Granular impact cratering by liquid drops: Understanding raindrop imprints through an analogy to asteroid strikes

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When a granular material is impacted by a sphere, its surface deforms like a liquid yet it preserves a circular crater like a solid. Although the mechanism of granular impact cratering by solid spheres is well explored, our knowledge on granular impact cratering by liquid drops is still very limited. Here, by combining high-speed photography with high-precision laser profilometry, we investigate liquid-drop impact dynamics on granular surface and monitor the morphology of resulting impact craters. Surprisingly, we find that despite the enormous energy and length difference, granular impact cratering by liquid drops follows the same energy scaling and reproduces the same crater morphology as that of asteroid impact craters. Inspired by this similarity, we integrate the physical insight from planetary sciences, the liquid marble model from fluid mechanics, and the concept of jamming transition from granular physics into a simple theoretical framework that quantitatively describes all of the main features of liquid-drop imprints in granular media. Our study sheds light on the mechanisms governing raindrop impacts on granular surfaces and reveals a remarkable analogy between familiar phenomena of raining and catastrophic asteroid strikes.

liquid impacts | granular impact cratering | jamming | liquid marble

Granular impact cratering by liquid drops is likely familiar to all of us who have watched raindrops splashing in a backyard or on a beach. It is directly relevant to many important natural, agricultural, and industrial processes such as soil erosion (1, 2), drip irrigation (3), dispersion of microorganisms in soil (4), and spray-coating of particles and powders. The vestige of raindrop imprints in fossilized granular media has even been used to infer air density on Earth 2.7 billion years ago (5). Hence, understanding the dynamics of liquid-drop impacts on granular media and predicting the morphology of resulting impact craters are of great importance for a wide range of basic research and practical applications.

Directly related to two long-standing problems in fluid and granular physics research, i.e., drop impact on solid/liquid surfaces (6–9) and granular impact cratering by solid spheres (10–16), liquid-drop impact on granular surfaces is surely more complicated. Although several recent experiments have attempted to investigate the morphology of liquid-drop impact craters (17–21), a coherent picture for describing various features of the impact craters is still lacking. Even for the most straightforward impact-energy (E) dependence of the size of liquid-drop impact craters, the results remain controversial and incomplete (17, 19, 20). Katsuragi (17) and Delon et al. (19) reported that the diameter of liquid-drop impact craters Dc scales as the 1/4 power of the Weber number of liquid drops, which yields $D_c \sim E^{1/4}$, quantitatively similar to the energy scaling for low-speed solid-sphere impact cratering (10, 11). However, because the energy balance of liquid-drop impacts is different from that of solid-sphere impacts, the energy scaling argument used for solid-sphere impact cratering cannot be applied to explain the 1/4 power. Instead, Katsuragi argued that the power arises from the scaling of the maximal spreading diameter of the impinging drop, which coincidently follows the same 1/4 scaling with $E$ (22). However, a later study by Nezzaoui and Skurlys showed that $D_c$ is not equal to the maximal spreading diameter and a different scaling with $D_c \sim E^{0.18}$ was found (20). Although covering a larger dynamic range of $E$, Nezzaoui and Skurlys only investigated the scaling dependence on $E$ and failed to provide a full scaling for $D_c$. The origin of the strange 0.18 scaling in liquid-drop impact cratering is still unclear. Finally, in addition to the diameter of impact craters, other important properties of liquid-drop impact craters such as the depth of impact craters and the shape of granular residues inside craters have not been systematically explored so far.

The challenges faced in the study of liquid-drop impact on granular surfaces are mainly due to the large number of relevant parameters involved in the process, the inability of existing methods for resolving the 3D structure of impact craters, and the difficulty in extending the dynamic range of $E$ in experiments. Here, we investigate the dynamics of liquid-drop impacts on granular surfaces across the largest range of impact energy that has been probed so far, which covers more than four decades from the drop deposition regime to the drop terminal velocity regime. Through a systemic study using different liquid drops and granular particles at various ambient pressures, we obtain a full dimensionless scaling for the diameter of liquid-drop impact craters. Surprisingly, we find that this scaling follows the well-established Schmidt–Holsapple scaling rule associated with asteroid impact cratering (23). Moreover, by combining high-speed photography with high-precision laser profilometry, we nonintrusively measure the depth of impact craters underneath

