# Exoplanet orbital eccentricity: Multiplicity relation and the Solar System

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**The known population of exoplanets exhibits a much wider range of orbital eccentricities than Solar System planets and has a much higher average eccentricity. These facts have been widely interpreted to indicate that the Solar System is an atypical member of the overall population of planetary systems. We report here on a strong anticorrelation of orbital eccentricity with multiplicity (number of planets in the system) among cataloged radial velocity (RV) systems. The mean, median, and rough distribution of eccentricities of Solar System planets fits an extrapolation of this anticorrelation to the eight-planet case rather precisely despite the fact that no more than two Solar System planets would be detectable with RV data comparable to that in the exoplanet sample. Moreover, even if regarded as a single or double planetary system, the Solar System lies in a reasonably heavily populated region of eccentricity–multiplicity space. Thus, the Solar System is not anomalous among known exoplanetary systems with respect to eccentricities when its multiplicity is taken into account. Specifically, as the multiplicity of a system increases, the eccentricity decreases roughly as a power law of index –1.20. A simple and plausible but ad hoc and model-dependent interpretation of this relationship implies that ∼80% of the one-planet and 25% of the two-planet systems in our sample have additional, as yet undiscovered, members but that systems of higher observed multiplicity are largely complete (i.e., relatively rarely contain additional undiscovered planets). If low eccentricities indeed favor high multiplicities, habitability may be more common in systems with a larger number of planets.**

orbital eccentricities | dynamical evolution | Solar System | radial velocity

Solar System orbital eccentricities are unusually low compared with those of exoplanets. This fact is one of the most frequently noted major surprises revealed by the discovery and early explorations of the exoplanet population orbiting Sun-like stars and has been widely interpreted to indicate that the Solar System is not a representative example of a planetary system (reviews by refs. 1–3 and references therein). Many planetary formation theories developed before the discovery of exoplanets suggested planets would have eccentricities similar to the Solar System planets (4, 5). Several attempts have been made to accurately model the dynamical evolution of planetary systems since then, with the goal of explaining the observed eccentricity distribution (6–10). These papers invoke planet–planet interactions as the primary mechanism determining the distribution of orbital eccentricities. The most recent of these papers (10) concludes that there would be a dependence of eccentricity on multiplicity (the number of planets in the system) in this scenario. We use existing radial velocity (RV) exoplanet data to test that prediction.

Our dataset consists of 403 of the 441 cataloged RV exoplanets obtained since the 1990s (exoplanet.org). Of these, 127 are members of known multiple-planet systems with multiplicities of up to six. The data are sufficient to allow an estimate of the relationship of eccentricity to multiplicity. It has been noted that eccentricity in two-planet systems tends to be lower than in single-planet systems (11). This paper explores the relation at higher multiplicities and notes its unexpected and surprising consistency with the Solar System case.

The dataset is discussed in the next section. We then show the trend in eccentricity with multiplicity and comment on possible sources of error and bias. Next we measure the mean, median, and probability density distribution of eccentricities for various multiplicities and fit them to a simple power-law model for multiplicities greater than two. This fit is used to make a rough estimate of the number of higher-multiplicity systems likely to be contaminating the one- and two-planet system samples due to as yet undiscovered members under plausible, but far from certain, assumptions. Finally, we conclude with some discussion of the implications of this result.

## The Dataset

Only RV exoplanet data obtained from exoplanet.org are used for this analysis. All of the RV exoplanets listed on the website that have a measured eccentricity are included in the analysis. If the eccentricity of the planet was not listed or if it was given as zero, the exoplanet was excluded from our sample. Thirty-eight systems were excluded on the latter basis, of which 29 had their eccentricity constrained to zero in the orbital fit. Table 1 lists the number of RV planets in each multiplicity bin.

