Growth rate of crystalline ice and the diffusivity of supercooled water from 126 to 262 K

Yuntao Xu, Nikolay G. Petrik, R. Scott Smith, Bruce D. Kay, and Greg A. Kimmel

*Chemical Physics & Analysis, Physical Sciences Division, Physical & Computational Sciences Directorate, Pacific Northwest National Laboratory, Richland, WA 99352

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Understanding deeply supercooled water is key to unraveling many of water’s anomalous properties. However, developing this understanding has proven difficult due to rapid and uncontrolled crystallization. Using a pulsed-laser-heating technique, we measure the growth rate of crystalline ice, \( G(T) \), for 180 K < \( T < 262 \) K, that is deep within water’s “no man’s land” in ultrahigh-vacuum conditions. Isothermal measurements of \( G(T) \) are also made for 126 K ≤ \( T ≤ 151 \) K. The self-diffusion of supercooled liquid water, \( D(T) \), is obtained from \( G(T) \) using the Wilson–Frenkel model of crystal growth. For \( T > 237 \) K and \( P ∼ 10^{-8} \) Pa, \( G(T) \) and \( D(T) \) have super-Arrhenius (“fragile”) temperature dependences, but both cross over to Arrhenius (“strong”) behavior with a large activation energy in no man’s land. The fact that \( G(T) \) and \( D(T) \) are smoothly varying rules out the hypothesis that liquid water’s properties have a singularity at or near 228 K at ambient pressures. However, the results are consistent with a previous prediction for \( D(T) \) that assumed no thermodynamic transitions occur in no man’s land.

Water is not a typical liquid, and its unusual properties have been discussed by many authors (1–13). Anomalies are observed in thermodynamic response functions (such as the isobaric heat capacity and the isothermal compressibility) and dynamic properties (such as diffusion, viscosity, and dielectric relaxation) over a wide range of pressures, \( P \), and temperatures, \( T \). These anomalies become more pronounced for supercooled water, and it is generally believed that the unusual properties arise as water adopts a more ice-like structure as the temperature decreases. However, the underlying reasons are fiercely debated (5, 11, 14–17). At ambient pressures, many of the anomalous properties appear to diverge at a temperature, \( T_a \), around 228 K leading to the “stability-limit conjecture” in which the liquid becomes unstable below \( T_a \) (2). Several other models have been proposed to explain the thermodynamic anomalies including the liquid–liquid critical point model (3), the critical-point free scenario (18), and the singularity-free hypothesis (19). However, it is noteworthy that both thermodynamic and dynamic properties apparently diverge at \( \sim T_s \) (1, 2). To explain the similarities in the thermodynamics and dynamics, Adam–Gibbs theory is attractive because it connects changes in the entropy of a liquid to its dynamics (20). Using this theory, Starr et al. (6) predicted that the self-diffusion of water, \( D(T) \), undergoes a dynamic crossover from super-Arrhenius to Arrhenius behavior (often called a “fragile-to-strong” transition) at temperatures just below the homogeneous nucleation temperature, \( T_{n,k} \), at \( \sim 235 \) K. However, purely dynamical explanations have also been proposed (5, 21).

Differentiating between various theories has proven difficult because the onset of rapid crystallization at \( T_{n,k} \) has prevented experiments below this point (5, 22). Another approach involves heating low-density amorphous (LDA) ice above its glass transition temperature (\( \sim 136 \) K) before crystallization at \( \sim 150 \) K (5, 10). The heretofore-inaccessible temperature range between \( \sim 150 \) and \( 235 \) K at ambient pressures is known as “no man’s land” (5, 23). Experiments on water confined to nanoscale drops provide entry into no man’s land (24, 25) but are complicated by interfacial effects (10). Considerable insight has also been gained by exploring water’s unusual properties over a wide range of pressures (3, 5, 7, 23).

