Magnetotransport signatures of Weyl physics and discrete scale invariance in the elemental semiconductor tellurium

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The study of topological materials possessing nontrivial band structures enables exploitation of relativistic physics and development of a spectrum of intriguing physical phenomena. However, previous studies of Weyl physics have been limited exclusively to semimetals. Here, via systematic magnetotransport measurements, two representative topological transport signatures of Weyl physics, the negative longitudinal magnetoresistance and the planar Hall effect, are observed in the elemental semiconductor tellurium. More strikingly, logarithmically periodic oscillations in both the magnetoresistance and Hall data are revealed beyond the quantum limit and found to share similar characteristics with those observed in ZrTe5 and HfTe5. The log-periodic oscillations originate from the formation of two-body quasi-bound states formed between Weyl fermions and opposite charge centers, the energies of which constitute a geometric series that matches the general feature of discrete scale invariance (DSI). Our discovery reveals the topological nature of tellurium and further confirms the universality of DSI in topological materials. Moreover, introduction of Weyl physics into semiconductors to develop “Weyl semiconductors” provides an ideal platform for manipulating fundamental Weyl fermionic behaviors and for designing future topological devices.

Significance

Recent intensive investigations have revealed unique electronic transport properties in solids hosting Weyl fermions, which were originally proposed in high-energy physics. Up to now, the discovered Weyl systems have been limited to semimetal compounds. Here we demonstrate that the elemental semiconductor tellurium is a Weyl semiconductor, with typical Weyl signatures, including the negative longitudinal magnetoresistance, the planar Hall effect, as well as the intriguing logarithmically periodic magneto-oscillations in the quantum limit regime. Such Weyl semiconductors offer a simple platform for the exploration of novel Weyl physics and topological device applications based on semiconductors and moreover confirm the universality of discrete scale invariance in topological materials.

Weyl semiconductor | tellurium | negative longitudinal magnetoresistance | planar Hall effect | log-periodic oscillations

The discovery of nontrivial topological phases has reshaped our understanding of solid-state materials (1–3). Besides band structure measurements, transport properties can also provide some unique signatures to identify these exotic topological phases. For example, two well-known transport signatures in Dirac/Weyl semimetals are the chiral-anomaly induced negative longitudinal magnetoresistance (NLMR) (4–14) and planar Hall effect (PHE) (12, 15–20). Recently, a new type of quantum oscillations featuring log-periodicity, wherein the extrema magnetic fields of oscillations constitute a geometric series, was discovered in ZrTe5 (21) and HfTe5 (22) beyond the quantum limit. Physically, these unusual log-periodic oscillations can be attributed to the manifestation of discrete scale invariance (DSI) (21, 23, 24), whereby self-reproduction occurs at specific scales that form a geometric series. The DSI character of topological materials is rooted in the two-body quasi-bound states formed between linearly dispersed quasiparticles and opposite charge centers, with the energy levels satisfying a geometric progression (21). In theory, log-periodic oscillations are considered a universal feature of materials hosting Dirac/Weyl fermions with Coulomb attraction (21, 23, 24).

Until now, Dirac/Weyl physics has been generally regarded as unique to semimetals, with the Dirac/Weyl points located around the Fermi level. Nevertheless, considering the high tunability and compatibility with modern electronic industry, the semiconductor will be a better candidate material for designing topological electronic devices if exotic transport phenomena related to Dirac/Weyl fermions can be realized. Tellurium (Te) is a narrow band-gap semiconductor possessing strong spin-orbit coupling, and it lacks space inversion symmetry due to the characteristic chiral crystal structure. As an elemental semiconducting material, Te has been widely studied for its lattice dynamics, band structure, transport, and optical properties, with key discoveries transpiring several decades ago (25). Recently, the Weyl semimetal phase was theoretically proposed in Te by closing its bulk band gap with


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external perturbations (26, 27), which has renewed interest in exploring the possible topological characters of Te (26, 28–30).

In this work, we report two transport signatures (NLMR and PHE) to reveal Weyl-related topological properties in the self-hole-doped elemental semiconductor Te. The Weyl band characteristic further manifests as the logarithmically periodic magneto-oscillations in the quantum limit regime. Our results demonstrate that these “Weyl semiconductors” (semiconductors hosting Weyl fermions, as schematically shown in Fig. 1A) can serve as a simple and ideal platform for the exploration of Weyl physics and devices beyond conventional semimetal materials.

