1700, the variation depending mainly on the other atoms in the molecule.

I am particularly indebted to Dr. W. W. Coblentz for his suggestions and personal interest in the performance of these measurements, which were made in his laboratory in the Bureau of Standards. I wish to thank Dr. G. K. Burgess and Dr. C. A. Skinner for permission to work at the Bureau, and Mr. C. W. Hughes for aid in the laboratory.

* Johnston Scholar, Johns Hopkins University.

[1] W. W. Coblentz, *Carnegie Inst. Washington Pub.*, No. **35**, Part II, p. 117.

[2] F. K. Bell, *J. Amer. Chem. Soc.*, **47**, 2192 (1925).

[3] W. W. Coblentz, *J. Opt. Soc. Amer.*, **7**, No. 6 (1923); Bureau of Standards, *Scientific Papers* **188, 204, 282** and **401.**

[4] *Carnegie Inst. Washington Pub.*, No. **35**, p. 24.

[5] J. W. Ellis, *Physic. Rev.*, [2] **23**, 48 (1924).

[6] O. Reinkober, *Zs. Physik.*, **5**, 192 (1921).

[7] A. Kratzer, *Ibid.*, **3**, 289 (1920). See also Sommerfeld, *Atomic Structure and Spectral Lines*, Ch. VII, § 2.

[8] J. W. Ellis, *Physic. Rev.*, **22**, 200 (1923).

A POSSIBLE EXPLANATION OF THE RELATIVITY DOUBLETS AND ANOMALOUS ZEEMAN EFFECT BY MEANS OF A MAGNETIC ELECTRON

BY F. R. BICHOWSKY AND H. C. UREY

DEPARTMENT OF CHEMISTRY, JOHNS HOPKINS UNIVERSITY

Communicated December 22, 1925

In a recent note Uhlenbeck and Goudsmit[1] suggest that the fourth of the quantum numbers proposed by Pauli[2] to describe the quantum state of each electron in an atom is to be associated with the angular moment of rotation of the electron about its own axis. The electron thus assumed is essentially identical except in dimensions with the magneton of Parsons.[3] This idea had also occurred to us quite independently and for largely the same reasons as those given by these authors. We have, however, carried the idea somewhat further than the authors have reported in their brief note and will report these further considerations here.

It is known that the separation of the members of the multiplet spectral terms such as are concerned in the *D* lines of sodium must be connected with magnetic forces since the multiplicity of the term is invariably associated with an anomalous Zeeman effect and since at high magnetic fields the complex Zeeman pattern of the multiplet term simplifies into the normal Zeeman triplet (the Paschen Back effect). It was therefore proposed by Heisenberg[4] that the multiplet separation is due to the difference

in the mutual magnetic energy of the atom when the valence electrons were moving in the same sense as the resultant motion of the rest of the atom and when they were moving in whole or in part in the opposite sense to the supposed resultant motion of the rest of the atom. This model was calculated by Heisenberg on the simplifying assumption that the magnetic force on each valence electron could be calculated from the vector sum of the external field and the resultant field due to the relative motion of the kernel. The result was shown to be consistent with the observed Zeeman and Paschen-Back effect of the normal doublets and triplets but as was pointed out by Breit[5] gave an energy order for the p_1 and p_2 and analogous level opposite to that observed. We noticed that this error in sign would be corrected if the electron were rotating on itself and moving about a positive kernel of no magnetic moment.

It was shown by Millikan and Bowen[6] and also by Landé[7] that these optical doublets and triplets obey the same law for their separation as the so-called relativity X-ray doublets, that is to say that the energy separation of all of these levels is given by the formula

$$\Delta W = \frac{8\pi^4 e^8 m}{c^2 h^4 n^3 k_1 k_2} Z_i^2 Z_a^2 \Delta m,$$

where $k_1 - k_2 = 1$ and where Z_i and Z_a are the effective atomic numbers for the inner and outer parts of the orbit, respectively. This is identical with the formula originally proposed by Sommerfeld for the X-ray doublets on relativity considerations except that in his case $Z_i = Z_a$. Landé[8] attempted the explanation of these optical and X-ray doublets on the basis of a magnetic kernel and secured the formula

$$\Delta W = \frac{8\pi^4 e^8 m}{c^2 h^4 n^3 k^2} Z_i Z_a^2 \Delta m,$$

where Δm is the difference of the two quantum numbers specifying the orientation of the orbit to the axis of the kernel and is equal to 1. This formula has the same constant as Sommerfeld's formula but is proportional to Z_i instead of Z_i^2, or Z_i^3 instead of Z_i^4 for the X-ray levels, as is the case in Sommerfeld's formula and as is required by experiment. This model can therefore not account for the observed separation and some other source for the magnetic forces must be sought.

