Evolution of kinship structures driven by marriage tie and competition
Kenji Ito and Kunihiko Kaneko

The family unit and kinship structures form the basis of social relationships in indigenous societies. Families constitute a cultural group, a so-called clan, within which marriage is prohibited by the incest taboo. The clan attribution governs the mating preference and descent relationships by certain rules. Such rules form various kinship structures, including generalized exchange, an indirect exchange of brides among more than two clans, and restricted exchange, a direct exchange of brides with the flow of children to different clans. These structures are distributed in different areas and show different cultural consequences. However, it is still unknown how they emerge or what conditions determine different structures. Here, we build a model of communities consisting of lineages and family groups and introduce social cooperation among kin and mates and conflict or mating. Each lineage has parameters characterizing the trait and mate preference, which determines the possibility of marriage and the degree of cooperation and conflict among lineages. Lineages can cooperate with those having similar traits to their own or mates, whereas lineages with similar preferences compete for brides. In addition, we introduce community-level selection by eliminating communities with smaller fitness and follow the so-called hierarchical Moran process. We numerically demonstrate that lineages are clustered in the space of traits and preferences, resulting in the emergence of clans with the incest taboo. Generalized exchange emerges when cooperation is strongly needed, whereas restricted exchange emerges when the mating conflict is strict. This may explain the geographical distribution of kinship structures in indigenous societies.

social physics | kinship structure | incest taboo | multilevel selection

In human society, a family and kinship are formed by marriage and descent. In indigenous societies, families sharing a common ancestor are called a lineage. Lineages form a socially related group, called a clan, in which common culture is shared (1–3). Social relationships with others, such as cooperation, rivalry, or marriage, are mostly determined by the clans the parties belong to (1). In particular, people rarely marry within a clan (4–8). This prohibition of the marriage of “siblings as a category” is called the incest taboo, whose origin is thought to be cultural, rather than genetic (2). Furthermore, each clan has some rules on marriage as to which of the other clans it prefers in choosing mating candidates, as well as on the descent as to which clan their children belong to (2). With these rules among clans, they form a certain kinship structure. Indeed, the elucidation of such rules and corresponding kinship structures lies at the core of cultural anthropology studies (1). However, it is still unknown how and why the incest taboo and certain kinship structures emerge.

Lévi-Strauss classified kinship structures according to marriage and descent relationships (1, 2). Examples of structures are shown in Fig. 1, where two different arrows ⇀, → represent the flow of women and children, respectively. Here, the flow of children means that the flow of attributions of children, neither the location nor parental authority (9). For example, when children inherit a mother’s surname and live in a father’s village, children’s affiliation, which is determined by both surname and location, differs from both father’s and mother’s. Fig. 1 shows the classes of kinship structures: the incest structure, without the incest taboo (Fig. 1A); dual organization, a direct exchange of brides between two clans (Fig. 1B); restricted exchange, a direct exchange of brides with the flow of children to different clans (Fig. 1C); and generalized exchange, an indirect exchange of brides among more than two clans (Fig. 1D). Suppose men in clan A marry women in clan B. In a generalized exchange, men in clan B marry women in clan C, whereas in restricted exchange, they marry women in clan A. Thus, the flow of women as a whole is A ↦ B ↦ · · · ↦ X ↦ A in generalized exchange, whereas it is A ↦ B in restricted exchange. Descent relationships are observed by tracing the clan attribution of fathers and children. Generalized exchange is observed in India and China and leads to the emergence of status differentiation between social classes. Restricted exchange is mainly observed in Australia and leads to a stable and egalitarian social structure (2). Nonetheless, it is still unknown why different structures are observed.

Mathematically, kinship structures are characterized by defining the marriage cycle (Cm) and descent cycle (Cd) as the length of cycles of the flow of women and children, respectively. Cm = Cd = 1 in the incest structure (Fig. 1A). Lévi-Strauss defined the system as restricted exchange if Cm = 2, regardless of Cd. However, he called the system with Cm = 2 and Cd = 1 a dual organization (Fig. 1B), which he assumed to be the original...
Fig. 1. Examples of kinship structures. Each symbol A, A₁, A₂, B, · · · is a clan. When clans are identified by a single trait, we denote them by characters as A, B, · · ·, whereas when they are represented by two traits, they are denoted both by characters and indices as A₁, B₁, · · ·. The double arrow from clan X to Y (X⇒Y) shows the marriage rule indicating that women in clan X marry men in clan Y. The arrow from clan X to Y (X→Y) shows the descent rule indicating that children belong to clan Y when their fathers belong to clan X. Hereafter, we refer to the length of cycles of ⇒ and → needed to return to the original point, as marriage cycle Cm and descent cycle Cd. Structures are classified according to those cycles. (A) Cm = Cd = 1 for incest structure. (B) Cm = 2, Cd = 1 for dual organization. (C) Cm = 2, Cd ≥ 2 for restricted exchange. (D) Cm > 2, Cd = 1 for generalized exchange.