Here, we report the growth rate of crystalline ice, \( G(T) \), using a pulsed-heating technique (26) for 181 K ≤ \( T < 262 \) K and measured isothermally for 126 K ≤ \( T ≤ 151 \) K. Because \( G(T) \) is proportional to \( D(T) \) (27–29), we also determine \( D(T) \). For \( T > 235 \) K, we reproduce the previously measured super-Arrhenius behavior (30). For 180 K < \( T < 235 \) K, \( G(T) \) and \( D(T) \) cross over to Arrhenius dependences with large activation energies and pre-factors. For 126 K ≤ \( T < 180 \) K, \( G(T) \) and \( D(T) \) are also Arrhenius-like but with lower activation energies. These results for \( D(T) \), which are inconsistent with a singularity at or near \( T_s \), agree with the prediction of a fragile-to-strong crossover using the thermodynamic properties of water and the Adam–Gibbs theory (6, 20).

Results

Fig. L4 shows a schematic of the pulsed-heating experiments (26). The crystalline ice growth rates were measured for layered ice/water samples (Sample Preparation and Figs. S1 and S2). First, 75-monolayer (ML) films of polycrystalline ice (CI) were prepared on Pt(111) in ultrahigh vacuum. Next, 25-ML films of amorphous solid water (ASW) were deposited to create metastable, layered CI-ASW films. To enhance the sensitivity of the measurements, the CI films were composed of pure H2O, whereas the ASW layers had \( \sim 9.5\% \) HDO in H2O. With the steady-state temperature set at 90 K, the platinum substrate was heated with IR pulses from a Nd: YAG laser (\( \lambda = 1,064 \) nm, \( \sim 10 \) ns, 1–10 Hz). Facile heat transfer from the substrate raised the temperature of the adsorbed water.

Significance

Water is ubiquitous, but its physical properties are anomalous compared with most liquids. Because the anomalies become enhanced upon cooling, understanding the behavior of deeply supercooled water is critical. Unfortunately, experiments below \( \sim 236 \) K at ambient pressure are difficult due to uncontrolled crystallization. Using a pulsed-laser-heating technique, we have determined the crystalline-ice growth rate and liquid-water diffusivity for temperatures between 180 and 262 K in ultrahigh-vacuum conditions. The fact that both of these quantities are smoothly varying rules out the hypothesis that water’s properties have a singularity at or near 228 K. However, the results are consistent with a previous prediction for the diffusivity that assumed no thermodynamic transitions occur in the supercooled region.


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1To whom correspondence may be addressed. Email: gregory.kimmel@pnnl.gov or bruce.kay@pnnl.gov.

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films to their maximum value, $T_{\text{max}}$, in $\sim 20$ ns, and subsequent diffusion of heat into the platinum rapidly cooled the samples back to cryogenic temperatures in between pulses. The black line in Fig. 1B shows the calculated temperature, $T(t)$, for a typical heat pulse. $T(t)$ was calibrated by measuring the water desorption per pulse from the films after they had crystallized (Temperature Calibration) (26).

The rapid heating transforms the ASW layer into supercooled liquid water with a concomitant increase in the diffusion leading to CI growth at the ice–water interface at the rate, $G(T)$ (e.g., Fig. 1B, blue line). Infrared reflection absorption spectroscopy (IRAS) was used to monitor the ice growth. Note that the IRAS spectra were acquired at 90 K after a specified number of heat pulses, $N_p$, had been applied to the sample. Fig. 1C shows how the OH-stretch region of the IRAS spectra evolves as the film crystallizes. By construction, HDO molecules are found only in the (initially) liquid layer, and, as a result, the crystallization of this layer is more readily apparent in the OD-stretch region (Fig. 1D). The amount of ice in the films can be determined by fitting the IRAS spectra to a linear combination of the initial and final spectra (Fig. S3). Fig. 1E shows the ice growth vs. the (normalized) number of laser pulses for $T_{\text{max}} = 217$, 237, and 259 K. For all three temperatures, the crystallization increases approximately linearly vs. the number of heat pulses, $N_p$, had been applied to the sample. Fig. 1C shows how the OH-stretch region of the IRAS spectra evolves as the film crystallizes.

