

Quantum nonlocality does not exist

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Quantum nonlocality is shown to be an artifact of the Copenhagen interpretation, in which each observed quantity has exactly one value at any instant. In reality, all physical systems obey quantum mechanics, which obeys no such rule. Locality is restored if observed and observer are both assumed to obey quantum mechanics, as in the many-worlds interpretation (MWI). Using the MWI, I show that the quantum side of Bell's inequality, generally believed nonlocal, is really due to a series of three measurements (not two as in the standard, oversimplified analysis), all three of which have only local effects. Thus, experiments confirming "nonlocality" are actually confirming the MWI. The mistaken interpretation of nonlocality experiments depends crucially on a question-begging version of the Born interpretation, which makes sense only in "collapse" versions of quantum theory, about the meaning of the modulus of the wave function, so I use the interpretation based on the MWI, namely that the wave function is a world density amplitude, not a probability amplitude. This view allows the Born interpretation to be derived directly from the Schrödinger equation, by applying the Schrödinger equation to both the observed and the observer.

Bell's theorem | Einstein–Podolsky–Rosen experiment | multiverse | indistinguishability

Nonlocality is a standard example of a quantum mechanical property not present in classical mechanics. A huge number of papers are published each year in the major physics journals [e.g., 5 in *Physical Review Letters* (PRL) in 1997 and 23 in PRL in 2004], purporting to clarify the meaning of "nonlocality." The phenomenon of nonlocality was first described in 1935 by Einstein, et al. (1), in their classic paper, "Can quantum mechanical description of physical reality be considered complete?"

The basic idea in Einstein, et al.'s paper (1) is best described in the well-known formulation in terms of two electrons and their spins. We have two spin 1/2 particles, and the two-particle system is in the rotationally invariant singlet state with zero total spin angular momentum. Thus, if we decide to measure the particle spins in the up–down direction, we would write the wave function of such a state as

$$|\Psi\rangle = \frac{|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2}{\sqrt{2}}, \quad [1]$$

where the direction of the arrow denotes the direction of spin, and the subscript identifies the particle. If we decide to measure the particle spins in the left–right direction, the wave function would be written in a left–right basis as

$$|\Psi\rangle = \frac{|\leftarrow\rangle_1|\rightarrow\rangle_2 - |\rightarrow\rangle_1|\leftarrow\rangle_2}{\sqrt{2}}. \quad [2]$$

Nonlocality arises if and only if we assume that the measurement of the spin of a particle "collapses the wave function" from the linear superposition to either $|\uparrow\rangle_1|\downarrow\rangle_2$ or $|\downarrow\rangle_1|\uparrow\rangle_2$ in [1]. If such a collapse occurs, then measuring the spin of particle 1 would fix the spin of particle 2. The spin of particle 2 would be fixed instantaneously, even if the particles were allowed to separate to large distances. If at the location of particle 1, we make a last-minute decision to measure the spin of particle 1 in the left–right direction rather than the up–down direction, then

instantaneously the spin of particle 2 would be fixed in the opposite direction to that of particle 1—if we assume that [2] collapses at the instant we measure the spin of particle 1. The purported mystery of quantum nonlocality lies in trying to understand how particle 2 changes—instantaneously—in response to what has happened in the location of particle 1.

There is no mystery. There is no quantum nonlocality. Particle 2 does not know what has happened to particle 1 when its spin is measured. State transitions are entirely local in quantum mechanics. All these statements are true because quantum mechanics tells us that the wave function does not collapse when the state of a system is measured. In particular, nonlocality disappears when the many-worlds interpretation (2–5) is adopted. The many-worlds interpretation (MWI) dispels the mysteries of quantum mechanics. Collapse interpretations are nonlocal. So the standard argument that quantum phenomena are nonlocal goes like this: (i) Let us add an unmotivated, inconsistent, unobservable, nonlocal process (collapse) to local quantum mechanics; (ii) note that the resulting theory is nonlocal; and (iii) conclude that quantum mechanics is nonlocal.

I outlined the arguments in an earlier paper (6). Everett was the first to suggest (ref. 3, p. 149) that nonlocality would disappear in the MWI, but this paper is to my knowledge the first to prove what Everett claimed. Here I directly address Bell's inequality, which requires a derivation of the Born interpretation of the wave function. My derivation starts from the standard MWI idea that the wave function is not a probability amplitude, but instead a "world density amplitude," which is to say $|\psi|^2$ is proportional to the density of universes in the multiverse. The problem is to derive the Born frequencies from this assumption. Previous derivations have been unsatisfactory, in my judgment, because an essential part of the physics has been left out. The physics that has been heretofore omitted has been quantum mechanical indistinguishability, applied to the experimenters and their experimental apparatus. From the MWI viewpoint, humans and their equipment are quantum mechanical objects no less than atoms and are thus subject to indistinguishability no less than atoms. Universes in the same quantum state are indistinguishable and