pattern of both generalized and other restricted exchanges. Here, we limit the use of restricted exchange to the case with Cm = 2 and Cd ≥ 2 (Fig. 1C), to distinguish it from the dual organization. He defined generalized exchange by Cm ≥ 3 and Cd = 1 (Fig. 1D). Structures with Cm > 2 and Cd > 1 are rarely observed* (Table 1). When Cd = 1, children inherit from and belong to the same clan as their father/mother, the rule of which is called “unilineal descent.” By contrast, when Cd ≥ 2, a child inherits the character (A, B, · · ·) (e.g., land) from his or her father and the index (1, 2, · · ·) (e.g., name) from his or her mother, as in Fig. 1C (2); this is called “bilateral descent.”

The advantage of the incest taboo has been discussed biologically and economically (12, 13). As for kinship structures, how marriage and descent rules are chosen were numerically investigated, under given clan separation (14). Group-theory analysis, known as “kinship algebra,” reveals structures that satisfy transformation symmetry under marriage and descent rules (15–17). These studies, however, can explain neither the social origin of the incest taboo (including the distinction between fathers’ sisters’ daughters and fathers’ brothers’ daughters) nor the emergence and transition of kinship structures.

Note that an important point was not considered in the earlier theoretical studies: Marriage brings social unity. Indeed, field studies revealed that lineages cooperated and conflicted with each other according to their social relatedness and mate preferences. Socially related lineages mutually cooperate and constitute a clan (2). Each lineage proposes marriage to certain lineage(s) depending on its mating preference (2). After marriage, the lineage of the bridgroom and that of the bride cooperate (2, 18), as well as social kin that share common traits (19, 20). There also exist strong conflicts among rival lineages competing for mates (21–24).

Here, we introduce an agent-based model of indigenous societies adopting the multilevel evolution of lineages and communities. Lineage is a unit of the dynamics, and community is an ensemble of lineages within which the interaction of lineages can take place. Unmarried women are exchanged by marriage over lineages. We assign each lineage a trait i and a mate preference p as an optimal trait of the bridgroom. Marriage takes place according to the mating preference given by i, p. Lineages with similar traits cooperate with each other as well as with mates, whereas those with similar preferences compete for mates. Depending on the cooperation and conflict, the population of a lineage grows. By introducing mutations in i, p values, lineages with higher population growth are selected.

With numerical simulations, lineages were found to form clusters in (i, p) space under a certain condition. For each cluster, marriage occurs only with a certain different cluster, resulting in the incest taboo. Lineages in the same cluster cooperate with each other, as well as with those in the bride’s cluster, whereas conflict for brides occurs only within a cluster. Such clusters are regarded as clans. Thus, clan (= cluster of lineages) is an emergent property in this model. Next, by introducing multiple traits (and preferences), such as habitats, names, and occupations, we uncovered the transition from dual organization to generalized and restricted exchanges, depending on the strengths of cooperation and conflict.

The present paper is organized as follows. In the next section, we describe the model with one trait. Then, with evolution simulations, we show the emergence of clans and incest taboo and present the condition for it. Next, we introduce the model with two traits and demonstrate the emergence of kinship structures. In the last section, we discuss the correspondence of our results with ethnographic records.

Model 1: One-Trait Model

Fig. 2 shows a schematic of our model. Lineages grow by interacting with other lineages in the same community (Fig. 2A). Each lineage splits into two when its population reaches twice the initial and is eliminated when the population goes to zero. Communities split and are eliminated in the same way in which their populations change by doubling and elimination of lineages within. When a community splits into two, one community, selected at random from the system, is removed to fix the total population of communities, to introduce community-level selection by removing communities with lower fitness. This multilevel selection on lineages and communities follows the so-called hierarchical Moran process, as adopted in the studies of biological and social evolution (25–31).