In a recent article Pauli[2] showed that it is possible to classify the multiplet levels by assigning four quantum numbers, i.e., four degrees of freedom, to each electron and that this assignment was in general agreement with our knowledge of such levels and his R quantum number is always $\pm 1/2$. In view of the above it seems possible to assume that the new degree of freedom is due to the fact that the electron possesses an intrinsic magnetic field. We do not need to restrict ourselves to any particular model of an

electron which would make it magnetic. Indeed relativity treatment of any such model would require very special considerations. We will assume here that the electron is a charged magnetic doublet of the magnetic moment of one-half a Bohr magneton. For the present considerations we do not need to know the angular momentum though the magnitude of the magneto-mechanical effect suggests that its value is equal to $h/2\pi$.

In our calculations we will neglect the perturbing effect of other electrons in the atom, that is to say our calculation will be strictly applicable only to the case of an electron moving in an inverse square field. The calculation will apply very closely for the X-ray levels of the atom, however, and by using the theory of Landé in regard to the contribution to the energy by the inner and outer parts of a penetrating orbit, it can be applied to the optical levels.

In securing the mutual energy of the system of a magnetic electron moving in the positive field of the nucleus, we can start from the electromagnetic equation that the magnetic force exerted on a unit magnetic pole moving in the electric field is

$$\mathbf{F} = -\frac{\mathbf{v} \times \mathbf{d}}{c},$$

where $\mathbf{v}$ is the vector velocity of the magnetic pole and $\mathbf{d}$ the electric intensity at the point. This force would cause a precession of the rotating electron in the same way as an external magnetic field causes a precession of the electron orbit of the hydrogen atom and the additional energy would be

$$\Delta E = \frac{e\mathbf{H}}{4mc} p_e \cos \alpha$$

$$= -\frac{p_e e}{4mc} \frac{|\mathbf{v} \times \mathbf{d}|}{c} \cos \alpha,$$

where p_e is the mechanical angular momentum of the electron and α is the angle between p_e and the angular momentum of the electron moving about the nucleus. The factor $1/4$ is used in place of $1/2$ since we are assuming that the mechanical moment is twice the magnetic moment of the electron, both being measured in quantum units. We assume that the quantum number for the rotation of the electron is 1. The same result can be secured if we assume that the rotational quantum number of the electron is $1/2$ and that its magnetic moment is $1/2$ a Bohr magneton.

In the formula for ΔE we can substitute for $\mathbf{v}$ and $\mathbf{d}$

$$\mathbf{v} = r\varphi\varphi_1 + r\mathbf{r}_1$$

$$\mathbf{d} = -\frac{Ze}{r^2}\mathbf{r}_1.$$

Then

$$\Delta E = \frac{p_e Z_{e'}}{4mc^2} \frac{r^2\varphi}{r^3} \cos\alpha$$

$$= \frac{p_e Z_{e'}}{4mc^2} \frac{p_\varphi}{r^3} \cos\alpha,$$

p_φ being the angular momentum of the electron about the nucleus. We regard this as a perturbing function and take the mean value over the unperturbed motion, i.e., the orbit of a non-magnetic electron moving in the field of a positive nucleus.

$$\overline{\frac{1}{r^3}} = \frac{1}{b^3} = \frac{64\pi^6 e^6 m^3 Z^3}{n^3 k^3 h^6}$$

where b is the semi major axis of the ellipse and n and k are the principal and subordinate quantum numbers. The final form for the energy is

$$\Delta W = \overline{\Delta E} = p_e \frac{8\pi^5 e^8 Z^4 m}{c^2 h^5 n^3 k^2} \cos\alpha.$$

If we set $p_e = \frac{h}{2\pi}$ and $\cos\alpha = \pm 1$,

$$\Delta W = \pm \frac{4\pi^4 e^8 Z^4 m}{c^2 h^4 n^3 k^2},$$

the positive sign being used when p_e and p_φ have the same direction and the negative sign when they are oppositely directed. This formula is the same as that derived by Landé except that the separation of the levels is now proportional to the fourth power of Z instead of the third power.

If following Landé,[9] we write this formula

$$\Delta W = \pm \frac{4\pi^4 e^8 Z_i^4 m}{c^2 h^4 n_i^3 k^2}$$

for the penetrating optical levels, where Z_i is the effective atomic number and n_i the effective total quantum number for the inner part of the orbit and then multiply by the factor

$$\frac{t_i}{t_a} = \frac{n_i^3 Z_a^2}{n_a^3 Z_i^2},$$

where t_i and t_a are the time spent in the inner and outer parts of the path, respectively, and Z_a and n_a are the effective atomic numbers and effective total quantum number, respectively, for the outer part of the orbit, we secure the formula

$$\Delta W = \pm \frac{4\pi^4 e^8 m Z_i^2 Z_a^2}{c^2 h^4 n_a^3 k^2}.$$

This differs from Landé's formula only in having Z_i in the second power instead of the first and as he has shown this agrees approximately with the experimental data on the separation of doublet and triplet levels.