Significance

I show that quantum nonlocality is an artifact of the assumption that observers obey the laws of classical mechanics, whereas observed systems obey quantum mechanics. Locality is restored if observed and observer both obey quantum mechanics, as in the many-worlds interpretation (MWI). Using the MWI, I show that the quantum side of Bell's inequality is entirely local. Thus, experiments confirming "nonlocality" are actually confirming the MWI. The mistaken interpretation of Bell's inequality depends on the idea that the wave function is a probability amplitude, but the MWI holds that the wave function is a world density amplitude. Assuming the wave function is a world density amplitude, I derive the Born interpretation directly from Schrödinger's equation.

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hence if interchanged, nothing has happened. I use this fact to derive the Born frequencies. In outline, the indistinguishability allows probabilities in the Bayesian sense to be assigned to the likelihood that we will be in a particular universe observing a particular sequence of paired electron spins, and Bayesian probability theory tells us how to calculate the most likely frequencies from these probabilities. I show that these most likely frequencies are the Born frequencies.

The Disappearance of Nonlocality in the MWI

To see how nonlocality disappears in detail, let us analyze the measure of the spins of the two particles from the many-worlds perspective. Let $M_i(\dots)$ denote the initial state of the device that measures the spin of the i th particle. The ellipsis denotes a measurement not yet having been performed. We can for simplicity assume that the apparatus is 100% efficient and that the measurement does not change the spin being measured (putting in a more realistic efficiency and taking into account the fact that measurement may affect the spin slightly would complicate the notation but the conclusions would be unchanged). That is, if each particle happens to be in an eigenstate of spin, a measurement of the i th particle changes the measuring device—but not the spin of the particle—as

$$\begin{aligned} \mathcal{U}_1 M_1(\dots) |\uparrow\rangle_1 &= M_1(\uparrow) |\uparrow\rangle_1 \\ \mathcal{U}_1 M_1(\dots) |\downarrow\rangle_1 &= M_1(\downarrow) |\downarrow\rangle_1 \end{aligned} \quad [3]$$

$$\begin{aligned} \mathcal{U}_2 M_2(\dots) |\uparrow\rangle_2 &= M_2(\uparrow) |\uparrow\rangle_2 \\ \mathcal{U}_2 M_2(\dots) |\downarrow\rangle_2 &= M_2(\downarrow) |\downarrow\rangle_2, \end{aligned} \quad [4]$$

where \mathcal{U}_i are linear operators that generate the change of state in the measurement apparatus, corresponding to the measurement. The operators \mathcal{U}_i are actually unitary, but this is not essential to the argument. What is essential is linearity.

In particular, if particle 1 is in an eigenstate of spin up, and particle 2 is in an eigenstate of spin down, then the effect of the \mathcal{U}_i s together is

$$\mathcal{U}_2 \mathcal{U}_1 M_1(\dots) M_2(\dots) |\uparrow\rangle_1 |\downarrow\rangle_2 = M_1(\uparrow) M_2(\downarrow) |\uparrow\rangle_1 |\downarrow\rangle_2 \quad [5]$$

even if particles 1 and 2 are light years apart when their spin orientations are measured. Similarly, the result of measuring the i th particle in the eigenstate of spin left would be $\mathcal{U}_i M_i(\dots) |\leftarrow\rangle_i = M_i(\leftarrow) |\leftarrow\rangle_i$ and for an eigenstate of spin right would be $\mathcal{U}_i M_i(\dots) |\rightarrow\rangle_i = M_i(\rightarrow) |\rightarrow\rangle_i$, which will generate equations for spins left and right analogous to Eqs. 3–5.

Now consider the effect of a measurement on the two-particle system in the Bohm state, that is, with total spin zero. This state is [1] or [2] with respect to an up/down or left/right basis, respectively. The result is completely determined by linearity and the assumed correct measurements on single electrons in eigenstates. For example, the effect of measurements in which both observers happen to choose to measure with respect to the up/down basis is

$$\begin{aligned} \mathcal{U}_2 \mathcal{U}_1 M_2(\dots) M_1(\dots) &\left[\frac{|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2}{\sqrt{2}} \right] \\ &= \mathcal{U}_2 M_2(\dots) \left[\frac{M_1(\uparrow) |\uparrow\rangle_1 |\downarrow\rangle_2}{\sqrt{2}} - \frac{M_1(\downarrow) |\downarrow\rangle_1 |\uparrow\rangle_2}{\sqrt{2}} \right] \\ &= \frac{M_2(\downarrow) M_1(\uparrow) |\uparrow\rangle_1 |\downarrow\rangle_2}{\sqrt{2}} - \frac{M_2(\uparrow) M_1(\downarrow) |\downarrow\rangle_1 |\uparrow\rangle_2}{\sqrt{2}}. \end{aligned} \quad [6]$$