Results

Electronic Band Structure of Te. The chiral structure of Te with no inversion symmetry is shown in Fig. 1B. The first-principles band structure of Te is shown in SI Appendix, Fig. S2 (see Materials and Methods for calculation details), which demonstrates that Te is a narrow-gap semiconductor exhibiting strong spin–orbit coupling. Its band gap (~0.38 eV) is near the corner of the Brillouin zone, that is, the H point (Fig. 1B), which is consistent with previous results (26, 31). Along the high symmetric H–L line, two Weyl points arise from the crossing of two spin-splitting valence bands, as shown in Fig. 1C. They are located below the Fermi level by ~0.20 eV (designated W1) and ~0.36 eV (designated W2), respectively. The calculated band structure exhibits good overall agreement with our angle-resolved photoemission spectroscopy measurements (SI Appendix, Fig. S3 and Note 2), although the two Weyl points cannot be seen clearly due to limited spectroscopy resolution.

Te single crystals were grown via physical vapor deposition (Materials and Methods). The as-grown crystals are normally hole-doped and possess a typical carrier density on the order of $10^{16}$ cm$^{-3}$, consistent with previous reports in which samples were grown using a similar method (30, 32). The hole-doped character is further verified via the angle-resolved photoemission spectroscopy results (SI Appendix, Fig. S3B and Note 2). Since there are no detectable impurities in our Te crystals, such self-hole-doping characteristics may result from Te vacancies which act as acceptors (33). Our first-principles calculation further demonstrates that the presence of vacancies indeed prompts the Fermi level to shift toward the valence bands (SI Appendix, Fig. S4). Below we will present the comprehensive magnetotransport measurement results for a typical Te single-crystal sample, in which signatures of Weyl-related physics are clearly observed.

NLMR Effect. Fig. 2A shows the temperature ($T$)-dependent resistance ($R$) curve of sample 6, which exhibits typical behaviors of a doped semiconductor (as detailed in SI Appendix, Note 3). Fig. 2B and C show the curves for the magnetoresistance [MR, defined as $(R(B) - R(0))/R(0) \times 100\%$] across a temperature range of 25 to 100 K under perpendicular and parallel fields, respectively. Further insights into the temperature dependence of the MR properties (across a wide temperature range of 2 to 100 K) are presented in SI Appendix, Fig. S5 and discussed in SI Appendix, Note 4. For the perpendicular case ($B \perp L$), the MR is positive and exhibits clear linear behavior (Fig. 2B). In contrast, markedly negative MR behavior is demonstrated for $B \parallel L$ (Fig. 2C). At $T = 25$ K, the negative MR achieves a magnitude up to ~22% at 14 T, which is comparable to the NLMR effect reported for several Weyl semimetals such as WTe$_2$ (11) and Co$_3$Sn$_2$S$_2$ (7). The negative MR degrades accordingly with increasing temperature due to the thermal effect but remains observable up to ~100 K.

The occurrence of NLMR in the presence of parallel magnetic and electric fields serves as an important signature of the chiral anomaly (13, 34) in material systems hosting Weyl fermions.

SI Appendix

Materials and Methods

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The occurrence of NLMR in the presence of parallel magnetic and electric fields serves as an important signature of the chiral anomaly (13, 34) in material systems hosting Weyl fermions.
Theoretically, the longitudinal magnetoconductivity can be described using the following equation (8):

$$\sigma(B) = (1 + C_w B^2) \cdot \sigma_{\text{WAL}} + \sigma_N,$$

where $C_w$ is a positive parameter representing the chiral anomaly contribution to the conductivity and $\sigma_{\text{WAL}}$ and $\sigma_N$ are the conductivities imparted by the weak antilocalization effect and conventional nonlinear band contributions around the Fermi level, respectively. Here $\sigma_{\text{WAL}} = \sigma_0 + \alpha \mu B$, $\sigma_N = \rho_0 + \alpha B \cdot \mu^2$, and $\sigma_0$ is zero-field conductivity. As demonstrated in Fig. 2D, Inset (SI Appendix, Fig. S8A), the experimental data can be well-fitted via Eq. 1, and the extracted $C_w$ increases monotonically with decreasing temperature down to 15 K (Fig. 2D). The chiral coefficient $C_w$ can be depicted using the formula $C_w \propto v_F \tau_\parallel / (T^2 + \mu^2/\pi^2)$ (10), the theoretically expected temperature-dependent behavior for the chiral anomaly, where $v_F$ is the Fermi velocity, $\tau_\parallel$ is the chirality-changing scattering time, and $\mu$ is the chemical potential relative to the Weyl points.