Significance

We provide a quantitative understanding of raindrop impacts on sandy surfaces—a ubiquitous phenomenon relevant to many important natural, agricultural, and industrial processes. Combining high-speed photography with high-precision laser profilometry, we investigate the dynamics of liquid-drop impacts on granular surfaces and monitor the morphology of resulting impact craters. Remarkably, we discover a quantitative similarity between liquid-drop impacts and asteroid strikes in terms of both the energy scaling and the aspect ratio of their impact craters. Such a similarity inspires us to apply the idea developed in planetary sciences to liquid-drop impact cratering, which leads to a model that quantitatively describes various features of liquid-drop imprints.

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the impinging drop. The measurement reveals that liquid-drop impact craters and asteroid impact craters exhibit a self-similar shape despite their enormous length difference over seven orders of magnitude. These remarkable findings inspire us to apply the physical insight developed for asteroid impact cratering to the problem of liquid-drop impact cratering. The insight, in combination with the concepts of liquid marble (24) and particle jamming transition (25, 26), leads to a simple coherent model that quantitatively captures all of the main features of liquid-drop imprints in granular media including the diameter and the depth of impact craters and the shape of granular residues.

**Results: Liquid-Drop Impact Dynamics**

In our experiments, we release a stationary water drop of diameter $D$ ranging from 1.4 to 4.6 mm from a height $h$. $D$ is chosen to represent the size range of natural raindrops (5, 27). The drop falls vertically in air onto a granular bed comprising $d_{sand} = 90 \pm 15 \mu m$ glass beads with volume fraction $\phi = 0.60$. To adjust the range of impact energy $E$, we vary $h$ from 1.8 mm up to 12 m, allowing a 4.6-mm drop to reach 98% of its terminal velocity (Materials and Methods).

The dynamics of liquid-drop impact on a granular surface are captured using high-speed photography as illustrated in Fig. 1 for the strike of a water drop at three different $E$ (Movies S1–S3). Upon impact, the drop penetrates into the top layer of the granular bed (Fig. 1 A, F, and K). After the initial impact, the drop can be treated as an incompressible fluid. The downward motion of the top part of the drop causes drop deformation and spreading.

At low $E$, the spreading liquid lamella moves horizontally along the surface of a shallow crater (Fig. 1B). The lamella retracts after reaching the maximum spreading diameter and entrains a layer of granular particles on its surface. Because the lamella’s surface-to-volume ratio reduces as it recedes, particles at the interface are gradually pushed into the liquid bulk, resulting in a “liquid marble” armored with a thick layer of granular particles (Fig. 1 C and D) (24). Above $E = 1.9 \times 10^{-5}$ J, the marble can even bounce off the granular bed (Fig. 1D). The jumping height of the marble is nonmonotonic with increasing $E$. As the spreading diameter increases, the lamella traps more particles, which increases the weight of the marble and reduces the jumping height.

Increasing $E$ further, the rim of the spreading lamella develops a fingering instability (Fig. 1G). After reaching the maximum spreading diameter, the fingers start to retract and gradually push particles at interface into the bulk (Fig. 1 H and J). The process continues until the concentration of particles within the retracting lamella becomes so high that the receding motion is completely arrested due to the jamming of particles. The jamming transition occurs before the fingers can fully retract back to a sphere, which leads to an asymmetric liquid marble with finger protrusions on its surface (Fig. 1 I and J). The length of these protrusions increases with $E$. At this intermediate $E$, the marble stops bouncing off the surface.

At even larger $E$, a water crown is formed along the wall of a deep crater (Fig. 1 L and M). The crown detaches from the granular surface at the edge of the crater. Above $E = 9.7 \times 10^{-5}$ J, the rim of the crown becomes unstable and disintegrates into secondary droplets (Fig. 1N). This violent splashing process effectively mixes granular particles with the liquid. Finally, the crown, fully loaded with particles, retracts and falls flat on the surface (Fig. 1O).