It may appear from Eq. 6 that it is the first measurement to be carried out that determines the split into the two worlds represented by two terms in [6]. This is false. In fact, if the measurements

are carried out at space–time events that are space-like separated, then there is no Lorentz invariant way of determining which measurement was carried out first. At space-like separation, the measuring operators \mathcal{U}_1 and \mathcal{U}_2 commute, and so we can equally well perform the measurement of the spins of the electrons in reverse order and obtain the same splits,

$$\begin{aligned} \mathcal{U}_1 \mathcal{U}_2 M_1(\dots) M_2(\dots) &\left[\frac{|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2}{\sqrt{2}} \right] \\ &= \mathcal{U}_1 M_1(\dots) \left[\frac{M_2(\downarrow) |\uparrow\rangle_1 |\downarrow\rangle_2}{\sqrt{2}} - \frac{M_2(\uparrow) |\downarrow\rangle_1 |\uparrow\rangle_2}{\sqrt{2}} \right] \\ &= \frac{M_1(\uparrow) M_2(\downarrow) |\uparrow\rangle_1 |\downarrow\rangle_2}{\sqrt{2}} - \frac{M_1(\downarrow) M_2(\uparrow) |\downarrow\rangle_1 |\uparrow\rangle_2}{\sqrt{2}}, \end{aligned} \quad [7]$$

the last line of which is the same as that of [6] (except for the order of states, which is irrelevant).

The effect of measurements in which both observers happen to choose to measure with respect to the left/right basis is

$$\begin{aligned} \mathcal{U}_2 \mathcal{U}_1 M_2(\dots) M_1(\dots) &\left[\frac{|\leftarrow\rangle_1 |\rightarrow\rangle_2 - |\rightarrow\rangle_1 |\leftarrow\rangle_2}{\sqrt{2}} \right] \\ &= \mathcal{U}_2 M_2 \left[\frac{M_1(\leftarrow) |\leftarrow\rangle_1 |\rightarrow\rangle_2}{\sqrt{2}} - \frac{M_1(\rightarrow) |\rightarrow\rangle_1 |\leftarrow\rangle_2}{\sqrt{2}} \right] \\ &= \frac{M_2(\rightarrow) M_1(\leftarrow) |\leftarrow\rangle_1 |\rightarrow\rangle_2}{\sqrt{2}} \\ &\quad - \frac{M_2(\leftarrow) M_1(\rightarrow) |\rightarrow\rangle_1 |\leftarrow\rangle_2}{\sqrt{2}}. \end{aligned} \quad [8]$$

A comparison of [6] or [7] with [8] shows that if two space-like-separated observers fortuitously happen to measure the spins of the two particles in the same direction—whatever this same direction happens to be—both observers will split into two distinct worlds, and in each world the observers will measure opposite spin projections for the electrons. However, at each event of observation, both of the two possible outcomes of the measurement will be obtained. Locality is preserved, because indeed both outcomes are obtained in total independence of the outcomes of the other measurement. The linearity of the operators \mathcal{U}_1 and \mathcal{U}_2 forces the perfect anticorrelation of the spins of the particles in each world. Because the singlet state is rotationally invariant, the same result would be obtained whatever direction the observers happened to choose to measure the spins.

The Crucial Third Measurement

In the experiment in ref. 1, there is a crucial third measurement: the comparison of the two observations made by the spatially separated observers. In fact, the relative directions of the two spin measurements have no meaning without this third measurement. Once again, it is easily seen that initialization of this third measurement by the two previous measurements, plus linearity, implies that this third measurement will confirm the split into two worlds. In the Copenhagen interpretation, this third measurement is not considered a quantum measurement at all, because the first measurements are considered as transferring the data from the quantum to the classical regime. However, in the MWI, there is no classical regime; the comparison of the data in two macroscopic devices is just as much a quantum interaction as the original setting up of the singlet state. Furthermore, this ignored third measurement is actually of crucial importance: It is performed after information about the orientation of the second device has been carried back to the first device (at a speed less than that of light!). The orientation is coded with correlations of the spins of both electrons, and these

correlations (and the linearity of all operators) will force the third measurement to respect the original split. These correlations have not been lost, for no measurement reduces the wave function: The minus sign between the two worlds is present throughout Eqs. 1–8.

