering even attracted the genius of Albert Einstein (1879–1955), whose short paper on the topic was read before the Prussian Academy on January 7, 1926 (9). Although Einstein did not continue with this line of research, his son Hans Einstein (1904–1973) became an important innovator in applied hydraulic engineering studies of rivers, including important contributions to understanding the role of secondary currents in river flows (10).

Although Lazarus and Constantine’s (1) theory acknowledges this tradition in the study of meandering alluvial rivers, it emphasizes that not all sinuous threadlike flows involve the migration of meander bends that so impressed da Vinci, Einstein, and many others. Migrating meandering rivers are held to be a subset of a larger range of phenomena that includes quasistatic forms: the sinuous drainage channels on tidal mudflats as well as the sinuous lunar and Venusian rilles that are generally ascribed to the flow of very fluid lavas (11). Lazarus and Constantine’s (1) theory employs a model of cellular topography in which flow finds a path of least resistance across a planar domain with a given slope S. Furthermore, 40,000 model simulations reveal that, when the resistance R exceeds S, both channel sinuosity and its variance increase with increasing R/S. An independent, corroborating test of this relationship is achieved by recognizing that R/S can be thought of as a kind of Froude number, which is defined for channelized flows as the ratio of the mean flow velocity to a gravitational wave velocity (square root of flow depth multiplied by gravitational acceleration). By substituting into this expression the well-known Gaukler–Manning formula that relates mean flow velocity to slope, flow depth, and the inverse of a roughness measure, Manning’s n, a floodplain Froude number is generated that redefines both the slope and n as applicable to the floodplain rather than to a river channel, which results in an expression for S/R that is calculated for 20 rivers from around the world. When the original R/S values generated by the model are inverted to S/R values, it is found that the channel sinuosity values associated with floodplain S/R dataset obtained from this Froude number convention plot right along with the model-generated ensemble minima for sinuosity.

Lazarus and Constantine emphasize that their approach generates a set of simplest-case explanations as a means of isolating rock weakness exerts a primary control on sinuosity.

and clarifying, “...more complicated, specific or dynamic factors driving planform evolution” (1). The authors agree that rock weakness exerts a primary control on sinuosity for rivers cut into bedrock like the one shown in Fig. 1. However, Lazarus and Constantine note that their simple formulation has a potential for great applicability because R and S can be determined from remote sensing data, digital terrain maps, light detection and ranging surveys, and other studies that bypass time-consuming and expensive in-channel data collection.

Finally, it is relevant to observe that the Gaukler–Manning formula used in Lazarus and Constantine’s theory (1) has the same physical basis as an equation published in 1776 by Antoine de Chezy (1718–1798), who had been engaged to design a canal to supply water to the city of Paris. Chezy recognized that because the gravitational force acting on water moving through a channel (proportional to depth × slope) would have to be opposed by an equal but opposite resisting force (proportional to velocity squared × a measure of resistance), velocity can be equated to the square root of the product of slope multiplied by depth divided by the measure of resistance. However, because this approach is based on Newton’s third law, this equation can only apply at one instant in time. Thus, in covering both quasistatic channels and mobile channels, Lazarus and Constantine’s (1) theory definitely applies to the former because they do not change, but it can only apply to the latter at a particular instant in time relative to the overall evolution of the river. This issue is similar to one raised by geologist J. Hoover Mackin (1905–1968) in his criticism of an explanation for meandering very similar to that proposed by Scholz (4). According to Mackin (12), any account of winding rivers in terms of the slope of a valley floor or other surface leaves out a factor that would be very important to a geologist: the origin of that valley floor or surface. Evolving, mobile rivers, as opposed to quasistatically aligned, create the valley floors and surfaces upon which they flow at any moment in time. From this geological viewpoint, an explanation for the pattern of any evolving river must include something about the historical development of that river. The Finke River (Fig. 1), for example, is superimposed across an ancient mountain range of folded rocks. Because the mountains originated between 400 and 300 million years ago and have since been further warped and deeply eroded, the causative character of the topographic surface to which the Finke River adjusted itself and the river’s subsequent evolutionary history can only be determined through careful geological reconstruction.