The dynamics of liquid-drop impacts illustrated by high-speed photography provide essential information for understanding the morphology of liquid-drop impact craters. Based on the dynamics, we will develop a simple theoretical understanding of various features of liquid-drop impact craters in Theory and Discussion. We shall first show our experimental results on the morphology of liquid-drop impact craters.

**Results: Morphology of Impact Craters**

After impact, water gradually drains into the granular bed and various fascinating crater topologies are observed at the end (Fig. 2 A–F). To fully characterize the morphology of liquid-drop imprints, we need to consider three main features of impact craters, i.e., the diameter of impact craters, the depth of impact craters, and the granular residues left in the center of impact craters.

**Diameter of Impact Craters.** We characterize the size of an impact crater by measuring its diameter, $D_c$ (Fig. 2C). Plotting $D_c$ versus $E$ reveals a power-law scaling with an exponent of $0.17 \pm 0.01$ (Fig. 3A), consistent with Nefzaoui and Skurtys’s result (20). This scaling is visibly different from the $1/4$ power-law scaling associated with the impact craters created by low-speed solid spheres. The $1/4$ scaling of solid-sphere impacts arises when $E$ lifts granular particles of volume $\sim D_c^3$ to a height of $\sim D_c$ against the gravity (10, 11, 23).

Surprisingly, the $0.17$ scaling is quantitatively similar to the Schmidt–Holsapple (S-H) scaling from hypervelocity impact cratering associated with asteroid strikes (23):

$$D_c \sim g^{-0.17} \cdot \rho U^{0.34} \cdot E^{0.32}$$

where $U$ and $\rho$ are the impact velocity and the density of the projectile, respectively, and $g$ is the gravitational acceleration. $\rho$ emerges in Eq. 1 when we convert $U$ into the impact energy $E$.

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**Fig. 1.** Impact of a water drop on a granular surface. Snapshots from high-speed movies showing the impact of a 3.1-mm water drop with $E = 7.8 \times 10^{-6}$ J (A–E) (Movie S1), $E = 6.0 \times 10^{-5}$ J (F–J) (Movie S2), and $2.3 \times 10^{-4}$ J (K–O) (Movie S3). For the low $E$, the time elapsed after the initial impact is $t = 1.1$ ms (A), 4.5 ms (B), 13.8 ms (C), 32.8 ms (D), and 84.0 ms (E). For the intermediate $E$, $t = 0.3$ ms (F), 5.7 ms (G), 11.9 ms (H), 19.4 ms (I), and 56.8 ms (J). For the high $E$, $t = 0.3$ ms (K), 1.0 ms (L), 1.9 ms (M), 6.4 ms (N), and 29.1 ms (O). Scale bars: 3.0 mm. Water in the liquid–granular mixtures gradually drains into the bed on the time scale of a second.
Eq. 1 inspired us to apply the full S-H scaling to our data. Remarkably, we find that the variation of $D_k$ with different $D$ collapses to a constant, $C = D_c/((\rho g)^{0.17} D^{3.2} E^{0.17}) = 1.74 \pm 0.15$ (Fig. 3B). Moreover, we tested the scaling using nine different liquids and seven different granular particles at two different ambient pressures. The results all conform to Eq. 1 (Fig. S1). Particularly, the $D_k$ scaling is independent of or only weakly depends on liquid properties such as density, viscosity, or surface tension.

**Depth of Impact Craters.** The quantitative similarity between liquid-drop impact cratering and asteroid impact cratering also extends to the aspect ratio of their impact craters, $\alpha = d_i/D_c$. Here, $d_i$ is the depth of crater, defined as the vertical distance between the rim and the bottom floor of the crater.

Previous studies reported the depth of crater in the presence of granular residues (17, 21), which, however, does not reflect the true bottom of a crater underneath the granular residues. Here, to detect $d_i$ without the optical obstruction of granular residues in the center of crater, we focus on the range of $E$ where the liquid marble bounces off the surface (Fig. 1D). The landing of the marble does not trigger further granular avalanche and, therefore, does not modify the crater depth at later times (Movie S1). Even though $E$ with jumping marbles does not cover the full dynamic range of our experiments, it still extends for almost two decades, allowing us to measure $d_i$ in a sufficient range comparable to other impact cratering experiments (11, 28).