To see explicitly how this third measurement works, represent the state of the comparison apparatus by $M_c[(\dots)_1(\dots)_2]$, where the first entry measures the record of the apparatus measuring the first particle, and the second entry measures the record of the apparatus measuring the second particle. Thus, the third measurement acting on eigenstates of the spin-measurement devices transforms the comparison apparatus as

$$\begin{aligned} \mathcal{U}_c M_c[(\dots)_1(\dots)_2] M_1(\uparrow) &= M_c[(\uparrow)_1(\dots)_2] M_1(\uparrow) \\ \mathcal{U}_c M_c[(\dots)_1(\dots)_2] M_1(\downarrow) &= M_c[(\downarrow)_1(\dots)_2] M_1(\downarrow) \\ \mathcal{U}_c M_c[(\dots)_1(\dots)_2] M_2(\uparrow) &= M_c[(\dots)_1(\uparrow)_2] M_2(\uparrow) \\ \mathcal{U}_c M_c[(\dots)_1(\dots)_2] M_2(\downarrow) &= M_c[(\dots)_1(\downarrow)_2] M_2(\downarrow), \end{aligned}$$

where for simplicity I have assumed the spins will be measured in the up or down direction. Then for the state [1], the totality of the three measurements together—the two measurements of the particle spins followed by the comparison measurement—is

$$\begin{aligned} &\mathcal{U}_c \mathcal{U}_2 \mathcal{U}_1 M_c[(\dots)_1(\dots)_2] M_2(\dots) M_1(\dots) \times \\ &\times \left[\frac{|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2}{\sqrt{2}} \right] \\ &= M_c[(\uparrow)_1(\downarrow)_2] \frac{M_2(\downarrow) M_1(\uparrow) |\uparrow\rangle_1 |\downarrow\rangle_2}{\sqrt{2}} - \\ &- M_c[(\downarrow)_1(\uparrow)_2] \frac{M_2(\uparrow) M_1(\downarrow) |\downarrow\rangle_1 |\uparrow\rangle_2}{\sqrt{2}}. \end{aligned}$$

Heretofore I have assumed that the two observers have chosen to measure the spins in the same direction. For observers who make the decision of which direction to measure the spin in the instant before the measurement, most of the time the two directions will not be the same. The experiment could be carried out by throwing away all observations except those in which the chosen directions happened to agree within a predetermined tolerance. However, this would waste most of the data. The Aspect–Clauser–Freedman experiment (7, 8) is designed to use more of the data by testing Bell’s inequality for the expectation value of the product of the spins of the two electrons with the spin of one electron being measured in direction \hat{n}_1 and the spin of the other in direction \hat{n}_2 . If the spins are measured in units of $\hbar/2$, the standard quantum mechanical expectation value for the product is

$$\langle \Psi | (\hat{n}_1 \cdot \sigma_1) (\hat{n}_2 \cdot \sigma_2) | \Psi \rangle = -\hat{n}_1 \cdot \hat{n}_2, \quad [9]$$

where $|\Psi\rangle$ is the singlet state [1]/[2]. In particular, $\hat{n}_1 = \hat{n}_2$ is the assumed setup of the previous discussion. Because the MWI shows that local measurements in this case always give +1 for one electron and –1 for the other, the product of the two is always –1 in all worlds, and thus the expectation value for the product is –1, in complete agreement with [9].

To show how [9] comes about by local measurements splitting the universe into distinct worlds, I follow [9] and write the singlet state [1]/[2] with respect to some basis in the \hat{n}_1 direction as

$$|\Psi\rangle = (1/\sqrt{2}) (|\hat{n}_1, \uparrow\rangle_1 |\hat{n}_1, \downarrow\rangle_2 - |\hat{n}_1, \downarrow\rangle_1 |\hat{n}_1, \uparrow\rangle_2). \quad [10]$$

Let another direction \hat{n}_2 be the polar axis, with θ the polar angle of \hat{n}_1 relative to \hat{n}_2 . Without loss of generality, we can choose the other coordinates so that the azimuthal angle

of \hat{n}_1 is zero. Standard rotation operators for spinor states then give (9)

$$\begin{aligned} |\hat{n}_1, \uparrow\rangle_2 &= (\cos \theta/2) |\hat{n}_2, \uparrow\rangle_2 + (\sin \theta/2) |\hat{n}_2, \downarrow\rangle_2 \\ |\hat{n}_1, \downarrow\rangle_2 &= -(\sin \theta/2) |\hat{n}_2, \uparrow\rangle_2 + (\cos \theta/2) |\hat{n}_2, \downarrow\rangle_2, \end{aligned}$$

which yields

$$\begin{aligned} |\Psi\rangle &= (1/\sqrt{2}) [-(\sin \theta/2) |\hat{n}_1, \uparrow\rangle_1 |\hat{n}_2, \uparrow\rangle_2 \\ &+ (\cos \theta/2) |\hat{n}_1, \uparrow\rangle_1 |\hat{n}_2, \downarrow\rangle_2 \\ &- (\cos \theta/2) |\hat{n}_1, \downarrow\rangle_1 |\hat{n}_2, \uparrow\rangle_2 \\ &- (\sin \theta/2) |\hat{n}_1, \downarrow\rangle_1 |\hat{n}_2, \downarrow\rangle_2]. \end{aligned} \quad [11]$$

In other words, if the two devices measure the spins in arbitrary directions, there will be a split into four worlds, one for each possible permutation of the electron spins. Just as in the case with $\hat{n}_1 = \hat{n}_2$, normalization of the devices on eigenstates plus linearity forces the devices to split into all of these four worlds, which are the only possible worlds, because each observer must measure the electron to have spin +1 or –1.