The five- and six-planet systems, one of each, were combined into one bin so that there was sufficient data for a statistical analysis. Note that the total number of planets with a given multiplicity is not necessarily a multiple of the multiplicity of the system because not all planets in some systems have measured eccentricities. In the cases where a certain parameter of an exoplanet or exoplanet system is under debate, we used the exoplanet.org value. For instance, the multiplicity of a system may be three planets based on observations; however, the motion of those three planets may imply that there are additional companions, and therefore the multiplicity of the system may be listed as four planets rather than three on exoplanet.org. In all such cases, we simply adopt the data listed on the website for

### Significance

The Solar System planets have near-circular orbits (i.e., unusually low eccentricity) compared with the known population of exoplanets, planets that orbit stars other than the Sun. This fact has been widely interpreted to indicate that the Solar System is an atypical member of the overall population of planetary systems. We find a strong anticorrelation of orbital eccentricity with the number of planets (multiplicity) in a system that extrapolates nicely to the eight-planet, Solar System case despite the fact that no more than two Solar System planets would be detectable in the sample in which the anticorrelation was discovered. Habitability may be more common in systems with a larger number of planets, which have lower typical eccentricities.

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**Table 1. Number of planets in dataset for given multiplicity**

| Multiplicity (no. planets in system) | Total number of planets with given multiplicity* |
|---|---|
| 1 | 276 |
| 2 | 81 |
| 3 | 25 |
| 4 | 12 |
| 5 or 6 | 9 |

*This value is not necessarily a multiple of the system multiplicity since not all exoplanets have measured eccentricities.

consistency. The data were taken from the website over the course of several weeks in February and March 2014. We used RV data only for our analysis since the planets in that subset of the dataset typically have known and relatively reliably measured eccentricities.

## A Trend in Multiplicity Versus Eccentricity

Fig. 1 shows the cumulative eccentricity distribution function in exoplanet systems and the Solar System for systems of various multiplicities. There is a clear trend toward lower eccentricities in higher-multiplicity systems. This trend is perhaps most noticeable in the top (high eccentricity) half of the plot. Fig. 2 shows the eccentricity vs. semimajor axis relation at each multiplicity and also displays the strong tendency toward lower eccentricity at higher multiplicity.

The strength and nature of the anticorrelation of orbital eccentricity with multiplicity is even more dramatically revealed by plotting the mean and median eccentricity as a function of multiplicity, as shown in Fig. 3. Two features of this figure are worthy of special note (*i*) The divergence of the one-planet systems, and two-planet systems to a lesser extent, from what appears to be a power-law relation at higher multiplicities is noticeable in both the mean and median curves. (ii) The Solar System fits the eccentricity trend at higher multiplicity quite well despite the fact that the RV data quality for the exoplanetary systems would only allow the detection of Jupiter and perhaps Saturn. In other words, it would be more statistically consistent to plot the Solar System point in these figures at a multiplicity of one or two, as also shown in the figure. In either case, the Solar System does not appear to have unusually low orbital eccentricities with respect to exoplanet systems, but rather the Solar System planet eccentricities are consistent with the exoplanet eccentricity distribution when multiplicity is taken into account.

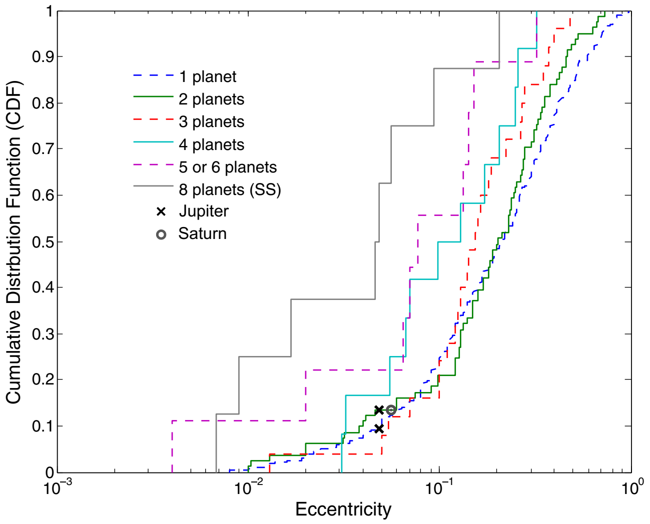

**Fig. 1.** Cumulative eccentricity distributions in RV exoplanet systems and the Solar System for various multiplicities. There is a trend toward lower eccentricities at higher multiplicities. The abbreviation "SS" is for Solar System planets. A black x is shown for where Jupiter would appear on the one- and two-planet distributions, and a gray circle is shown for Saturn on the two-planet curves. This demonstrates that even if the Solar System were detected via RV as a one- or two-planet system, it would still be consistent with the data.

