LETTER

Reply to Aksentijevic: It is a matter of what is countable and how neurons learn

Aksentijevic (1) raises issues with our recent report (2) that bear further clarification because they reflect some fundamental confusion about the nature of random sequences. First, it is essential to distinguish statistics for individual elements (i.e., patterns of length one) vs. statistics for patterns consisting of more than one element (i.e., higher order patterns). Aksentijevic's comments reflect some confusion between these two. The independent Bernoulli process only guarantees that the statistics for patterns of length one are fully random in the usual sense (e.g., independent and identically distributed), where the mean time and waiting time are indeed equal. Once one starts looking at higher order patterns, different statistical structures can, and do, emerge. For example, in a sequence of length three, pattern HH can happen twice but pattern HT cannot. No "assumption of self-correction" is required here. Different waiting times, clustering, or spreading of pattern occurrences are simply consequences intrinsic to the patterns' composition (3, 4). Although somewhat difficult to comprehend precisely, we hope it is recognized that we did not fudge our random sequences to create structures where none actually exist. These differences are mathematical facts that are readily observed and replicable by anyone.

The second major point raised by Aksentijevic (1) seems to accept that higher order patterns do exist in random sequences, and offers an alternative framework based on detecting changes across these patterns. We do not disagree that sensitivity to changes over time, and associated attentional effects, might play an important role here. However, the fundamental feature of human cognition that is at play in our model or in a change-detection model is the integration of information over time (5). It is this integration over time that then creates a sensitivity to higher order structures within a sequence, as opposed to a memoryless system that would only be sensitive to the first-order structure. Moreover, structural complexity and randomness are abstract mathematical concepts, which we do not assume the neurons in our model are initially equipped with. Nevertheless, our model demonstrates how higher order structures can be learned without supervision by merely observing independent coin tosses one at a time. This was the essential insight of our report, and one that we hope can be recognized as mathematically sound and a promising basis for further investigation into the precise nature of the temporal integration processes taking place in the human mind.

Yanlong Sun[a,1], Randall C. O'Reilly[b], Jack W. Smith[a], and Hongbin Wang[a,1]
[a]*Center for Biomedical Informatics, Texas A&M University Health Science Center, Houston, TX 77030; and* [b]*Department of Psychology and Neuroscience, University of Colorado, Boulder, CO 80309*

1 Aksentijevic A (2015) No time for waiting: Statistical structure reflects subjective complexity. *Proc Natl Acad Sci USA* 112:E3159.
2 Sun Y, et al. (2015) Latent structure in random sequences drives neural learning toward a rational bias. *Proc Natl Acad Sci USA* 112(12):3788–3792.
3 Sun Y, Wang H (2010) Gambler's fallacy, hot hand belief, and time of patterns. *Judgm Decis Mak* 5(2):124–132.
4 Sun Y, Wang H (2010) Perception of randomness: On the time of streaks. *Cognit Psychol* 61(4):333–342.
5 O'Reilly RC, Wyatte D, Rohrlich J (2014) Learning through time in the thalamocortical loops. arXiv:1407.3432.

Author contributions: Y.S., R.C.O., J.W.S., and H.W. wrote the paper.

The authors declare no conflict of interest.

[1]To whom correspondence may be addressed. Email: ysun@tamhsc.edu or hwang@tamhsc.edu.

 www.pnas.org/cgi/doi/10.1073/pnas.1508265112