Using Many-Worlds and Bayes–Laplace Probability Theory to Derive the Born Interpretation

The fact that the splits are determined by the nature of the measurement apparatus is the key to deriving the Born interpretation wherein the squares of the coefficients in [11] are the “probabilities” of an observed occurrence of the four respective outcomes in [11]. Note that all two or four outcomes actually happen: The sums in [11] (or [1] or [2]) are in 1–1 correspondence with real universes. Because the observers are unaware of the other versions of themselves after a measurement, ignoring the existence of the other versions necessarily means a loss of information available to one observer, and it is this loss of information that results in probabilities. The information is still in the collection of observers—time evolution is unitary—but it is now divided between the four versions, who are now mutually incommunicado.

Note that I have placed the word probabilities above in quotation marks. I do this because many physicists are confused about the meaning of the word probability. Most think that probability means the relative frequency of some event among some collection of events, for example the relative number of times we measure the spin to be up, divided by the total number of times we measure the spin in the vertical direction, in the limit as the number of measurements approaches infinity. This is not the meaning given to probability by the physicist founder of probability theory, Pierre-Simon de Laplace, for whom probabilities are a numerical measure of human ignorance and not an objective feature of nature (10). In the case of quantum mechanics, the ignorance in question is ignorance of the other universes of the multiverse. The frequency interpretation of probability was introduced into physics by Maxwell, who obtained this mistaken idea from Adolphe Quételet, who has been called “one of the most destructive fellows in the history of thought” (ref. 11, p. 241). Maxwell had an excuse: He was only 19 y old when he encountered Quételet’s mistaken idea, via an article by John Herschel (ref. 12, p. 587). Maxwell used the frequency interpretation in his statistical physics work, where it was hugely successful. So when the Born interpretation was first presented in the late 1920s, the original Laplacean meaning of probability had been forgotten, and the frequency interpretation of probability was adopted to give a meaning to the square of the modulus of the wave function.

However, the course of atoms and their spins in the multiverse is exactly and completely determined by the deterministic wave equation. Further, the fact that it is the square of the modulus of the wave function that gives the best estimate of the probability density—in the sense of a numerical value of human ignorance—is due to the deterministic nature of the wave equation itself.

To see this, let me show that quantum mechanics is just classical mechanics required to be globally deterministic. The most general expression of classical mechanics is the Hamilton–Jacobi equation

$$\frac{\partial S}{\partial t} + H\left(x_i, \frac{\partial S}{\partial x_i}, t\right) = 0. \quad [12]$$

The most general Hamiltonian we need to consider is

$$H = \sum_{i=1}^k \frac{(\vec{\nabla}_i S)^2}{2m_i} + V(x_1, x_2, \dots, x_{3N}, t), \quad [13]$$

where there are k particle types, each type with mass m_i , and each particle type has l_i particles. The operator $\vec{\nabla}_i$ is, for each i , the differential operator in $3l_i$ dimensions. If V is an attractive potential, the trajectories can cross, resulting in a breakdown of the equation at a caustic singularity. This can be prevented by adding to the potential V the “quantum” potential (ref. 13, pp. 51–52, and ref. 14):

$$U = -\left(\frac{\hbar^2}{2}\right) \sum_{i=1}^k \frac{1}{m_i} \left(\frac{\nabla_i^2 R}{R}\right). \quad [14]$$

The new function R satisfies the continuity equation

$$\frac{\partial R^2}{\partial t} + \sum_{i=1}^k \vec{\nabla}_i \cdot \left(R^2 \frac{\vec{\nabla}_i S}{m_i}\right) = 0. \quad [15]$$

These two equations, the Hamilton–Jacobi equation with potential $V + U$ and Eq. 15, can be combined into a single equation if we define a function ψ by the expression (ref. 13, pp. 51–52, and ref. 14)

$$\psi \equiv R \exp(iS/\hbar). \quad [16]$$

Then the function ψ is easily seen to satisfy the single equation for the complex valued function ψ :

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2} \left[\sum_{i=1}^k \frac{\nabla_i^2 \psi}{m_i} \right] + V(x_1, x_2, \dots, x_{3N}, t) \psi. \quad [17]$$

Because Eq. 17 is linear, it cannot give rise to caustics and hence is globally C^2 . Because it is equivalent to the pair of classical equations, they also are globally C^2 . Eq. 17 is obviously just the Schrödinger equation. I have demonstrated that quantum mechanics is merely classical mechanics made globally deterministic.