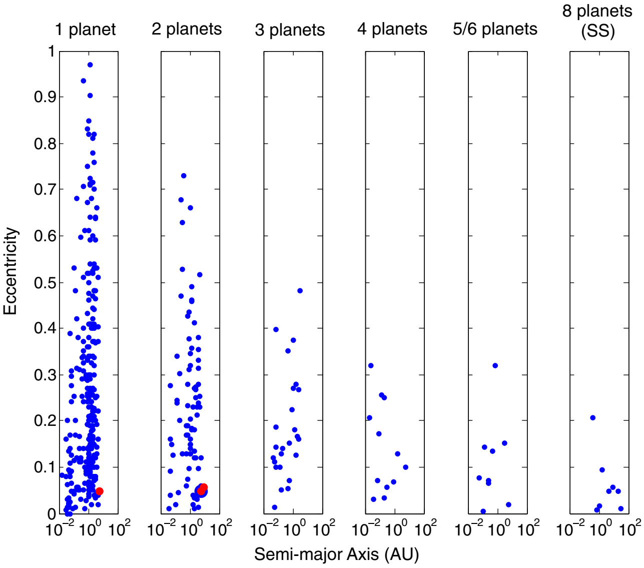

**Fig. 2.** Eccentricity versus semimajor axis going from low (left) to high multiplicity (right). A red dot is shown for where Jupiter would appear on the one- and two-planet distributions, and for Saturn on the two-planet distribution. This demonstrates that even if the Solar System were detected via RV as a one- or two-planet system, it would still be consistent with the data.

The uncertainties of the mean eccentricities shown in Fig. 3 were calculated by bootstrapping. The bootstrap method gave an uncertainty of approximately two thirds of the usual rms estimator, which is consistent with the limited extent of the eccentricity distribution tail. The uncertainties in the medians correspond to the one-third and two-thirds points in the distributions shown in Fig. 1 divided by $\sqrt{N-1}$, where $N$ is the number of points in the multiplicity bin.

A two-sample Kolmogorov–Smirnov (K–S) test was used to determine the significance of the differences in the eccentricity distributions of systems with different multiplicities. The test was applied both to the entire data sample and to the subsamples consisting of the highest 75% of the eccentricities in each multiplicity subsample. Use of the latter subset of the data is motivated by the possibility that low-multiplicity systems with low-eccentricity orbits may have undiscovered members and thus actually be of higher multiplicity.

In both cases, the $P$ values consistently decrease for larger difference in the multiplicities of the two samples being compared. This is consistent with the systematic trends visible in Figs. 1 and 3. The results of the tests for the full sample and the high-eccentricity subsamples are shown in Table 2. In the high-eccentricity subsample case, for samples that have multiplicities that differ by at least two planets, the K–S test yields a statistically significant difference ($P \leq 0.05$). At higher multiplicities, the significance $(1 - \mathrm{p})$ of the difference between distributions with adjacent multiplicities generally decreases. This may well be an artifact of the smaller sample sizes at higher multiplicities rather than an actual convergence of the eccentricity distributions.