Within this $E$ range, $d_i$ increases linearly with $D_c$, which leads to a constant crater aspect ratio $\alpha = 0.20 \pm 0.01$ (Fig. 4, Inset). As a comparison, simple craters from the moon, Mars, and Mercury also show an aspect ratio $\alpha = 0.20 \pm 0.03$ (29). Even though there is a seven-order-of-magnitude difference in lengths, liquid-drop impact craters and planetary craters show the same aspect ratio within experimental errors (Fig. 4). The angle of repose of granular materials $\theta_i$ sets an upper limit for $\alpha$. For $\theta_i = 26^\circ$—the angle measured from our experiments—we have $\alpha < \tan \theta_i/2 = 0.24$. However, the geometrical factor alone is not sufficient to explain the similarities and differences between impact cratering processes. The aspect ratio of impact craters from low-speed solid-sphere impacts is 0.12, substantially smaller than that of liquid-drop impact craters (Fig. 4, Inset). With strong scattering around 0.16, the aspect ratio of impact craters from hypervelocity solid-sphere impact experiments partially overlaps with that of liquid-drop impact craters (Fig. 4).

A theoretical understanding of the scaling of the diameter and the depth of impact craters and a discussion on the similarity between liquid-drop impact cratering and asteroid impact cratering will be presented below in Theory and Discussion.

**Granular Residues.** Finally, we also measure the size of granular residues $D_g$ in the center of impact craters (Fig. 2C). $D_g$ as a function of $E$ exhibits two different regimes (Fig. 5). At low $E$, $D_g$ slowly increases. Above certain threshold impact energy $E^*$, it starts to enlarge strongly and merges into a master curve.

The trend of $D_g$ can be qualitatively understood based on the impact dynamics. As shown in Fig. 1, a liquid marble coated with a layer of granular particles is formed at low $E$ during impacts. The thickness of the granular layer depends on the number of entrained particles. The liquid phase of the marble eventually drains into the granular bed and particles are left as a granular residue. With a small $E$, particles cover only the surface of the marble. Hence, when the liquid drains into the bed, a ring of particles is left (Fig. 2A). Because the maximal spreading diameter of the impinging drop increases with $E$, at larger $E$ an increasing number of particles are entrained at the lamella interface and pushed into the bulk of the liquid marble, which leads to a liquid marble with a thicker layer of granular particles. As a result, the hole at the center of the ring-shaped residues gradually fills up (Fig. 2B). At $E$ close to the transition impact energy $E^*$, particles completely saturate the marble, which leaves a solid pellet-shaped residue in the crater (Fig. 2C). Increasing $E$ above $E^*$, the receding lamella cannot fully restore back to a spherical shape due to the jamming of particles inside the marble.
During asteroid impacts, impact cratering, which may help to explain the origin of the S-H scaling into the physical picture derived from the studies of asteroid impact cratering and asteroid impact cratering more generally in analogy with the low-speed solid-sphere impact experiments (29). Group 4 is from liquid-drop impact cratering experiments (11). Group 4 is from liquid-drop impacts with \( D = 3.9 \) mm (blue diamonds), 3.1 mm (green triangles), 2.6 mm (red disks), and 1.4 mm (dark yellow squares). Circles are for \( D = 3.1 \) mm water drops impacting at one-tenth of the atmospheric pressure. (Insets) \( d_i/D_c \) of liquid-drop impact craters. The upper and lower dashed lines indicate the aspect ratio of planetary impact craters (0.20) and low-speed solid-sphere impact craters (0.12), respectively.

**Fig. 4.** Aspect ratio of liquid-drop impact craters. \( d_i \) versus \( D_c \) for four different impact cratering processes. Group 1 is from astronomical observations of asteroid impact craters on different planetary bodies (28). Group 2 is from hypervelocity solid-sphere impact experiments (28). Group 3 is from low-speed solid-sphere impact experiments (11). Group 4 is from liquid-drop impacts with \( D = 3.9 \) mm (blue diamonds), 3.1 mm (green triangles), 2.6 mm (red disks), and 1.4 mm (dark yellow squares). Circles are for \( D = 3.1 \) mm water drops impacting at one-tenth of the atmospheric pressure. (Insets) \( d_i/D_c \) of liquid-drop impact craters. The upper and lower dashed lines indicate the aspect ratio of planetary impact craters (0.20) and low-speed solid-sphere impact craters (0.12), respectively.