The Hamilton–Jacobi equation has been recognized since the 19th century as the most powerful mathematical expression of classical mechanics. However, it is clear that the Hamilton–Jacobi equation is a multiverse expression of classical mechanics. In the 19th century, this multiverse nature was ignored. However, the other worlds of the multiverse really do exist even in classical mechanics: It is the collision of the worlds that yields the caustics. If something can hit you, it exists.

Eq. 15 is a conservation equation for these universes, and it is expressed in standard form for a conservation equation, which therefore allows us to recognize that R^2 is proportional to the density of universes (only “proportional” because the wave function can be multiplied by a constant without any change in the physics, a necessary consequence of linearity). If many universes exist—as they do—then there must be a quantity representing a density of universes. The function R^2 is this natural choice for this density.

The total number of what I term “effectively distinguishable” universes is the space integral of R^2 , and this integral may be infinite. We see that Schrödinger’s equation does not require the integral of R^2 to be finite, and there will be many cases of physical interest in which it is not. The plane waves are one important and indispensable example, and physicists use various delta function normalizations in this case. An infinite integral of R^2 for the wave function of the multiverse has been shown (15) to provide a natural and purely kinematic explanation for the observed flatness of the universe, if the universe is spatially a three-sphere, as I have argued (15) that it must be if unitarity is to be preserved in black hole evaporation.

However, for quantum nonlocality problems, the integral of R^2 will be finite, and if we pose questions that involve the ratio of the number of effectively distinguishable worlds with a given property to the total number of effectively distinguishable worlds, it is convenient to normalize the spatial integral of R^2 to be 1.

With this normalization, $R^2 d^3X$ is then the ratio of the number of effectively distinguishable universes in the region d^3x to the total number of universes. In the case of spin up and spin down, there are only two possible universes, and so the general rule for densities requires us to have the squares of the coefficients of the two spin states be the total number of effectively distinguishable—in this case obviously distinguishable—states. Normalizing to 1 gives the ratio of the number of the two spin states to the total number of states.

Consider a measurement of [1] or [11] with $\theta = \pi/2$. In either case, the initial state of the observer is the same, and there is no way even in principle of distinguishing the two or four final states of [1] or [11], respectively. Because there is no difference between the initial states of the observers, there is no difference in the terms of the expression except for the labels I have given them, and the labels can be interchanged, leaving the physics invariant. This interchange of labels forms a group and shows that the probabilities assigned to each state must be the same. This transformation group argument for assigning a probability distribution is originally due to Henri Poincaré; see refs. 16 and 17 for a modern discussion. Thus, the invariance of the physics under the relabeling of the states yields the “principle of indifference”: We must assign equal probabilities to each of two or four states, respectively, and so the probabilities must be 1/2 or 1/4, respectively. These are seen to be the relative numbers of distinguishable universes in these states. In summary, it is the indistinguishability of the initial states of the observers in all two or four final states that forces us to equate the probabilities with the relative number of distinguishable universes in the final state. The same argument gives the same equation of the probability of the general orientation state in [11] with arbitrary θ with the squares of the coefficients of the states in [11] with the relative number of effectively distinguishable universes in the final states.

Note that this does not give the Born interpretation in the usual sense of probabilities mean relative frequencies as the number of observations approaches infinity. In Laplacean probability theory, the relative frequency is a parameter to be estimated from a probability, not a probability itself (see refs. 16 and 17 for a detailed discussion of this point). However, the most probable value of the relative frequency has been shown (ref. 16, pp. 336–339, 367–368, 393–394, and 576–578 and ref. 17, pp. 106–110) to be equal in classical physics to the probability (in the Laplacean sense) that the event will occur.

A proof in quantum physics that in the limit of a very large number of trials, the measured relative frequencies will approach the probabilities—the measure of human ignorance of the other universes of the multiverse—proceeds as follows. The proof depends crucially on the indistinguishability of the initial states of the observers and on the actual existence of the many worlds. Indistinguishability also is an essential idea in Deutsch's similar proof (4) for the Born interpretation: Deutsch in effect assumed that two systems with the same R^2 are physically equivalent; they can be interchanged with no effect on the physics. This is exactly the same notion of indistinguishability. For simplicity I assume that the spins of a series of electrons are measured and that the spins of all of the measured electrons are spin up before the measurement. I also assume that the measuring apparatus is at an arbitrary angle θ with respect to the vertical in all of the universes. In this case the Laplacean probabilities for measuring spin up along the axis of the apparatus are $p_{\uparrow,\theta} = \cos^2(\theta/2) \equiv p$ and for measuring spin as antialigned with the axis are $p_{\downarrow,\theta} = \sin^2(\theta/2) \equiv q$, respectively, for $0 \leq \theta \leq \pi/2$. The probability $\text{prob}(r | N)$ that an observer in a particular universe will, after N measurements of N different electrons but with all in the spin-up state, see the electron as having spin aligned with the apparatus r times is

