LETTER

Random sampling of skewed distributions does not necessarily imply Taylor's law

Cohen and Xu (1) claim that random samples of any skewed distributions with four finite moments would give rise to Taylor's law (TL). In fact, skewed distributions do not necessarily generate data following TL. Some highly skewed distributions can generate random data rejecting the law. Here, I show examples for this using beta, lognormal, and Poisson distributions (the last one is used for comparison).

I followed the same random sampling and calculation procedure (1): Applied to each of the 10,000 copies of 100 × 100 random matrices generated from a specific skewed distribution, the ordinary least-squares regression is used to calculate the parameters of TL and test the evidence of the law by checking the associated P values of coefficients b and c in the models $\log(v_j) = \log(a) + b \log(m_j)$ and $\log(v_j) = \log(a_1) + b_1 \log(m_j) + c \log(m_j)^2$, respectively. Here, m_j and v_j are the sample mean and variance for the j-column vector in a copy of the random matrices. If $P(b) < 0.05$ and $P(c) > 0.05$, TL is confirmed (1–3). I calculated the supporting rate of TL as the ratio between the number of matrix copies in which TL is confirmed and 10,000. To check the impacts of varying parameters in the skewed distributions, for beta distribution, scale parameter was fixed to $\beta = 0.5$ and shape parameter varied as $\alpha \in [0.01, 0.5]$. For lognormal distribution, the location parameter was fixed to $\mu = 1$ and scale parameter varied as $\sigma \in [0.001, 10]$. For Poisson distribution, the intensity parameter varied as $\lambda \in [1,100]$. For each parameter point (e.g., $\{\alpha = 0.5; \beta = 0.5\}$ in beta distribution), 10,000 random matrices were simulated. The supporting rate of TL and mean skewness (over the 10,000 copies) were calculated. These specific parameter intervals can generate positively skewed or symmetric distribution curves (thus the fitted coefficient $b \geq 0$).

The results show that there are unimodal and bimodal relationships between the skewness and the supporting rate of TL for beta and lognormal distributions, respectively (Fig. 1 *A* and *C*). When the distribution has very low or no skewness (close to zero), the supporting rate is low. However, when the distribution has very high skewness ($\sigma \to 10$ or $\alpha \to 0.01$; Fig. 1 *B* and *D*), the supporting rate becomes very low again (Fig. 1*C*) or even reaches zero (Fig. 1*A*). By contrast, the supporting rate of TL in Poisson distribution changes monotonically with skewness (Fig. 1*E*), which is in turn monotonically controlled by the parameter λ (Fig. 1*F*).

Conclusively, there is a complex and nonmonotonic association between skewness and the occurrence of TL in random data from some probability distributions. Highly skewed distributions could reject TL sometimes (Fig. 1 *A* and *C*). The reason is that random sampling of skewed distributions generates random data with a significant coefficient b, but, meanwhile, the coefficient c is likely to be significant, thus denying the law.

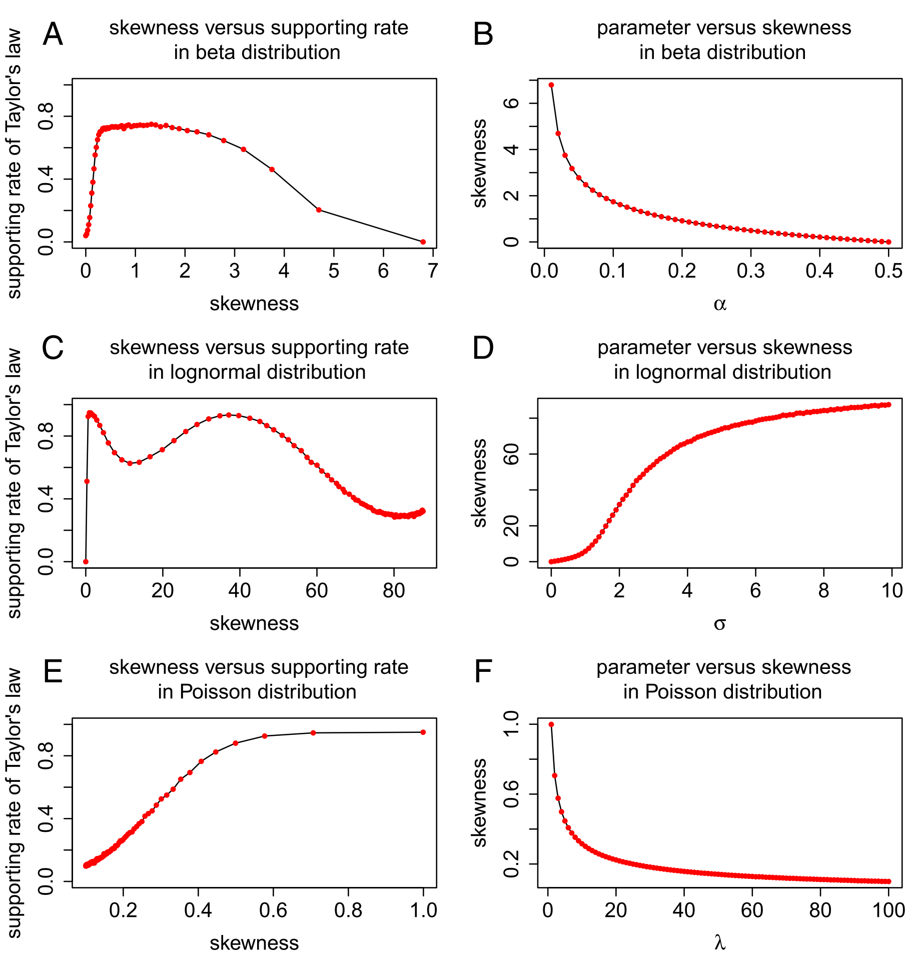

Fig. 1. Relations between changing skewness vs. supporting rate (*A*, *C*, and *E*) and parameter vs. skewness (*B*, *D*, and *F*) of TL in beta (*A* and *B*), lognormal (*C* and *D*), and Poisson (*E* and *F*) distributions. For each data point (in red), 10,000 random matrices with a size of 100 × 100 are simulated and used to calculate mean skewness and the supporting rate.

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