To gain further insight into the observed negative MR, we also studied its angular-dependent character. Fig. 2E shows the MR for different angles (θ) of B with respect to I as measured at 25 K. Rotating the magnetic field away from B∥I prompts the positive MR magnitude to decrease accordingly. Signatures of negative MR occur for $\theta < 25^\circ$ and peak at $\theta \sim 0^\circ$ (B∥I). In addition, the $C_w$ obtained from the fitted data reveals linear dependence of $\cos^2 \theta$ (Fig. 2F and SI Appendix, Fig. S8B), which is consistent with observations in topological semimetals (12). Similar angular-dependent MR behavior is observed at temperatures down to 2 K, although the negative MR is largely outweighed by the enormous positive MR background attributed to magnetic freeze-out (35) (SI Appendix, Fig. S9).

Thus far, the negative MR observed in Te crystals under B∥I could be well explained via the chiral anomaly, reproducing previous observations of Weyl semimetals (for data from additional samples see SI Appendix, Figs. S10 and S11). Other possible origins, such as weak localization and current jetting, were examined carefully and eventually ruled out (SI Appendix, Note 6).

**PHE.** The PHE, which manifests as the appearance of in-plane transverse voltage when the in-plane magnetic field is not exactly parallel or perpendicular to the current, is another important transport signature of Weyl physics (19, 20). The configuration for measuring the PHE is depicted in Fig. 3A. Inset. Even though a conventional Hall component may still exist in such a setup due to misalignment between the actual rotation plane and the sample plane, it can be eliminated by simply averaging the Hall resistances measured under positive and negative magnetic fields. Fig. 3A shows the symmetrized planar Hall resistivity $\rho_{xy}$ vs. $\varphi$ measured at 25 K over a range of magnetic fields ($\varphi$ is the angle between the directions of the in-plane magnetic field and the current). The in-plane anisotropic MR [AMR, defined as $(R(\varphi) - R(\varphi = 90^\circ))/R(\varphi = 90^\circ) \times 100\%$] was measured simultaneously, and the results are plotted in SI Appendix, Fig. S13A. Both the planar Hall resistivity and the in-plane AMR display a 180° periodic angular dependence, with the AMR reaching its maximum and minimum at 90° and 0°, respectively, while $\rho_{xy}$ does so at 135° and 45°, respectively. These observations are well consistent with typical characters of the PHE.

Theoretically, PHE arising from the chiral anomaly could be fitted via the following equation (17) (SI Appendix, Note 7):

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where the first term is the intrinsic PHE due to the chiral anomaly and the second and third terms are, respectively, the in-plane anisotropic MR and longitudinal offset caused by the Hall configuration misalignment. The quantity \( \Delta \rho^{\text{chiral}} = \rho_\perp - \rho_\parallel \) is the anisotropic resistivity originating from the chiral anomaly. Fig. 3A shows that Eq. 2 provides an excellent fit for the experimental \( \rho_{xy} \) across a range of magnetic fields. For a more quantitative PHE analysis, we extracted the field-dependent \( \Delta \rho^{\text{chiral}} \) at 25 K. The plot in Fig. 3C clearly demonstrates that the obtained \( \Delta \rho^{\text{chiral}} \) increases monotonically with increasing magnetic field. For \( B < 3 \) T, \( \Delta \rho^{\text{chiral}} \) is proportional to \( B^2 \). In the relatively high field regime (\( B > 4.5 \) T), \( \Delta \rho^{\text{chiral}} \) exhibits nearly linear variation with \( B \) and displays no evidence of saturation up to \( B = 14 \) T. Similar field dependence has been previously reported for the PHE in topological semimetals (17, 18).