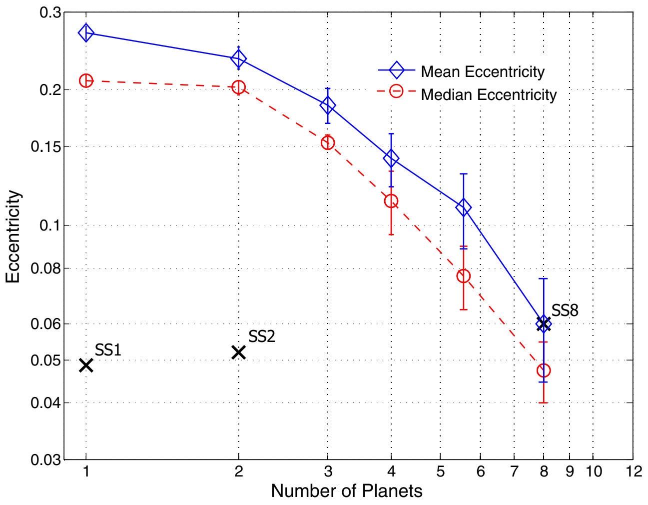

**Fig. 3.** Mean and median eccentricities in RV exoplanet systems and the Solar System as a function of multiplicity (number of planets in the system). As the number of planets increases, eccentricity decreases. A plateau in eccentricity at low multiplicity is noticeable especially in the median. This is possibly due to contamination of the one-planet data with higher-multiplicity systems. If the Solar System were detected via RV, it is mostly likely that only Jupiter (one planet) or Jupiter and Saturn (two planets) would be detected. The mean eccentricity in these scenarios is marked on the plot by a black x and labeled SS1 and SS2, respectively. If the entire Solar System were detected, it would appear in the eight-planet bin—this case is labeled SS8. Note that the SS8 point is consistent with the trend in the mean, whereas the SS1 and SS2 points are not (which should suggest to a hypothetical alien RV exoplanet hunter studying the Solar System from afar that it contains more than one or two planets).

Systematic errors in the eccentricity distribution of exoplanets are possibly due to the techniques used for measuring this parameter and biases associated with exoplanet detection methods. Shen and Turner (12) studied the systematic bias of RV determinations of eccentricity using the conventional $\chi^2$ fitting and showed that such measurements are biased high and significantly so for low values of $N^{1/2}K/\sigma$, where $K$ is the velocity semi-amplitude and σ is the typical uncertainty of each of the $N$ velocity measurements. One might then be concerned that higher-multiplicity systems receive more telescope time and thus have a higher value of this parameter and a more accurately determined eccentricity values than lower-multiplicity systems. If this were the case, one might observe a multiplicity–eccentricity trend due to observational bias rather than a true physical phenomena. To ensure that this effect is negligible for our dataset, we conducted a careful analysis of the measured $N^{1/2}K/\sigma$ values and found that there is no significant effect in our sample. Furthermore, higher-multiplicity systems tended to have $N^{1/2}K/\sigma$ values similar to those of lower-multiplicity systems. Shen and Turner (12) only expect eccentricity to increase for $N^{1/2}K/\sigma \gtrsim 15$, and most of the measurements in our sample have higher $N^{1/2}K/\sigma$ values than that threshold. For these reasons, we do not believe that the multiplicity–eccentricity trend we reported here is influenced by this potential bias.

A probable shortcoming of the data is undetected companions, especially in low-multiplicity systems. Given the eccentricity–multiplicity relation, this would add low-eccentricity planets to the low-multiplicity subsamples. Indeed, there does appear to be an excess of low eccentricities in the low-multiplicity systems relative to a smooth extrapolation of their higher-eccentricity distributions (see Fig. 1), and this excess of low-eccentricity orbits is particularly noticeable in the single-planet and two-planet distributions. This raises the question of whether or not the turnover in the trend at low multiplicities in Fig. 3 is real or due to contamination of the low multiplicity by high-multiplicity systems with so-far-undiscovered planets. In *Analysis*, we give a crude and model-dependent estimate of the number of higher-multiple systems contaminating the one- and two-planet distributions by fitting a power law to the higher-multiplicity data.

## Analysis

For each multiplicity, the probability density distributions of eccentricities were derived from the cumulative eccentricity distributions and are shown in Fig. 4. The probability density distributions were obtained by taking the derivative of polynomials that were fit to the cumulative eccentricity distribution function (the data shown in Fig. 1). This procedure yields a heavily smoothed estimate of the true differential distributions. Second-order polynomials were fit to the data, except in the one-planet case. A K–S test indicated that a second-order polynomial was inconsistent ($P < 0.01$) with the one-planet data, and a third-order polynomial was necessary to obtain an acceptable fit.

If we knew the true occurrence rate of planetary systems as a function of multiplicity, these probability density distributions would allow determination of the likelihood of a planet with any given eccentricity belonging to a system of a particular multiplicity. While that conditional is clearly not satisfied at the present time, it is clear that if one wants to find companions to existing exoplanets, one- and two-planet systems with low eccentricities are relatively good targets for further investigation.

