LETTER

Soft wetting: Models based on energy dissipation or on force balance are equivalent

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In Newtonian mechanics, an overdamped system at steady state is governed by a local balance of mechanical stress but also obeys a global balance between injected and dissipated energy. In the classical literature of purely viscous drop spreading, apparent differences in "dissipation" and "force" approaches have led to unnecessary debates, which ultimately could be traced back to different levels of mathematical approximation (1). In the context of wetting on a soft solid, Zhao et al. (2) interpret their experiments by a model based on viscoelastic dissipation inside the substrate. It is claimed that this global dissipation model is fundamentally different from the local mechanical model presented by Karpitschka et al. (3). The purpose of this letter is to demonstrate that (*i*) the models in refs. 2 and 3 are in fact strictly equivalent and (*ii*) the apparent difference can be traced back to an inconsistent approximation made in ref. 2.

Following the analysis in ref. 2, there is a step where the dissipation per unit volume is integrated over depth (equations 44 and 45 in the SI Appendix of ref. 2). The analysis provides no information on the explicit depth dependence; the integral is estimated to scale with the wavenumber as $^1/_k$, arguing that this is the extent by which the deformation penetrates into the layer. Such an approximation is inconsistent, however, since the finite thickness h_o induces a screening of the modes of wavenumber $k \lesssim {}^1/_{h_o}$. This is a key point, since this estimation underlies the scaling laws presented in the main text.

To resolve this issue explicitly, we propose performing the depth integral at the very start of the analysis and compute the dissipation P as

$$P = \int d^2x \sigma_{ij} \frac{\partial \dot{u}_i}{\partial x_j} = \oint ds\, \sigma_{ij} n_j \dot{u}_i = \int_{-\infty}^{\infty} dx \sigma(x,t) \dot{h}(x,t). \quad [1]$$

In the last step, and for the rest of the analysis, we strictly follow ref. 2 by keeping track only of the normal displacement $h(x - vt)$ and the normal traction $\sigma(x - vt)$. We then proceed with the exact same formula for σ proposed in refs. 2 and 3 and obtain an expression in terms of the Fourier transform $h(k)$:

$$P = -v \int \frac{dk}{2\pi} \frac{\mu(kv)}{K(k)} (ik) |h(k)|^2 = v(\gamma \sin\theta)^2 \int \frac{dk}{2\pi} \frac{k\, K(k) G''(kv)}{|K(k)\gamma_s k^2 + \mu(kv)|^2}, \quad [2]$$

where $\mu(\omega) = G'(\omega) + iG''(\omega)$ is the viscoelastic modulus and $K(k)$ is the spatial Green's function that accounts for the finite layer thickness. In the second step, we used the explicit form of $h(k)$. This result differs from the estimation given in equation 45 in the SI Appendix of ref. 2. However, balancing this explicitly integrated formula for the dissipation with the power injected by capillary forces, one exactly recovers equations 22–23 in ref. 3, even up to the prefactors.

As expected, the dissipation route proposed in ref. 2 leads, once derived consistently, to the same prediction as the mechanical approach (3) under the same assumptions: small solid surface deformations to use the Green's function formalism, and constant solid surface tension, decoupled from strain. For fully quantitative comparison with experiments, future work should go beyond these restrictions.

1 Bonn D, Eggers J, Indekeu J, Meunier J, Rolley E (2009) Wetting and spreading. *Rev Mod Phys* 81:739–805.
2 Zhao M, et al. (2018) Geometrical control of dissipation during the spreading of liquids on soft solids. *Proc Natl Acad Sci USA* 115:1748–1753.
3 Karpitschka S, et al. (2015) Droplets move over viscoelastic substrates by surfing a ridge. *Nat Commun* 6:7891.

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