**Fig. 5.** Morphology of granular residues, \( D_g \) versus \( E \). Crater morphologies shown in Fig. 2 A–F are indicated. Stars mark the transition impact energy \( E^* \) between the low- and high-\( E \) regime for each drop size. Solid lines are from the liquid marble model (Eq. 3). The dashed line is \( D_g(E^*) \) calculated by combining the liquid marble model (Eq. 3) with the jamming criterion (Eq. 5).

Thus, we propose a simple formula for the coefficient of energy conversion: \( f = (\pi D_c^2/\pi D_t^2) \), where \( \pi D_t^2 \) provides the only relevant area for normalization. The fraction of energy for ejecting particles is then \( E_{\text{eject}} = f \cdot E \), with \( f < 1 \) automatically satisfied by construction. \( E_{\text{eject}} \) is consistent with recent experiments that estimate the momentum of ejected particles (21). Finally, an energy scaling argument similar to that used for solid-sphere impacts can be applied: instead of \( E \), \( E_{\text{eject}} \) lifts granular particles in a crater of volume \( V_c \) to a height determined by \( d_1 \), i.e., \( E_{\text{eject}} \approx \phi \rho_{\text{sand}} V_c g d_1 \), where \( \phi = 0.60 \) is the volume fraction of the bed and \( \rho_{\text{sand}} \) is the particle density. If we approximate the crater as a paraboloid and replace \( d_1 = aD_c \), then \( V_c = \pi aD_c^3/8 \). Taken together, we successfully recover the S-H scaling:

\[
D_c \approx \left( \frac{\pi}{8} \alpha^2 \phi \rho_{\text{sand}} \right)^{-1/6} \left( [g\phi]^{-1/6} D_c^{1/3} E^{1/6} \right). \tag{2}
\]

Moreover, with \( \alpha = 0.20 \pm 0.01 \) for liquid-drop impact craters (Fig. 4), we have the dimensionless prefactor \( C = (\pi/8)\alpha^2 \phi \rho_{\text{sand}}/\rho \approx 1.86 \pm 0.04 \), quantitatively matching our measurement \( C = 1.74 \pm 0.15 \) (Fig. 3B). Hence, the scaling analysis provides a quantitative description for both the diameter and the depth of the liquid-drop impact craters.

**Discussion on the Analogy Between Liquid-Drop Impact Cratering and Asteroid Impact Cratering.** It should be clear from the above derivation that the energy partitioning of liquid-drop impact cratering and asteroid impact cratering share a quantitative similarity. The forms of energy dissipation in the two processes are obviously different. For asteroid impacts, the impact energy is primarily dissipated by shock-wave heating of the asteroid and the target during the initial stage of the impact event (32). For liquid-drop impacts, it dissipates mainly through the deformation and viscous dissipation of the liquid drops. However, the ratio of the energy dissipation over the total impact energy seems to follow the same quantitative trend in the two processes. Hence, it would be interesting to check if the energy conversion coefficient of asteroid impact cratering is also inversely proportional to the surface area of impact craters, i.e., \( f \sim 1/D_c^2 \). Without shock-wave heating or projectile deformation, a large energy partitioning does not occur in low-speed solid-sphere impact cratering. Most of the impact energy is thus directly converted into the kinetic energy of granular particles for creating impact craters, which leads to the \( 1/4 \) power as dictated by the energy scaling (10, 11).