We assign a pair of a trait and a preference (i, p) to each lineage i, which is culturally transmitted to the next generation. When a lineage splits into two, daughter lineages inherit (i, p) of the mother with slight variation, because cultural traits are modified when transmitted (32). Specifically, (i, p) are “mutated” each step by adding a small noise component η, following a uniform distribution in [−μ, μ]. This variation corresponds to a genetic mutation in biology, but the inheritance and variation of traits here are of a cultural basis. Human beings categorize social groups even without genetic relatedness (33).

We introduce cooperation among social kin and that by marriage (blue and orange solid lines in Fig. 2B). Here, relationships of lineages are recognized by comparing the trait and preference values with a tolerance parameter σ. Thus, lineages i and j cooperate when |i − j|/σ, |i − p|/σ, or |p − j|/σ is sufficiently small. The density of cooperators for each lineage i, denoted

*Levi-Strauss assumed that the so-called Murungin structure fulfills these conditions; however, this was later proved to be a mere generalized exchange with alternative pathways (3, 10, 11).

<table>
<thead>
<tr>
<th>Cm</th>
<th>Dual</th>
<th>Generalized (Murungin?)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Restricted</td>
<td></td>
</tr>
</tbody>
</table>
by friend, is calculated by summing the degree of cooperation across all lineages. A larger friend value implies that the lineage gains more cooperation, resulting in larger growth of its population, where \( d_i \) is a parameter that gives the decline of death rate by cooperation (or a disadvantage of noncooperation).\(^1\)

Mating conflict of lineages \( i \) and \( j \) occurs when \([|p_i - p_j|]/\sigma\) is sufficiently small (red dashed line in Fig. 2B). The strength of conflict depends only on the number of lineages with close preferences and is independent of that of preferred lineages, because the conflict occurs even when there are sufficient bridegrooms and mates (22). The density of rivals, denoted by rival, is calculated by summing the degree of conflict across all lineages. A larger rival value implies a larger suppression of the growth rate, where \( d_m \) is a parameter indicating the decline of the population by conflict.

There is a mating chance for each lineage in each step. Each lineage \( i \) likely chooses lineage \( j \) when \([|t_i - t_j|]/\sigma\) is sufficiently small. Then, unmarried men in lineage \( i \) and unmarried women in lineage \( j \) form couples in lineage \( i \). Thus, women move into their husbands’ lineage after marriage. Children belong to and inherit (\( t \) and \( p \)) of their father’s lineage.

The initial values of \( t \) and \( p \) are zero in this model. Thus, at first, every couple can get married, even within a lineage. However, no qualitative changes are observed under other initial conditions, such as the uniform distribution of \([t, p] \in [0, 1]^2\) or the Gaussian distribution. See Materials and Methods for details. Source codes are available online (34) The notations and parameter values adopted in the simulations are summarized in Table 2.

**Table 2. Parameters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>( r )</td>
<td>Intrinsic growth rate</td>
<td>4</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Mutation rate for ( t_i ) and ( p_i )</td>
<td>Variable</td>
</tr>
<tr>
<td>( d_c )</td>
<td>Decline of death rate by cooperation</td>
<td>Variable</td>
</tr>
<tr>
<td>( d_m )</td>
<td>Increment of death rate by mating conflict</td>
<td>Variable</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Tolerance for similarity</td>
<td>1.0</td>
</tr>
<tr>
<td>( \text{Pop.} )</td>
<td>Initial population in a lineage</td>
<td>5</td>
</tr>
<tr>
<td>( N_i )</td>
<td>Initial number of lineages in a village</td>
<td>50</td>
</tr>
<tr>
<td>( N_c )</td>
<td>Number of communities in a system</td>
<td>100</td>
</tr>
</tbody>
</table>

Emergence of Clans with the Incest Taboo. This one-trait model was simulated by changing the parameters shown in Table 2. \((t, p)\) values of lineages in a community after 500 steps of simulation are plotted in Fig. 3A–C, and their time series are shown in Fig. 3D–F. As in Fig. 3B and C, they form a few clusters under a certain range of parameter values. Here, we used the X-mean method to optimize the number of clusters by adopting the Bayesian information criterion (35).