Fig. 3B shows the temperature-dependent PHE behavior (see SI Appendix, Fig. S13B for the AMR data taken simultaneously). With increasing temperature, the extracted \( \Delta \rho^{\text{chiral}} \) decreases accordingly until it becomes negligible at \( T \sim 100 \) K (Fig. 3D). Such behavior is remarkably consistent with that of the \( C_\omega \) shown in Fig. 2D, suggesting the same underlying physical origin of both the PHE and NLMR in the present Te crystals (see SI Appendix, Note 4 for further discussion).

The above-discussed experimental observations of the NLMR and PHE in Te crystals adhere well to the chiral-anomaly mechanism in materials possessing Weyl fermions. On the other hand, it has been theoretically proposed that both the NLMR and PHE can be observed in a system with nonzero Berry curvature at the Fermi level (36, 37). One notices that the nonzero Berry curvature is also induced by either inversion or time-reversal symmetry breaking, which is the same to the realization of Weyl fermions (3, 38). Therefore, both mechanisms originate from symmetry breaking. To explore the band topology of Te, the Berry curvature (\( \Omega \)) of the highest occupied valence band is calculated. The Fermi level (\( E_\text{F} \)) is set at \( \sim 2 \) meV below the valence band maximum according to the typical carrier densities of samples at 25 K (SI Appendix, Fig. S14). Nonzero Berry curvature on the Fermi surface, shown in Fig. 1D, is dominated by the contribution of the Weyl point \( W_1 \) (see SI Appendix, Fig. S14B and Note 8). This gives a strong evidence of correlation between the observed NLMR and Weyl physics. Recent theoretical works have predicted that Te can be turned into a Weyl semimetal via closing the band gap by applying high pressure (26, 27). In contrast, here we reveal that the pristine semiconductor Te can be considered a “Weyl semiconductor,”

\[
\rho_{xy} = -\Delta \rho^{\text{chiral}} \sin \varphi \cos \varphi + b \Delta \rho^{\text{chiral}} \cos^2 \varphi + c. \tag{2}
\]
since Weyl fermions and related exotic transport properties can be directly realized without gap closing.

**Log-Periodic MR/Hall Oscillations.** Unusual properties of material systems under the quantum limit, wherein all of the carriers are condensed to the lowest Landau level, have been a topic of interest since their discovery (21, 22, 39–44). As a promising candidate for “Weyl semiconductor” with relatively small carrier density and unique band topology, Te offers a suitable platform for the study of Weyl physics beyond the quantum limit. For the density and unique band topology, Te offers a suitable platform for the study of Weyl physics beyond the quantum limit. For the density and unique band topology, Te offers a suitable platform for the study of Weyl physics beyond the quantum limit. For the density and unique band topology, Te offers a suitable platform for the study of Weyl physics beyond the quantum limit. For the density and unique band topology, Te offers a suitable platform for the study of Weyl physics beyond the quantum limit. 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To reveal the feature of these unexpected magneto-oscillations, we conducted magnetotransport measurements of sample 6 in two high-magnetic-field facilities (Materials and Methods). Fig. 4A shows the MR data taken under static fields at 1.8 K, which exhibit oscillations with more periods. Following previous studies (21, 22), a suitable background (SI Appendix, Fig. S1A) obtained via smoothing was subtracted to better distinguish the oscillations (Materials and Methods). The results are presented in Fig. 4A, Bottom with dashed vertical lines marking the positions of peaks/dips. Interestingly, the interval between adjacent dashed lines grows larger as the magnetic field increases. For better presentation, the extremal fields (\(B_n\)) corresponding to these peaks/dips were extracted, and \(1/B_n\) and \(\log(B_n)\) were plotted as a function of the extremal index \(n\), respectively (Fig. 4C). Here the peaks and dips are assigned to integers and half-integers, respectively. The clear deviation from the linear dependence of \(1/B_n\) on \(n\) further demonstrates the inability of the Shubnikov–de Haas effect to account for the observed oscillations. In contrast, the well-defined linear behavior observed for the \(\log(B_n)\) vs. \(n\) curve indicates the log-periodicity. Note that the point at \(\sim 3.1\) T deviates from the linear trend, possibly because this field is below the quantum limit, or perhaps as a result of some unavoidable anomalies during the background subtraction (SI Appendix, Note 9). Fig. 4B shows the MR data taken across a wider field range at 2.5 K using pulsed fields. These data exhibit more oscillations than do the data shown in Fig. 4A, while both the low field data and the extracted \(B_n\) are consistent across both datasets (see also Fig. 4C).