$$\begin{aligned} \text{prob}(r | N) &= \sum_k \text{prob}(r, S_k | N) \\ &= \sum_k \text{prob}(r | S_k, N) \times \text{prob}(S_k | N), \end{aligned} \quad [18]$$

where the summation is over all of the 2^N sequences of outcomes S_k , each of which actually occurs in some universe of the multiverse, after N measurements in each of these now 2^N distinct universes. The first term in the second line of [18] will equal one if S_k records exactly r measurements of the spin in the θ direction and will be zero otherwise. Because the N electrons are independent, the probability of getting any particular sequence S_k depends only on the number of electrons with spins measured to be in the θ direction and on the number with spins measured in the opposite direction. In particular, because the only sequences that contribute to [18] are those with r spins measured to be in the θ direction and those with $N - r$ spins to be in the opposite direction, we have

$$\text{prob}(S_k | N) = p^r q^{N-r}. \quad [19]$$

However, the order in which the r aligned spins and the $N - r$ antialigned spins are obtained is irrelevant, so the number of times [19] appears in the sum [18] will be C_r^N , the number of combinations. Thus, the sum [18] is

$$\text{prob}(r | N) = \frac{N!}{r!(N-r)!} p^r q^{N-r}. \quad [20]$$

The relative number of universes in which we would expect to measure aligned spin r times—that is to say, the expected value of the frequency with which we would measure the electron spin to be aligned with the axis of the measuring apparatus—is

$$\begin{aligned} \langle f \rangle &= \left\langle \frac{r}{N} \right\rangle = \sum_{r=0}^N \left(\frac{r}{N} \right) \text{prob}(r | N) \\ &= \sum_{r=1}^N \frac{(N-1)!}{(r-1)!(N-r)!} p^r q^{N-r} = p(p+q)^{N-1} = p, \end{aligned} \quad [21]$$

where the lower limit has been replaced by one, because the value of the $r = 0$ term is zero.

The sum in the second line of [21] has been evaluated by differentiating the generating function of the binomial series $\sum_{r=0}^N C_r^N p^r q^{N-r} = (p+q)^N$. That is, we have $\langle r^m \rangle = (p[d/dp])^m (p+q)^N$, where q is regarded as a constant in the differentiation, setting $p+q=1$ at the end. This trick also allows us to show that the variance of the difference between the frequency $f = r/N$ and the probability p vanishes as $N \rightarrow \infty$, because we have

$$\left\langle \left(\frac{r}{N} - p \right)^2 \right\rangle = \frac{pq}{N}. \quad [22]$$

In fact, all moments of the difference between f and p vanish as $N \rightarrow \infty$, because the generating function gives

$$\left\langle \left(\frac{r}{N} - p \right)^m \right\rangle \sim \frac{1}{N} + \text{higher order terms in } \frac{1}{N}. \quad [23]$$

So we have

$$\lim_{N \rightarrow \infty} \left(\frac{r}{N} \right) = p \quad [24]$$

in the sense that all of the moments vanish as $1/N$ as $N \rightarrow \infty$. This law of large numbers explains why it has been possible to believe, incorrectly, that probabilities are frequencies. This is not so, as Laplace emphasized over 200 y ago. It is, instead, that the quantum property of indistinguishability, applied to the observers, forces the measured frequencies to approach the probabilities.

This quantum property of indistinguishability also allows us to answer the question, "What were the two electron spins before a choice of the measurement directions was made?" The answer is given by the formalism, as indicated in Eqs. 1 and 2: All possible pairs exist. This must be the case, because any direction could have been chosen before a choice is made. The only reason this seems implausible is due to the neglect of the other universes—other observers—in the multiverse. The Hamilton–Jacobi equation asserts that there is an uncountable infinity of identical observers before any choice of the measurement basis is made. So each possible basis can be associated with one of these identical observers. However, because of indistinguishability, it is meaningless to say that a particular spin direction is associated with a particular universe. Rather, each spin direction is associated with them all. So, over the entire multiverse, all spin directions exist.

It cannot be emphasized too strongly that a probability cannot be an objective feature of reality, but instead a probability is a numerical expression of human ignorance of the actual state of affairs. However, we cannot improve our knowledge in the quantum mechanical case. Quantum indistinguishability and our ignorance of the other universes preclude an increase in knowledge.

Note that the above derivation of the measured frequencies requires the actual existence of the other universes of the multiverse. All of the sequences S_k really exist. The fact that the measured frequencies approach the probabilities requires that the indistinguishable versions of the physicist carry out the measurements simultaneously. So an observation of the approach of the frequencies to the probabilities is actually an observation of the effect of the simultaneous action in the multiverse of the analogs of the human observer.