A power law was fit to the median eccentricity vs. multiplicity relationship and is shown in Fig. 5. The fit is only for the higher-multiplicity data, specifically $M > 2$ where $M$ is the multiplicity or number of planets in the system. The one- and two-planet systems were excluded from the fit on the assumption that they are likely contaminated with planets that belong in higher-multiplicity bins due to so-far-undetected additional planets in those systems. If the eccentricity ($e$) versus multiplicity ($M$) relation obeys this power-law fit,

**Table 2. K–S test on eccentricity for various multiplicities**

| | One planet | Two planets | Three planets | Four planets | Five or six planets | Eight planets |
|---|---|---|---|---|---|---|
| P value for K–S test* | | | | | | |
| One planet | — | | | | | |
| Two planets | 0.34 | — | | | | |
| Three planets | 0.02 | 0.09 | — | | | |
| Four planets | 0.02 | 0.04 | 0.18 | — | | |
| Five or six planets | <0.01 | <0.01 | 0.05 | 0.46 | — | |
| Eight planets | <0.01 | <0.01 | <0.01 | 0.05 | 0.09 | — |
| *P* value for K–S test† | | | | | | |
| One planet | — | | | | | |
| Two planets | 0.57 | — | | | | |
| Three planets | 0.14 | 0.26 | — | | | |
| Four planets | 0.11 | 0.21 | 0.25 | — | | |
| Five or six planets | 0.02 | 0.02 | 0.16 | 0.64 | — | |
| Eight planets | <0.01 | <0.01 | <0.01 | 0.12 | 0.12 | — |

Dashes indicate coincidence of two sets of identical multiplicity data for which the *P* value has no significant meaning.

*Including highest 75% of eccentricities for each multiplicity shown in Fig. 1.

†Including entire sample.

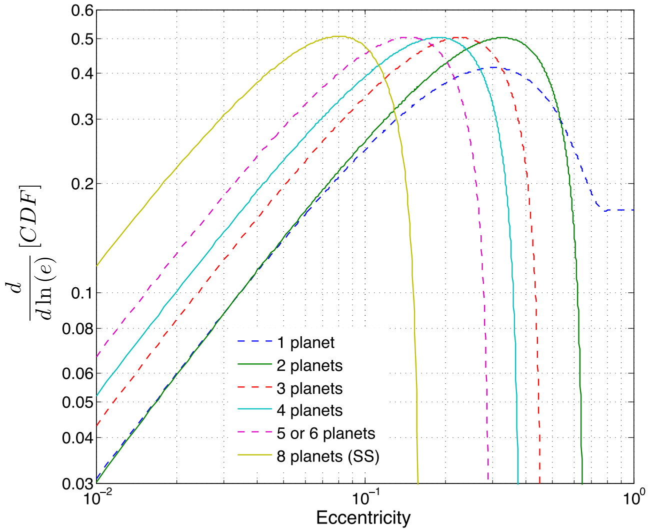

**Fig. 4.** Eccentricity probability density distributions for various multiplicities based on polynomial fits to the cumulative distribution functions (CDF). Lower-eccentricity systems are more likely to belong to higher-multiplicity systems. Second-order polynomials were fit to the cumulative distributions for all multiplicities except the single-planet systems, in which case a third-order polynomial was used.

$$e(M) \approx 0.584\,M^{-1.20}, \qquad [1]$$

then the relationship can be used to estimate the number of systems in the one- and two-planet bins that belong to higher-multiplicity systems. Proceeding on the basis of this hypothesis leads to the conclusion that ~80% of the one-planet systems and 25% of the two-planet systems belong to higher-multiplicity systems with as-yet-undiscovered members. While this scenario seems quite plausible, we emphasize that the estimates of companions in the one- and two-planet bins is dependent on the assumption that the true eccentricity–multiplicity trend behaves like a power law of constant index at all multiplicities and that the $M > 2$ subsamples are not significantly contaminated by yet higher multiplicity systems due to undiscovered members. It is also quite possible that the turnover or plateau seen in the eccentricity distribution is an actual physical phenomena.

The power-law fit can also be used to predict the median eccentricity of as-yet-undiscovered high-multiplicity RV systems. For example, we estimate that seven-planet RV systems, when discovered, will have a median eccentricity of 0.06 ± 0.01.

In addition to studying the relation between eccentricity and multiplicity, we also checked for a trend in semimajor axis with multiplicity but found no significant relationship with the exception of the trend noted by ref. 13 that extremely short period exoplanets ($\lesssim 10$ d) typically have lower orbital eccentricities, presumably due to tidal circularization.