Fig. 4D presents the Hall data obtained simultaneously with the MR data from Fig. 4A. The log-periodic character is also evident from a comparison of the index dependence of the \(1/B_n\) and \(\log(B_n)\) curves (Fig. 4E). The fast Fourier transform for \(\log(\Delta R_{xx})\) and \(\log(\Delta R_{xy})\) shows nearly identical periods, indicating the same origin for both the observed MR and Hall oscillations.

As aforementioned, the oscillations with peculiar log-periodicity have been experimentally observed only recently, and only in ZrTe5 (21) and its sister compound HfTe5 (22). The phenomenon is regarded as the manifestation of DSI in solid-state systems and is theoretically considered to be universal in topological materials hosting Dirac/Weyl fermions with Coulomb attraction (21–24).

The underlying mechanism can be summarized as follows (21, 24): Dirac/Weyl fermions in an attractive Coulomb potential possess a continuous scaling symmetry. In the supercritical case \(\alpha > 1/2\), \(\alpha = e^2/4\pi\varepsilon_0\hbar v_f\) is the fine-structure constant, where \(e\) is the elementary charge, \(\varepsilon_0\) is the vacuum permittivity, \(\hbar\) is the reduced Planck constant, and \(v_f\) is the Fermi velocity, these massless Dirac/Weyl fermions tend to form two-body quasi-bound states with opposite charge centers, like charged impurities. Owing to

**Fig. 4.** Log-periodic MR and Hall oscillations. (A and B) MR taken under static magnetic fields at 1.8 K and under pulsed fields at 2.5 K, respectively. (Bottom) The extracted oscillations after background subtraction. The positions of peaks and dips of oscillations are marked by dashed lines. (C) Index dependence of \(1/B_n\) and \(\log(B_n)\) from the MR oscillation data shown in A and B. (D) Hall resistance measured at 1.8 K and the extracted oscillations. Dashed lines mark the positions of peaks and dips. (E) Index dependence of \(1/B_n\) and \(\log(B_n)\) for the Hall oscillations. (F) Comparison of fast Fourier transform results for the MR oscillations from A and the Hall oscillations from D. The starting field of fast Fourier transform analysis was set to be \(\sim 7\) T, from which the log-periodic oscillations become distinct. The frequency corresponding to the peaks is \(3.1\) (2.8) for the MR (Hall) oscillations, corresponding to a period of \(\log(B_n)\) \(\sim 0.32\) (0.36).
the relativistic energy–momentum dispersion relation of a Dirac/Weyl band, the energies of the quasi-bound states constitute a geometric series that matches the general feature of DSI. The increasing magnetic field enables the quasi-bound states with different energies approaching the Fermi energy continuously. The as-induced scattering between free carriers and quasi-bound states thus influences the transport properties (21), manifesting as the log-periodic oscillations for the measured MR and Hall data.

Here we would like to attribute the observed log-periodic oscillations in Te crystals to the DSI after excluding another possible origin (SI Appendix, Note 9). The quasi-bound states are likely formed between Weyl fermions located near the top of valence bands and opposite charge centers. Since the self-hole-doping characteristic may result from Te vacancies acting as acceptors (33), such vacancies are likely to serve as the negative charge centers that exert Coulomb attraction on the positive Weyl fermions (see SI Appendix, Fig. S4 for the calculation results).