Collectively, these points indicate that our measurements are germane to bulk supercooled water. From experiments like those shown in Fig. 1E, the amount of ice growth per heat pulse, $G(T_{\text{max}})$, can be determined (Fig. 2, black circles). At the highest temperatures, $G(T_{\text{max}})$ changes relatively slowly with temperature, whereas for $T < 235$ K, $G(T_{\text{max}})$ decreases very quickly. An Arrhenius fit to the data yields an activation energy of $\approx 76$ kJ/mol (Fig. 2, dashed blue line). $G(T_{\text{max}})$ is related to $G(T)$ by an integral equation that explicitly treats the temperature as function of time, $T(t)$, at the ice/liquid interface during each heat pulse:

$$
\tilde{G}(T_{\text{max}}) \equiv \int_{T_{\text{pulse}}} G(T(t))dt.
$$

Because $T(t)$ is known (Temperature Calibration) (26), $\tilde{G}(T_{\text{max}})$ can be calculated using trial functions for $G(T)$ in Eq. 1 and then

![Figure 1](image-url)

**Fig. 1.** (A) Schematic of the experiments. Nanoscale films were heated with a laser pulse that raised the temperature for $\sim 20$ ns, creating a film of supercooled water on top of ice. Ice growth that occurred due to the pulsed heating was measured with IRAS. (B) $T(t)$ calculated for $T_{\text{max}} = 217$ K (black) and the corresponding $G(T)$ that matches the observations (blue). (C) and (D) IRAS spectra for an as-grown film (black), at intermediate stages of ice growth (blue), and after crystallization is complete (red). Both the OH-stretch (C) and the isolated HDO-stretch (D) regions show the crystallization. (E) The number of MLs of the initially amorphous layers crystallized vs. the number of heat pulses, $N_p$, for $T_{\text{max}} = 217$, 237, and 259 K. To facilitate comparison, $N_p$ has been normalized by the number of pulses required to crystallize 10 ML at each temperature.
compared with the experimentally measured values. Fig. 2 shows a form of $G(T)$ that reproduces the observed $G(T_{\text{max}})$ (Fig. 2, red and black lines, respectively).

Previous measurements of $G(T)$ have been made on macroscopic samples over a relatively small temperature range near the melting point (32–34), or on nanoscale films at cryogenic temperatures (35). At the melting point, $G(T) = 0$ and at temperatures just below this, $G(T)$ rapidly increases with decreasing temperature as the thermodynamic driving force for crystallization increases (27–29). As discussed below, $G(T)$ is expected to go through a maximum and then decrease at even lower temperatures due to a decrease in $D(T)$. It is important to note that, for macroscopic samples, the heat of fusion that is released upon crystallization and subsequent heat transfer from the ice/water interface limits the minimum temperature that is achieved at the interface (36). The local heating at the ice/water interface has precluded observation of the expected decrease in $G(T)$ with decreasing temperature in macroscopic samples. These effects are less important in the nanoscale films used here (Latent Heat of Fusion and Fig. S6), and the decrease in $G(T)$ expected at lower temperatures is clearly observed (Fig. 2). Nonetheless, the magnitude of $G(T)$ determined for macroscopic samples agrees reasonably well with the current results at comparable temperatures (Fig. 2, Inset).

Molecular dynamics simulations have been used to investigate the growth of CI in contact with the supercooled liquid with a variety of water models (37–43). An advantage of the simulations is that they can be done using various techniques to remove the latent heat generated by the crystallization (39, 40). As a result, $G(T)$ can be investigated as a function of the (known) interface temperature. In this sense, the calculations can be more directly related to our measurements of $G(T)$ than to earlier measurements of $G(T)$ on macroscopic samples, and several simulations have observed a maximum in $G(T)$ (38, 39, 42, 44). For example, simulations with TIP4P/ice, which has a melting point of 270 K, observed a maximum at 260 K and found that $G(T)$ decreased by ~50% by 245 K (39). Both of these results are in qualitative agreement with our measurements.