When clusters are formed as shown in Fig. 3, lineages in a cluster prefer those in one of the other clusters as their mates, which is determined by comparing the trait and preference values of cluster centers. Thus, people get married not within a cluster but across clusters. Marriage exchange among clusters emerges. In Fig. 3B, two clusters—namely, A (red) and B (blue)—conduct direct exchange as \( A \leftrightarrow B \), whereas in Fig. 3C, three clusters—namely, A (red), B (green), and C (blue)—conduct indirect exchange as \( A \Rightarrow B \Rightarrow C \Rightarrow A \). We define these clusters as clans. Here, lineages in the same clans cooperate because of social relatedness, and those in different clans are united by marriage. Because siblings of the same sex belong to the same clan, a father’s brother belongs to the same clan, and, thus, marriage with a father’s brother’s daughter is prohibited, when the incest taboo is organized. In contrast, because siblings of the opposite sex belong to a different clan by the move of brides to husbands’ lineages, a mother’s brother belongs to the mate’s clan, and, thus, marriage with a mother’s brother’s daughter is promoted. Such distinction between the cross and parallel cousins is observed in the different social relationships, as reported in ethnographic records (2). The clans here are fluid compositions; this observation is consistent with that of a previous study (36). In this manner, the social kinship with marriage exchange in indigenous societies spontaneously emerges.

The preferential relationships, resulting in the incest taboo, are sustained by multilevel selection. The relationships are cyclic unless there would be some lineages without having or sending brides. Then, we define the length of the marriage cycle \((C_m)\) by counting the number of clans engaged in the cycle. Time series of \( C_m \) are shown in Fig. 4 with and without the community-level selection, in addition to lineage-level selection. Thus, the number of communities in the system is one in Fig. 4A and 100 in Fig. 4B. If there is just one community, the structure of marriage exchange will soon collapse because of random drifts in trait \( t \) and preference \( p \). Whereas if there exist many communities competing with each other, collapsed communities will be eliminated, and, as a whole, the structure with \( C_m \geq 2 \) is sustained, when \( d_c \) and \( d_m \) are sufficiently large (orange line in Fig. 4B).

By imposing this multilevel selection, we simulated the model over 500 steps and 50 times for every condition to obtain \( C_m \). Recall that the existence of the incest taboo is equivalent to \( C_m \geq 2 \). Fig. 5 shows the phase diagram of the incest taboo. Here, society is defined to achieve the incest taboo when more than 90% of communities satisfy \( C_m \geq 2 \) (see SI Appendix, Fig. S1 for the dependence of \( C_m \) on \( d_c \) and \( d_m \)). As \( d_c \) increases, there is more pressure to eliminate isolated lineages without cooperators so that lineages form clusters. In contrast, large \( d_m \) creates pressure to eliminate lineages with many mating rivals, so that lineages become diverged. Hence, if both \( d_c \) and \( d_m \) are sufficiently large, lineages are clustered and diverged, leading to the formation of clans with marriage exchange. Fig. 5 also shows that as \( \mu \) increases, it becomes more difficult to establish
the incest taboo. For larger \( \mu \), the changes in traits and preferences are larger, so that the structure of marriage exchange is more fragile.

Here, lineages spontaneously form clans and the incest taboo emerges. The structures with various \( C_m \) values emerge as in Fig. 3. In this model, however, children always belong to their fathers' clans and, thus, \( C_t = 1 \). Thus, this model is insufficient for modeling some kinship structures, such as the restricted exchange shown in Fig. 1C.

**Model 2: Two-Trait Model**

Recalling that children can belong to different clans from parents by inheriting traits from both parents, as in Fig. 1C, we extended the model to have two culturally independent traits and preferences, \( t = (t_1, t_2) \) and \( p = (p_1, p_2) \). These traits are inherited maternally or paternally. Because there are only two sexes, there exist only two pathways of inheritance. If several traits are inherited via father/mother, they can be effectively regarded as one trait. Thus, the two-trait model is sufficient for considering kinship structures with multiple traits.

In the previous model, children belong to and inherit \( t, p \) from their father's lineage. In reality, however, a child can inherit some traits from father and other traits from mother, as observed in many indigenous societies (2, 37, 38) (e.g., Fig. 1C). For example, he or she inherits land from the father and the name of the mother. We assume that children inherit \( t_1, p_1 \) from their fathers. Next, each lineage can choose whether they inherit \( t_2, p_2 \) from the father or mother. The former (latter) corresponds to a unilinear (bilateral) descent rule. Suppose men in lineage \( i \) marry women in lineage \( j \). If lineage \( i \) adopts a unilinear descent rule, children possess traits and preferences \( t', p' \). By contrast, if a bilateral descent rule is adopted, lineage \( i \) adopts the trait and preference to \( t = (t_1, t_2), p = (p_1, p_2) \). For both cases, children's lineages and fathers' lineages' kin cooperate. In the initial generation, either of the rules is assigned at random, and the rule would switch with a probability \( \mu_{sd} = 0.01 \).