For the log-periodic oscillations, the scale factor (\(\lambda\)) for the DSI can be defined as \(B_p/B_{n+1}\). As such, this quantity is estimated to be \(\sim 2.33\) for sample 6, as based on linear fitting of the \(\log(B_p)\) vs. \(n\) plot (Fig. 4C). This value of \(\lambda\) is consistent with that obtained from the fast Fourier transform analysis, which is \(\sim 2.10\) (\(\sim 2.28\)) for the MR (Hall) oscillations (Fig. 4F). In ref. 21, the semiclassical quantization condition leads to the DSI with \(\lambda = c^2/\alpha\). Here \(\alpha = \sqrt{(Z\alpha)^2 - k^2}\), which can be simplified to \(\alpha = \sqrt{c^2 - 1}\) when we consider the charge number \(Z = 1\) and the lowest angular momentum channel with \(k = \pm 1\) (21). Thus, the fine-structure constant \((\alpha)\) is calculated to be \(\sim 7.5\), far exceeding 1/137 due to the small Fermi velocity of Te crystals. According to \(\alpha = \sqrt{2}\), the Fermi velocity \(v_F\) is calculated to be \(\sim 2.9 \times 10^5\) m/s, which is comparable to the value \((\sim 1.9 \times 10^5\) m/s) estimated from the calculated band structure. The small \(v_F\) ensures the supercritical condition, the formation of quasi-bound states with DSI (21).

With increasing temperature, the amplitudes for both the MR and Hall oscillations decline accordingly before becoming indiscernible at temperatures above 15 K (SI Appendix, Fig. S16 and Note 10). Similar behaviors are also observed in other samples (SI Appendix, Fig. S17). This can be attributed to the thermal excitation that weakens the Coulomb attraction. However, compared to the cases of ZrTe5 and HfTe5, the critical temperature above which the oscillations disappear is much smaller for the present Te crystals, implying the relatively small binding energy of the as-formed quasi-bound states (as discussed in SI Appendix, Note 10).

Conclusion

The present work clearly demonstrates the realization of Weyl-related properties in a narrow-gap semiconductor with strong spin–orbit coupling and without inversion symmetry, thus greatly broadening the scope of topological materials. Furthermore, the highly tunable electronic performance of Weyl semiconductors enables further manipulation of Weyl fermions through various means commonly adopted in semiconductor electronics, such as electrostatic gating and optical illumination. Moreover, the observation of the log-periodic magneto-oscillations further confirms the universality of DSI in topological materials hosting Dirac/Weyl fermions with Coulomb attraction, thus endowing Te with a novel platform for the study of intriguing ground states beyond the quantum limit. The successful introduction of Weyl physics into semiconductor systems therefore offers a new dimension for the future design of topological semiconductor devices.

Materials and Methods

Growth of Te Single Crystals. High-quality Te single crystals were grown using a physical vapor transport technique. High-purity (99.9999% pure) Te powder was loaded into a quartz tube, and a small amount of C powder was added to remove trace oxygen. The quartz tube was heated to 1,000 °C over the course of 5 h and subsequently maintained at 1,000 °C for 1 h. Then, the temperature was cooled to a range of 20 °C/h to 400 °C and 30 °C/h to the hot and the cold zone, respectively. After maintaining the set temperature for 2 wk, the tube was slowly cooled to room temperature, and rod-like silvery crystals (typical dimensions \(5 \times 0.3 \times 0.1\) mm\(^3\)) were obtained (Fig. 2A, Inset).

Structure and Composition Characterization. The structure of Te single crystals was measured by a Rigaku SmartLab X-ray diffractometer (XRD) at room temperature using Cu-Kα radiation (1.5418 Å) at room temperature. The large curved imaging plate detector with a 210° aperture allows a two-dimensional diffraction imaging over a broad 2\(\theta\) range, such that the detection of many lattice planes is possible without breaking the single crystal. As also shown in SI Appendix, Fig. S1A, the measured data match well with the standard powder XRD pattern of Te. SI Appendix, Fig. S1B shows the high-resolution scanning transmission electron microscopy (Thermo Fisher Temis G2 60-300) imaging and selective area electron diffraction pattern of the exposed surface, which both further demonstrate the high quality of the single crystal at the atomic scale. The crystalline long axis, that is, the growth direction, is the [0001] direction. The lattice parameters were determined to be \(a = 4.48\) Å for the [0001] direction. The chemical composition of the crystals was measured by an energy dispersive spectrometer (INCA detector; Oxford Instruments) attached to a field-emission scanning electron microscope (Sirion 200; FEI) operated at 20 kV. No impurity elements were detected (SI Appendix, Fig. S1C).