Finally, it is also interesting to compare liquid-drop impact cratering and asteroid impact cratering more generally in terms of hydrodynamic similarity and the states of matter.
Firstly, it is known that the important dimensionless number governing asteroid impact cratering is the inverse Froude number, \( Fr^{-1} = gD/2U^2 \) (23). For typical asteroid impacts, \( 10^{-6} < Fr^{-1} < 10^{-2} \), which overlaps well with our liquid-drop impact experiments \( 2 \times 10^{-5} < Fr^{-1} < 0.1 \). Secondly, in studying asteroid impact cratering, the impacted surface is frequently modeled as a Bingham fluid (32). On the other hand, granular materials display a typical Bingham fluid behavior (33). More importantly, during asteroid strikes the impact pressure can rise as high as \( 10^7 \) GPa and the temperature may increase above 2,000 °C. Under such extreme conditions, asteroids of normal composition have already been liquefied if not vaporized (32). Hence, liquid drops provide a better model than solid spheres for high-energy asteroids. This important analogy has been overlooked in many previous attempts in search of the link between asteroid impact cratering and low-speed solid-spheres impact cratering (11, 12, 15, 16, 34, 35).

**Model for Granular Residues.** The model for granular residues can be divided into two parts: (i) Based on the liquid marble model, we will show a quantitative understanding of the size of granular residues at low \( E \). (ii) Using the concept of the jamming transition, we will calculate the transition energy \( E^* \) between the low- and high-energy regimes (Fig. 5).

(i) As shown previously, the slow increase of \( D_g \) at low \( E \) is due to the formation of liquid marbles (Fig. 2 A–C). \( D_g \) in this regime is equal to the diameter of liquid marbles. A simple model can thus be constructed based on the liquid marble model proposed by Aussillous and Quéré (24). Firstly, the number of entrained particles at the lamella–granular bed interface \( N \) is proportional to the maximal contact area between the lamella and the bed. Therefore, \( N \approx (\pi D_g^2/\delta_{\text{Sand}}^2) \). The volume of the liquid marble is simply the sum of the volume of the drop and the volume of entrained particles: \( V_m = V_{\text{drop}} + V_{\text{sand}} = \pi D_g^3/6 + N\pi \delta_{\text{Sand}}^3/6 \). If we assume the marble is spherical, then the effective diameter of the liquid marble is \( D_m = (6V_m/\pi)^{1/3} \). For \( D_m \ll \kappa^{-1} \), the liquid marble maintains a spherical shape, where \( \kappa^{-1} = (\gamma/\rho_{\text{Sand}}g)^{1/2} \) is the capillary length, \( \gamma \) is the surface tension of the liquid, and \( \rho \) is the density of the liquid–granular mixture. The diameter of the liquid marble, and therefore the diameter of the granular residue, is simply \( D_g = D_m = (6V_m/\pi)^{1/3} \). However, for \( D_m \gg \kappa^{-1} \), the marble deforms into a puddle under the force of gravity. The thickness of the puddle is given by \( 2\kappa^{-1} \). If we approximate the shape of the puddle as an oblate ellipsoid, then the diameter of the marble is given by \( D_g = (3V_m/\pi\kappa^{-1})^{1/2} \).

In summary, we have

\[
D_g = \begin{cases} 
C_1 \cdot (6V_m/\pi)^{1/3} & \text{if } D_m \ll \kappa^{-1}, \\
C_2 \cdot (3V_m/\pi\kappa^{-1})^{1/2} & \text{if } D_m \gg \kappa^{-1}, 
\end{cases}
\]  

where we add two proportionality constants \( C_1 \) and \( C_2 \) to account for the fact that \( D_m \) is close to \( \kappa^{-1} \) between the two limiting cases and the approximation taken for the shape of the puddle. Replacing \( D_g \) in \( V_m \) using the S-H scaling (Eq. 2), we finally reach \( D_g(E) \). The results quantitatively agree with our measurements (solid lines in Fig. 5) with the fitting parameters \( C_1 = 1.1 \) and \( C_2 = 1.55 \pm 0.15 \) on the order of 1.

(ii) Increasing \( E \) further, at the transition impact energy \( E^* \), the retraction of the lamella is arrested before it can fully restore back to a sphere, which leads to asymmetric granular residues with quickly enlarging \( D_g \) and results in a cross-over from the low-energy liquid marble regime to the high-energy regime (Fig. 5). As discussed previously, the resistance against the capillary retraction comes from the jamming of entrained particles. Thus, we can identify \( E^* \) as the “jamming energy” of liquid-drop impact process. Note that the particles entrained at the liquid interface are gradually pushed into the interior of the receding liquid lamella due to the strong capillary retraction. Hence, the jamming occurs in the bulk of liquid marble rather than only at its interface (36).