**Emergence of Kinship Structures.** Lineages are clustered into clans in \((t, p)\) space as with one-trait model. For the two-trait model, descent relationships of clans emerge, as well as those of marriage. Fig. 6 shows examples of the final state after 500 steps of simulation. In Fig. 6A, there are three clans where only the first trait is clustered into three groups (Fig. 6A, 1), whereas the second is not (Fig. 6A, 2). The green clan prefers red, the red prefers blue, and the blue prefers green. The relationship of marriage exchange forms a three-period cycle. Thus, generalized exchange in Fig. 1D emerges. In Fig. 6B, there are four clans, where each trait is clustered into two, and thus two-by-two clans are formed (Fig. 6B, 1 and 2). Considering the two-dimensional distance of preference relationships, red and purple clans prefer each other, as do blue and green clans. Here, most lineages adopt bilateral descent rules. A child of a father in a red clan and a mother in a purple clan inherits \( t_1 \) from red and \( t_2 \) from purple and, thus, belongs to a green clan. Hence, restricted exchange in Fig. 1C emerges. Such structures are classified as shown in Fig. 1, by computing the marriage cycle (\( C_m \)) and descent cycle (\( C_d \)).

Fig. 7 shows the phase diagram of kinship structures. We use the classification in Fig. 1. Incest taboo is not generated (orange in Fig. 7) when \( d_m \) is small, as in the previous model. As \( d_m \) increases, dual organization (green) appears, and, thus, the incest taboo emerges. Then, more sophisticated structures as generalized exchange (red) and restricted exchange (purple) emerge. When both \( d_m \) and \( d_a \) are large, population suppression is so strong that it is hard to achieve high cooperation and low competition to overcome the suppression. Hence, all communities are extinct in the upper right region in Fig. 7 (blue). In this model, other structures with \( C_m > 2 \) and \( C_d > 1 \), such as the
so-called Murungin structure, are scarcely observed. This time, various structures emerge, depending on the relative weight of $d_c$ and $d_m$. As the relative weight of $d_m$ increases, the emergent structure generally changes from the dual organization to generalized exchange and then to restricted exchange, because a larger $d_m/d_c$ favors reducing mating rivals at the expense of cooperators. Furthermore, this phase diagram is robust against the choice of the initial values. For example, even if generalized or restricted exchange is set initially, it will soon collapse when $d_c$ and $d_m$ are small.

Restricted exchange needs the separation of two traits by each cluster, whereas generalized exchange needs that of only a single trait, as shown in Fig. 6. With larger $\mu$, clan separation is more fragile, as shown in Fig. 5, and thus restricted exchange is replaced by generalized exchange (see SI Appendix, Fig. S2 for diagrams with larger and smaller $\mu$ values).

**Discussion**

We have shown that the incest taboo emerges spontaneously by considering the cooperation of kin and mates, as well as the mating conflict of rivals. Furthermore, all of the kinship structures observed in the indigenous society emerge in the model with two traits and preferences. When clans are formed as clusters of lineages with close values of traits and preferences, marriage and descent relationships of clusters. Kinship structures emerge as the marriage and descent relationships of clusters. Because women change lineages after marriage, a distinction between the cross and parallel cousins is made, such as mother’s brother’s daughters and father’s brother’s daughters, which some ethnographic records emphasized, but previous biological or mathematical studies ignored (2). Here, as a result of the dynamics and clustering of lineages, the social categories of siblings and the incest taboo simultaneously emerge.

A small “mutation rate” $\mu$ facilitates the emergence of the incest taboo and sophisticated kinship structures. The speed of change in cultural traits is known to be faster (i.e., $\mu$ is larger) in societies with mass teaching by teachers than in those with education within families (32). Furthermore, a small $\mu$ indicates that people likely marry according to the mate preferences of parents, which requires strong lineage connections. As the societies are centralized and parental influence is decreased, $\mu$ increases. Then, sophisticated kinship structures disappear, as shown in our model.

Generalized exchange and dual organization emerge when cooperation is important, whereas restricted exchange emerges when the avoidance of mating conflict is more important. This suggests that dual organization is similar to generalized exchange rather than restricted exchange, in contrast to Lévi-Strauss’s classification. If the mating conflict is little, the community with a small number of clans united by marriage is fitted. As mating conflict becomes stronger, it would be better to separate clans within villages to avoid conflicts. Then, restricted exchange emerges. In this case, however, each clan has more than one noncooperating clan, such as $B_1$ for $A_1$ in Fig. 6. Thus, restricted exchange emerges only when the avoidance of conflict is more important than cooperation, i.e., $d_m/d_c$ is sufficiently large.

