RETROSPECTIVE

Jean Bourgain, problem solver

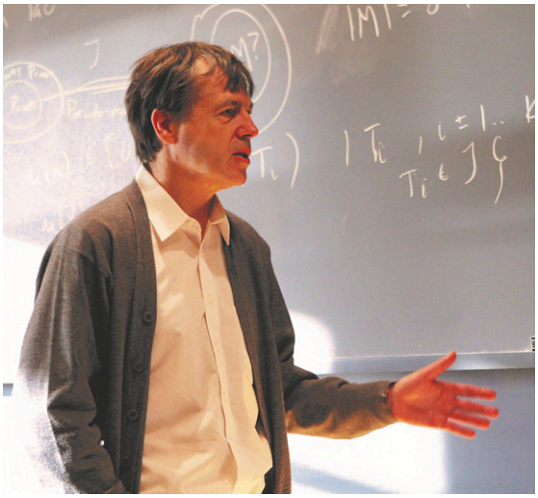
Terence Chi-Shen Tao[a,1]

Jean Bourgain, a truly exceptional and prolific problem solver who transformed multiple areas of mathematics, died on December 22, 2018, aged 64, after a prolonged battle with cancer.

Jean received almost every award in the field of mathematics, ranging from the Fields Medal in 1994 (widely regarded as the premier award for mathematicians under the age of 40) to a baronetcy by the Belgian government in 2015. He was elected to the National Academy in 2011, but as a colleague of his noted after his death, "Even considering this, it is hard not to think that in some ways the weight of his contributions has still somehow gone underappreciated within the larger mathematical and scientific community."

Jean's early work was in the geometry of high-dimensional (or infinite-dimensional) spaces, as well as closely related objects such as large (or infinite) matrices. For instance, he showed how to efficiently approximate a high-dimensional convex body using randomly selected points in that body, and (with Tzafriri) showed that starting from an arbitrary large matrix (obeying some mild conditions), one could select a large number of columns to create a submatrix that was well behaved mathematically (e.g., it had a well-behaved matrix inverse). In both cases, Jean was motivated by purely mathematical considerations, but both of these results have since found major application in the analysis of "big data"; for instance, the column selection that is guaranteed by the Bourgain–Tzafriri theorem is now used as a basic tool in extracting important features of high-dimensional datasets.

A recurring theme in Jean's work was the application of techniques from harmonic analysis to other fields of mathematics, often to such a striking extent that these fields were transformed by the assimilation of these techniques. Roughly speaking, harmonic analysis is the study of how a mathematical function (e.g., one used to describe a physical field or wave, one representing a set of numbers or points) can be decomposed into more fundamental components or "harmonics" (e.g., sinusoidal waves), and how those components interact with each other to reconstruct the original function, or various interesting transforms



Jean Bourgain, 1954–2018. Image courtesy of Institute for Advanced Study/Andrea Kane.

of that function. As just one example of the areas of mathematics impacted in this fashion, Jean developed and then applied powerful inequalities and conceptual frameworks from harmonic analysis to control the solutions of various nonlinear dispersive equations that arise in physics, such as the nonlinear Schrodinger equation or the Korteweg–de Vries equation, allowing one to understand these solutions over very long periods of time, even if the initial conditions for this solution were very rough or very large in amplitude. More recently, Jean and his colleagues developed another set of harmonic analysis inequalities, now known as decoupling theorems, that settled a long-standing central problem in analytic number theory (the Vinogradov main conjecture) that (roughly speaking) concerned the number of ways a whole number could be split up into the sum of perfect powers.

Even from his earliest days, Jean was a noted problem solver; according to legend, he would frequently ask unsuspecting young mathematicians what they were working on, and in many cases, they would then find the solution to their most difficult

[a]Department of Mathematics, University of California, Los Angeles, CA 90095-1555
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[1]Email: tao@math.ucla.edu.
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problems in a couple of pages of neatly handwritten text slid underneath their office door. (The one joint paper I myself had with Jean originated more or less in this fashion, although, in this case, the problem took Jean a few months to solve and the two pages of text arrived by fax.) A review of one of his articles notes that the "40-year-old problem" solved in that text was casually mentioned to him by another mathematician, "but if [it] had not, it might well have gone unsolved for another century."

Jean's papers were somewhat notorious for their conciseness, as a line in one of Jean's arguments he might consider to be a routine application of what was (to him) a standard technique would often be equivalent to a page or so of explanation in any other paper. A review of one of Jean's papers, by a leading expert in that field, commented "The reviewer found the paper rather hard to read, and indeed had to seek assistance at one point. Readers are advised not to take some of the assertions made in the course of the proofs too literally, and, in particular, to abandon any preconceived notion as to the meaning of the symbol '~'." As a graduate student, I myself struggled for many frustrating months with one of Jean's papers; however, once I absorbed the "bag of tricks" he was using to go through these steps so quickly, I was able to use these tricks to greatly advance my own research. In time, I began to look forward to reading Jean's latest paper, and now recommend that my own students go through at least one of his papers.

His prodigious research output (over 500 papers across four decades) was not slowed at all when he was diagnosed with cancer in 2015; indeed, he would say (not entirely jokingly) that by freeing him from travel and administrative obligations, he actually had more time to devote to research. Indeed, Jean continued to produce many striking results in his last few years, such as the solution of the Vinogradov main conjecture mentioned previously.

In Jean's own words: "If you have a question which is generally perceived as unapproachable, it's often you don't even quite know where to look to find a solution. From that point of view, we are . . . stranded in the desert, hopelessly lost. At the moment you get this insight, all of a sudden you'll escape the desert and things open up for you. Then we feel very excited. These are the best moments. They make all the suffering, with absolutely no progress, they really make it worth it."