## Discussion

The distribution of orbital eccentricities as a function of multiplicity provides an important previously unidentified constraint (or clue) for planetary system formation and evolution models. The observational evidence for the multiplicity–eccentricity relation can perhaps be explained by planet–planet interactions and dynamical evolution. The general trend of decreasing eccentricity with increasing multiplicity was predicted by ref. 10 using dynamical evolution simulations. Although the relationship reported here is qualitatively similar to that prediction, the observed dependence of $e$ on $M$ is steeper and extends to smaller eccentricities than the one produced by the simulations carried out by Juric and Tremaine (10).

Although we are aware of no reason to suspect that the strong dependence of orbital eccentricity on multiplicity among RV exoplanet systems reported here is due to some defect or bias in the available data, it is undeniable that it is based on an inhomogeneous and statistically ill-conditioned data set. However, even if it were the case that the trend shown in Fig. 3 is somehow a spurious nonphysical one, the empirical correlation is still an important one that merits detailed future study because such a situation would imply that either a large number of low-eccentricity, low-multiplicity exoplanetary systems or high-eccentricity, high-multiplicity ones have been systematically missed in RV searches.

The rather precise consistency of the eccentricity distribution, as well as its mean and median, of the eight-planet Solar System with the observed correlations presented above is rather puzzling because existing RV data have neither the precision nor the duration required to detect Solar System planets other than Jupiter and, perhaps, Saturn. Thus, the fair comparison with the RV sample would regard the Solar System as a multiplicity one or two case, as illustrated in Figs. 2 and 3. There are at least two possible interpretations. One is that this is simply a statistical fluke and that the comparison of the Solar System to exoplanetary systems would be quite different if the RV data used to construct our sample were of sufficient quality to detect all eight Solar System planets. Another is that the comparison is valid because much better RV data would not result in the detection of a significant number of additional planets in the $M > 2$ systems.

Intriguingly, the Solar Systems position in eccentricity–multiplicity space is not particularly unusual even if it is regarded as an $M = 1$ or $M = 2$ case, as illustrated in Fig. 2, which shows that the Solar System lies in a reasonably densely populated region of the space at $M = 1$ or $M = 2$ as well as fitting an extrapolation to $M = 8$.

## Conclusions

We find that the orbital eccentricities of the Solar System planets are consistent with those found in exoplanetary systems when multiplicity is taken into account. Specifically, we find that as the multiplicity of a system increases, the eccentricity decreases. This relation can be well fit by a power law at multiplicities greater

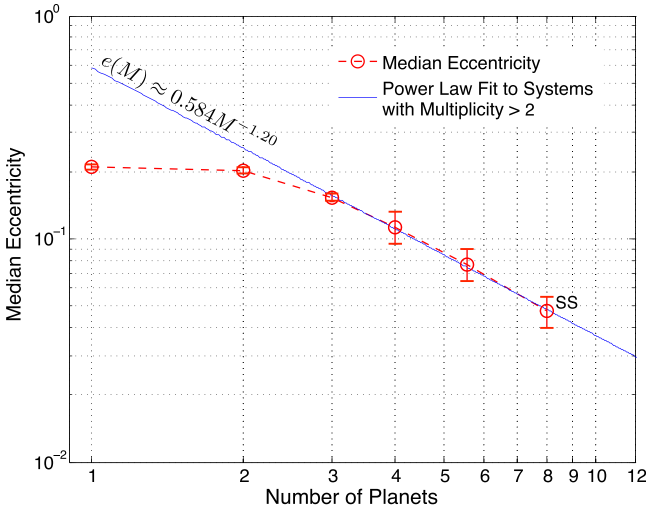

**Fig. 5.** Power-law fit to median eccentricity in systems with multiplicity greater than two planets. The fit suggests that the one- and two-planet data are contaminated with higher-multiplicity systems due to so-far-undiscovered members.

than two. A simplistic and model-dependent interpretation of this fit implies that ∼80% of one-planet systems and ∼25% of two-planet systems are likely members of higher-multiplicity systems. The distribution of orbital eccentricities as a function of multiplicity provides an important previously unidentified constraint for planetary system formation and evolution models. Any theory that accounts for this trend would be adequate to explain the distributions of eccentricities seen both in our Solar System and in exoplanetary systems.

Because low eccentricity is arguably advantageous for habitability (14, 15), this relationship suggests that high-multiplicity systems may be more likely to host habitable exoplanets.

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