For $T$ less than ~180 K, $G(T)$ is small enough to preclude measuring it via pulsed heating. However, $G(T)$ can be measured directly in isothermal experiments at even lower temperatures (Isothermal Measurements) (35). Fig. 3 shows $G(T)$ measured isothermally for 126 K $\leq T \leq 151$ K (red diamonds). The figure also displays the pulsed-heating data (red circles), which has been scaled for comparison (Isothermal Measurements). Collectively, these data span 11 orders of magnitude in $G(T)$. It can be divided into three regions: (i) for $T > 235$ K, $G(T)$ varies slowly with temperature but is clearly non-Arrhenius; (ii) for ~180 K $< T < 235$ K, $G(T)$ exhibits Arrhenius behavior with an activation energy of ~76 kJ/mol; (iii) for $T \leq 151$ K, $G(T)$ is also Arrhenius but with a lower activation energy of ~47 kJ/mol. In Fig. 3, for $T < 225$ K, the red line (solid and dashed), which is the sum of two Arrhenius functions with activation energies appropriate for the intermediate- and low-temperature regions, fits the data over that entire temperature range and suggests that the low-temperature data matches smoothly with $G(T)$ determined from the pulsed-heating measurements.

Although $G(T)$ displayed in Figs. 2 and 3 is interesting in its own right, crystal growth is also intimately related to dynamics occurring in the supercooled liquid. In particular, crystal growth at a liquid/solid interface can be decomposed into a thermodynamic and a kinetic component (27, 28, 45). The thermodynamic component reflects the (net) probability that molecules from the liquid attach to the crystal. The kinetic component is related to the mobility of molecules at the liquid/solid interface, and it is often found that the growth rate is proportional to the diffusivity in the liquid (46, 47).
Therefore, our measurements of $G(T)$ can provide valuable insight into the dynamics of supercooled liquid water over a wide temperature range. When the temperature is well below the melting point and the thermodynamic driving force for crystallization is large, the growth rate can be described by the Wilson–Frenkel model:

$$G(T) = D(T)/\alpha \left[1 - \exp(-\Delta G_\text{c}(T)/k_B T)\right],$$  

[2]

where $\Delta G_\text{c}(T)$ is the difference in free energy between the liquid and crystalline phases and $\alpha$ is a constant (27–29). More information on the Wilson–Frenkel model is given in Wilson–Frenkel Growth Model. Because $D(T)$ and $\Delta G_\text{c}(T)$ are known for $T > 238$ K, we can use our measured $G(T)$ to determine $\alpha$ via Eq. 2 (Fig. S7). At lower temperatures where other techniques for measuring $D(T)$ fail, it can be calculated using Eq. 2 and the measured $G(T)$ (46). The blue line in Fig. 3 shows $D(T)$ calculated with this procedure. Above 238 K, it agrees with known values of $D(T)$ (Fig. 3, Inset), whereas at lower temperatures, $\Delta G_\text{c}(T)$ is weakly temperature-dependent and thus $D(T)$ closely tracks $G(T)$. There are several noteworthy aspects to these results. First, a power-law fit with a singularity at or near 228 K will not fit either $G(T)$ or $D(T)$ (Fig. S8). Thus, the stability-limit conjecture is incompatible with the current results. Second, the current results for $D(T)$ are incompatible with Vogel–Fulcher–Tammann (VFT) fits such as those used by Price et al. (30) and Smith and Kay (48) (Fig. S8). Instead, $D(T)$ exhibits a similar three-region temperature dependence as discussed above for $G(T)$. Based on the activation energy in the low-temperature region, the fragility index at the glass transition temperature ($T_g \sim 136$ K) is $\sim 18$, which is consistent with previous measurements and corresponds to an extremely strong liquid (49).