Magnetic and Transport Property Measurements. The magnetic properties were analyzed using a Quantum Design SQUID-VSM system with the magnetic field up to 7 T and the temperature down to 2 K. For electrical transport measurements, electron beam lithography was utilized to define the electrode pattern on the surface of the Te crystals, then 10 nm Pd and 50 nm Au were deposited as electrode materials using the e-beam evaporation method. Silver epoxy was subsequently used to affix the wires to the Pd/Au contacts. The measurements were performed in a Quantum Design Physical Property Measurement System with the magnetic field up to 14 T. The planar Hall measurements were performed via rotating the sample in a fixed magnetic field. The angle of 105° is the starting point for the measurement, and the mechanical backlash of the rotator normally leads to slight discontinuity at an angle of around 105° in the planar Hall data. High-field transport measurements were carried out in the High Magnetic Field Laboratory in Hefei (static field, 33 T) and the Wuhan National High Magnetic Field Center (pulsed field, 53 T). Due to the rod-like shape of Te crystals (Fig. 1F, Inset), the current was applied along the long axis (the [0001] direction) during all transport measurements conducted in this study.

Angle-Resolved Photoemission Spectroscopy Measurements. The angle-resolved photoemission spectroscopy measurements were performed at beamline 9U (Dreamline) of the Shanghai Synchrotron Radiation Facility with a Scienta Omicron DA30L analyzer. The angle resolution was 0.1°, and the combined instrumental energy resolution was better than 20 meV. The photon energy used in our experiments ranged from 25 to 90 eV, and the measurements were conducted at 7 to 20 K. The (1120) surface of the single crystals was cleaved, corresponding to the \(k_x\)-\(k_z\) plane in the hexagonal Brillouin zone. The Fermi level of the Te samples was compared to a gold film reference which was evaporated onto the sample holder. All of the measurements were conducted under a vacuum better than \(7 \times 10^{-10}\) mbar.

Electronic Band Structure Calculations. First-principles calculations were carried out within the framework of density functional theory using the Vienna Ab initio Simulation Package (45). All of the calculations were performed with a plane-wave cutoff of 450 eV on the \(9 \times 9 \times 9\) \(\Gamma\)-centered \(k\)-mesh, and the convergence criterion of energy was \(10^{-6}\) eV. The generalized gradient approximation with the Perdew–Burke–Ernzerhof functional (46) was adopted to describe electron exchange and correlation. During structure optimization, all atoms were fully relaxed until the force on each atomic position was smaller than 0.01 eV/Å. In order to obtain a reliable band gap, the HSE06 hybrid functional (47) was used in the band structure calculation. The rotational symmetry \(g = C_{2z}\) is a generator along the \(H\)–\(L\) line. Including the spin–orbit coupling, the rotational symmetry \(g\) satisfies \(g^2 = -1\), so its
eigenvalues are ±i. The first-principles calculated eigenvalues for g at two arbitrary momentums (k_1 and k_2) are shown in Fig. 1C. The eigenvalue switches between the two bands at k_1 and k_2. Thus, there must exist a band crossing point with twofold degeneracy between k_1 and k_2. Hence, the Weyl point of W_1 is protected by rotational symmetry. It is well known that the Kramers degeneracy is a twofold degeneracy at time-reversal-invariant momentum. Here, the Weyl point of W_2 is at L point, which is a time-reversal-invariant momentum. Hence, the Weyl point of W_2 is protected by time-reversal symmetry. To accurately calculate the Berry curvature in the Brillouin zone, we have fitted a Wannier tight-binding Hamiltonian by using the WANNIER90 package (48). The p orbitals of Te are used as the initial projectors for Wannier Hamiltonian construction. The chirality of a Weyl point is calculated by integrating the Berry curvature on a surface enclosing the Weyl point.

Data Processing. Due to the nonlinearity of Hall data, the carrier density was extracted from the linear part of the low-field Hall resistivity. Attempts to use the two-carrier fitting model were unsuccessful. For the log-periodic oscillations, the background was obtained through smoothing the MR/Hall data. The second derivative method is a widely adopted approach to confirm the exact positions of the extrema fields (21). We thus compared the extracted oscillations obtained via these two methods and obtained consistent results (SI Appendix, Fig. S15), demonstrating the validity of our data processing.

Data Availability. All data are available in the main text or SI Appendix.

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