A simple analysis based on the liquid marble model can show that the number of entrained particles at the lamella–granular interface is not sufficient to jam the liquid marble at \( E^* \). To reach the jamming transition, the effect of liquid imbibition during the impact needs to be considered (19). We estimate the volume of imbibed liquid into the bed, \( V_{\text{imb}} \), based on the well-established Washburn–Lucas equation (37), which leads to the following equation specifically for liquid-drop impact cratering (SI Text and Fig. S2):

\[
V_{\text{imb}} = 0.058 A \cdot (\eta^2)^{-1/4} \cdot \rho_{\text{Sand}}^3/6 \cdot D_{g}^{5/4} \cdot E^{1/2},
\]

where \( \eta \) is the liquid viscosity and \( A \) is a proportionality constant of order 1. The jamming transition at \( E^* \) can then be expressed as

\[
\frac{V_m - V_{\text{imb}}}{V_m} = \frac{\pi \delta_{\text{Sand}} D_{g}^2/6}{\pi \delta_{\text{Sand}} D_{g}^2/6} = \phi_c,
\]

with the jamming volume fraction \( \phi_c \approx 0.55 \) (25, 26). Using the S-H scaling for \( D_g \), Eq. 5 quantitatively agrees with our measurement on \( E^* \) for different \( D \) with the fitting constant \( A = 1.19 \pm 0.02 \) (Fig. 6). Finally, in combination with the liquid marble model, we also reach \( D_g(E^*) \) (dashed line in Fig. 5) (see SI Text for additional comments).

**Conclusions**

When a liquid drop impacts on a granular surface, the impact energy is converted into the surface energy of the deformed drop, the internal energy of liquid and particles, and the kinetic energy of the spreading lamella and ejected particles. The process is notoriously complicated, involving high Reynolds hydrodynamics, shock compression in the impinging drop, fast granular flows, and capillary interactions between fluid and granular particles. Given the complexity, it is surprising that the simple model presented here can quantitatively capture the morphology of liquid-drop impact craters over a large range of impact energy.
Such a model will be considerably useful for predicting the outcome of raindrop impacts on granular media—a ubiquitous process occurring in numerous natural, agricultural, and industrial circumstances.

Moreover, our study reveals a quantitative similarity between raindrop impact cratering and asteroid impact cratering in terms of both the energy scaling and the aspect ratio of their impact craters. Compared with extensively studied low-speed solid-sphere impact cratering, liquid-drop impact cratering provides a better analogy to high-energy asteroid impact cratering. Apparently, one should be very cautious when drawing a close link to a better analogy to high-energy asteroid impact cratering. Applications of both the energy scaling and the aspect ratio of their impact craters can be further extended to low-speed liquid-drop impact cratering. In SI Text, we used several different liquids including methanol, ethylene glycol, mineral oil, water–glycerin mixtures, and SDS solutions as liquid drops to probe the effect of liquid viscosity and surface tension on the impact cratering process. Additional discussion of liquid-drop impact cratering on a wet granular bed is also presented in SI Text (fig. S3).

A Photon SA-X2 camera was used for high-speed imaging of drop impact dynamics. The morphology of impact craters was measured using a high-resolution laser profilometer (Kenyeiji LJ-V7060) with the resolution in the x-y plane at 20 μm and the resolution in depth at 0.4 μm. The camera and the profilometer were further combined to monitor the depth of crater during impacts. Experiments in the terminal velocity regime were conducted in an indoor laboratory with a high-height experimental platform. To prevent perturbation from air flows that lead to uncontrollable impact positions, we set up a PVC tube of 11.5 m in length and 20 cm in diameter. A free-falling drop travels inside the tube before it impacts on a granular bed underneath the bottom opening of the tube. The release heights in previous investigations are all below 3 m (17–21), which seriously limits the dynamic range of impact energy and thus the accuracy of the scaling relationship. Finally, we also performed one set of experiments at one-tenth of the atmospheric pressure to test possible effects of ambient air on the dynamics of liquid-drop impact cratering. The ambient air has been shown to play a significant role in liquid-drop impacts on solid surfaces (8, 9).

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