Discussion

Although using growth rates of the crystalline phase to extract the diffusivity in the liquid phase often works well (46, 47), it also fails in several cases, particularly near the glass transition temperature (50, 51). For example, the growth rate for tris-naphthylbenzene (TNB) tracks the self-diffusion at high temperatures but leads to an over-estimate of it by approximately an order of magnitude near $T_g$ (50). A similar level of uncertainty might apply for $D(T)$ shown in Fig. 3, particularly for the isostructural measurements at $T \leq 151$ K. Another potential issue with the Wilson–Frenkel model is related to the type of diffusion, namely, translational or rotational, that is involved in the ice growth. Although it is likely that both are involved, the relative importance of these, particularly near $T_g$, is currently debated (52, 53). On the other hand, the qualitative features displayed for $D(T)$—no singularity in the temperature dependence and a fragile-to-strong crossover at $\sim 235$ K—are unlikely to be changed by any specific shortcomings of the Wilson–Frenkel model.

The complicated form of $D(T)$ shown in Fig. 3 is qualitatively similar to a prediction by Starr et al. (6). Assuming a thermodynamically reversible connection between supercooled water and LDA, they used the entropy, specific heat, and enthalphy of water at 150 and 236 K to estimate the excess entropy, $S_\text{ex}$, and the configurational entropy, $S_\text{conf}(T)$, at intermediate temperatures, and then used Adam–Gibbs theory to calculate $D(T)$. With this approach, they predicted a fragile-to-strong transition (or dynamic crossover) just below 236 K and a second crossover at $\sim 187$ K to a lower activation energy, both of which agree with the current results. Fragile-to-strong crossovers have also been observed in molecular dynamics simulations at or near the corresponding maxima in $C_p$ as expected from Adam–Gibbs theory (6, 7). The second crossover at $\sim 180$ K, which is seen in both $G(T)$ and $D(T)$, might be associated with changes in the hydrogen-bonding network of the liquid water. For example, molecular dynamics simulations suggest that the average number of bonds, which is less than four at higher temperatures, approaches four at lower temperatures (54). Thus, one possibility is that the larger apparent activation energy for $T > 180$ K is associated with the mobility in the “defective” liquid and the decreasing number of bonds with increasing temperature. The smaller activation energy found for $T < 180$ K would then be associated with diffusion of molecules in a “fully” hydrogen-bonded fluid (i.e., with a fixed number of bonds).

The success of the Adam–Gibbs prediction for $D(T)$ supports the hypothesis that the unusual thermodynamics of liquid water are also responsible for its dynamic anomalies. Our results appear to be consistent with either the critical-point-free scenario or a liquid–liquid critical point at positive pressures, as both are consistent with a continuous change in entropy across no man’s land. However, because the current experiments are continuous in $G(T)$ and $D(T)$, they provide no evidence for a liquid–liquid transition line extending to negative pressures (55). At low temperatures, our $D(T)$ is considerably larger than the predictions of Starr et al. (6). However, as they noted, this is likely due to the breakdown of the Stokes–Einstein relationship, which is frequently observed for deeply supercooled liquids (56).

Conclusions

In summary, using a recently developed laser-heating technique, we have measured the growth rate of CI, $G(T)$, across a large temperature range of no man’s land at low pressure. When these data are analyzed using the venerable Wilson–Frenkel theory of crystal growth, we are able to extract the self-diffusion coefficient, $D(T)$, for supercooled water over the temperature range from 126 to 262 K. Over this range, $G(T)$ and $D(T)$ smoothly vary by $\sim 11$ orders of magnitude with no evidence of a singularity. Instead, they exhibit three characteristic temperature dependencies clearly indicating a fragile-to-strong crossover at $\sim 233$ K and a “strong-to-stronger” crossover at $\sim 180$ K, which are qualitatively similar to an earlier prediction based on Adam–Gibbs theory (6). This approach affords the opportunity to explore deeply supercooled liquids in regions of the phase diagram that were previously inaccessible by rapid crystallization.

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