The present study sheds light on why different kinship structures are adopted in different societies. Restricted exchange is mainly observed in hunter-gatherer societies, such as the Aboriginal in Australia and Yanomamo in the Amazon, whereas generalized exchange is observed in Chinese peasants, agricultural societies such as the Kachin in Myanmar, and fishery societies such as the Nivkh in Russia (2, 39–43). Studies on the Aboriginal and Yanomamo reported that conflicts over females often cause fights among lineages in the hunter-gatherer societies (21–23). By contrast, agriculture needs massive cooperation, including that for wars over land or food (21). In fishery societies, massive fishing and competition for access to fishing rights require cooperation (44, 45). To conclude, the tribes under stricter mating conflict conduct restricted exchange, whereas those requiring stronger cooperation conduct generalized exchange, as is consistent with the observation.

Of course, for a better understanding of the emergence of kinship structures, detailed analyses are needed on the degree of cooperation and mating conflict in each indigenous society—for example, by examining the cause of conflicts and deaths therein. Furthermore, it is necessary to clarify the relationship between lifestyles and parameters—$d_c$, $d_m$, and $\mu$.

The present model has some limitations. First, we did not consider the direct interaction of communities. However, massive wars between communities, for example, need a strong tie among lineages, and, thus, it can be implicitly included as the increase of $d_c$. Second, we use the same fixed value of $\sigma$ for measuring social relatedness and the possibility of marriage. In reality, lineages can propose marriage to more or fewer lineages than others. However, with sufficiently large $d_m$, those proposed to more lineages would be eliminated by stricter mating conflict, whereas those proposed to fewer lineages would suffer from lack of mates. Thus, even if we introduce the evolution of $\sigma_m$ for the possibility of marriage, it would remain finite. Hence, for simplicity, we set $\sigma_m = \sigma$. Third, our model cannot explain the emergence of social strata as Lévi-Strauss discussed (2) or relationships between kinship structures and social systems as Todd discussed (46). For such issues, one needs to consider economic activities and/or social factors besides kinship—for example,
In these forms, the suppression of population is relaxed from 1/(1 + d_f) to 1 by cooperation as friend increases from 0 to 1, whereas it is amplified from 1 to 1/(1 + d_m) with the increase of rival. Note that the results to be presented were qualitatively independent of these specific forms as long as cooperation enhances and conflict suppresses the population.

We adopted the following algorithm for the population change in lineages. For lineage i of time step n, the number of boys, girls, unmarried men, unmarried women, and couples are denoted by B_i^n, G_i^n, M_i^n, F_i^n, and C_i^n, respectively. The intrinsic growth rate is denoted by \( \lambda_i \). Then, the population change in lineage i is given by

\[
\lambda_i^n = r \times C_i^{n-1} - \lambda_i^n \quad \text{Poission}(\lambda_i^n), \quad G_i^n = \text{Poission}(\lambda_i^n), \quad t_f^n = t_f^{n-1} + n, \quad p_f^n = p_f^{n-1} + n. \]

\[
\text{friend}^n = \sum_{j=1}^{\#\Lambda} \exp \left( - \left( t_f^n - t_j^n \right)^2 / 2 \sigma^2 \right), \quad \text{rival}^n = \sum_{j=1}^{\#\Lambda} \exp \left( - \left( p_f^n - p_j^n \right)^2 / 2 \sigma^2 \right), \quad M_i^n = \frac{1 + d_m \times \text{rival}^n}{1 + d_f \times \left( 1 - \text{friend}^n \right)} B_i^n, \quad F_i^n = \frac{1 + d_m \times \text{rival}^n}{1 + d_f \times \left( 1 - \text{friend}^n \right)} G_i^n, \quad C_i^n = \min \left( M_i^n, \sum_{j=1}^{\#\Lambda} F_j^n \right) \]

Couples in each lineage give birth to children following the Poisson distribution, as given by Eq. 1. Noise component is added to \((t, p)\), following the uniform distribution in \([-\mu, \mu]\) as Eq. 2. As they grow up, the population is suppressed with regards to friend and rival, as given by Eqs. 3–6. People get married according to the traits and preferences of their lineages. After marriage, the women move into husbands’ lineage, as in Eq. 7. Here, we assumed monogamy, but the result is qualitatively independent of such a marriage system.

Data Availability. Source codes for these models can be found at https://github.com/Kenjiitaoclan.git.

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