Using an incorrect probability theory has prevented physicists from realizing that they have actually directly observed the effects of the other versions of themselves.

Many-Worlds Analysis of the Bell Experiment

Now let us use the MWI and Laplacean probability theory to analyze the Bell experiment.

The expectation value [9] for the product of the spins is just the sum of each outcome, multiplied respectively by probabilities of each of the four possible outcomes,

$$\begin{aligned} & (+1)(+1)P_{\uparrow\uparrow} + (+1)(-1)P_{\uparrow\downarrow} + \\ & + (-1)(+1)P_{\downarrow\uparrow} + (-1)(-1)P_{\downarrow\downarrow}, \end{aligned} \quad [25]$$

where $P_{\uparrow\downarrow}$ is the probability that the first electron is measured spin up and the second electron spin down and similarly for the other P s. Inserting these probabilities—the squares of the coefficients in [11]—into [25] gives the expectation value

$$\begin{aligned} & \frac{1}{2}\sin^2\theta/2 - \frac{1}{2}\cos^2\theta/2 - \frac{1}{2}\cos^2\theta/2 + \frac{1}{2}\sin^2\theta/2 \\ & = -\cos\theta = -\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2, \end{aligned} \quad [26]$$

which is the quantum expectation value, [9].

Once again it is essential to keep in mind the third measurement that compares the results of the two measurements of the spins and, by bringing the correlations between the worlds back to the same location, defines the relative orientation of the previous two measurements and in fact determines whether there is a twofold or a fourfold split. The way the measurement of [9] is actually carried out in the Aspect–Clauser–Freedman experiment is to let θ be random in any single run and for the results of each fixed θ from a series of runs be placed in separate bins. This separation requires the third measurement, and this local comparison measurement retains the correlations between the spins. The effect of throwing away this correlation information would be equivalent to averaging over all θ in the computation of the expectation value: The result is $\int_0^\pi \langle \Psi | (\hat{\mathbf{n}}_1 \cdot \sigma_1)(\hat{\mathbf{n}}_2 \cdot \sigma_2) | \Psi \rangle d\theta = 0$; i.e., the measured spin orientations of the two electrons are completely uncorrelated. This is what we would expect if each measurement of the electron spins is completely local, which in fact they are. There is no quantum nonlocality.

Bell's results (18–20) lead one to think otherwise. However, Bell made the tacit assumption that each electron's wave function is reduced by the measurement of its spin. Specifically, he assumed that the first electron's spin was determined by the measurement direction $\hat{\mathbf{n}}_1$ and the value of some local hidden variable parameters λ_1 : The first electron's spin is given by a function $A(\hat{\mathbf{n}}_1, \lambda_1)$. The second electron's spin is given by

an analogous function $B(\hat{\mathbf{n}}_2, \lambda_2)$, and so the hidden variable expectation value for the product of the spins would not be [13] but instead

$$\int \rho(\lambda_1, \lambda_2) A(\hat{\mathbf{n}}_1, \lambda_1) B(\hat{\mathbf{n}}_2, \lambda_2) d\lambda_1 d\lambda_2, \quad [27]$$

where $\rho(\lambda_1, \lambda_2)$ is the joint probability distribution for the hidden variables. By comparing a triple set of directions $(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \hat{\mathbf{n}}_3)$, Bell derived an inequality showing that the hidden variable [27] was inconsistent with quantum mechanical [9].

However, [27] assumes that the spin of each particle is a function of $\hat{\mathbf{n}}_i$ and λ_i ; that is, it assumes the spin at a location is single valued. This is explicitly denied by the MWI, as one can see by letting λ_i be the spatial coordinates of the i th electron. Bell's analysis tacitly assumes that the macroscopic world is a single-valued world. The automatic elimination of action at a distance by the MWI is a powerful argument for the validity of the MWI.

Conclusion

I have given several powerful arguments for the MWI: the restoration of locality of physics and the true origin of the Born interpretation. The main difficulty that many physicists have with the MWI is the required existence of the analogs of themselves. However, every time physicists measure a frequency and verify the quantum expectation value in the Bell inequality, they are actually seeing the effect of the analogs of themselves making the same measurements of the electron spin. The language of the frequency interpretation of probability has prevented physicists from seeing what is actually happening. It has prevented physicists from realizing that they are actually observing the effects in our universe of the other universes of the multiverse.

Have you ever seen Earth rotate on its axis? I have. I see it every day, when I see the earth's rotation expose the unmoving sun at dawn and cover the unmoving Sun from my view at dusk. Common language, however, says that the Sun sets and rises. Everyone believed this was so until Copernicus and Galileo taught us to see nature through the laws of physics. It is time to see the measurements of the electron spin frequencies through the laws of quantum mechanics, which apply not only to electrons, but also to the physicists who measure these spins.

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