We must choose for the azimuthal quantum number of an s orbit of silver the value $^1/_2$ since the experiments of Stern and Gerlach show that this atom has a total magnetic moment of one Bohr magneton which on our theory is contributed half by the electron rotating on itself and half by its rotation around the nucleus. We must, therefore, choose half quantum numbers for the azimuthal quantum numbers.

Our formula differs from Sommerfeld's only by a small difference then between his k_1k_2 and our k^2. Our k is the mean of his k_1 and k_2 so that $k^2 = k_1k_2 + {}^1/_4$. This difference is probably unimportant since it amounts to only 1 part in 9 for the 2 quantum orbits (the L shell relativity doublets) and is smaller for the doublets having higher values of the total quantum number. This seems to be a possible rational explanation of the so-called relativity doublet levels, both in the X-ray and optical region.

We see that the additional energy is positive when the angular momentum of the electron rotating on itself has the same sign as its angular momentum about the nucleus so that the objections of Breit to the original theory of Heisenberg do not apply to this theory; and so far as we can see Heisenberg's theory of the anomalous Zeeman effect for the doublet type of spectrum follows without any change whatever. It is also found that a partial explanation for the anomalous Zeeman effect and multiplicities of spectra from atoms containing more than one electron can be secured, but a complete dynamical description would require a more extensive study. The case of the S level of the alkaline earths is of considerable interest. Each electron must be characterized by $n_k = n_{1/2}$, since the removal of either leaves the ion in the normal state and as mentioned above it is necessary to assign these quantum numbers in order to meet the experiments of Stern and Gerlach.[10] Since the magnetic moment of the normal alkaline earth atom should be zero this means that the two electrons must be oriented in opposite directions. And according to the general scheme of Stoner,[11] this would lead us to conclude that Sommerfeld's[8] model for the helium atom should be correct. It is interesting that this model, though dynamically unstable, does give the correct ionizing potential for helium.

Also on the basis of the present theory we would expect the spectrum of helium to consist of a singlet and triplet series of lines just as in the case of the alkaline earths. It seems that the decision of the question as to whether the ortho helium spectrum is a doublet or triplet spectrum might be decisive.

One serious objection to half quantum numbers has been that above atomic number 68 the electron moving in a $^1/_2$ orbit would collide with

the nucleus if relativity mechanics holds. While a magnetic force acts on the electron moving about a positive charge, an electric force is exerted on the nucleus due to the rotation of a magnet about it. This force is equal to $-\frac{\mathbf{v} \times \mathbf{h}}{c}$, where $\mathbf{v}$ is the velocity of the magnet and $\mathbf{h}$ is the magnetic intensity at the nucleus. This force is in the opposite direction to the electrostatic force of attraction of a nucleus and electron and tends to force them apart if the p_a and p_φ have the same direction. It may be that this force will prevent the collision of nucleus and electron.

We have found it impossible to account for the fine structure of the hydrogen and helium lines by assuming a simple superposition of the relativity change in energy according to Sommerfeld's formula and the magnetic changes in energy according to the above formula, if half quantum numbers are used for the azimuthal quantum numbers. However, it appears that the proper form to apply the relativity theory to this case is possibly not that given above. Furthermore it does not seem impossible that the proper form of relativity mechanics suitable to this electron may give the correct fine structure of the hydrogen and helium lines.

We wish to state that in spite of the rather remarkable results secured for the relativity doublets in the optical and X-ray regions, we appreciate fully the very important fault of the theory, that in its present form it cannot account for the fine structure of the hydrogen and helium lines which is so beautifully explained on the basis of the Lorentz electron. Until this can be done we regard the theory as a purely tentative suggestion.

Note added in proof: Since this paper was written, Dr. W. H. McCurdy and Dr. W. A. MacNair[12] of the Johns Hopkins Physics Department have separated the ortho helium lines and find no evidence for a triplet structure.

[1] G. E. Uhlenbeck and S. Goudsmit, *Naturwissenschaften*, **47**, 953 (1925).
[2] W. Pauli, Jr., *Zs. Physik*, **31**, 765 (1925).
[3] A. L. Parsons, *Smithsonian Inst. Publ.*, **65**, No. 11 (1915).
[4] W. Heisenberg, *Zs. Physik*, **8**, 273 (1922).
[5] G. Breit, *Nature*, **112**, 396 (1923).
[6] R. A. Millikan and I. S. Bowen, *Physic. Rev.*, **23**, 1 (1924).
[7] A. Landé, *Zs. Physik*, **25**, 46 (1924).
[8] A. Sommerfeld, *Atomic Structure and Spectral Lines*, Methuen, **1923**.
[9] A. Landé, *Zs. Physik*, **24**, 88 (1924).
[10] O. Stern and W. Gerlach, *Ibid.*, **8**, 110 (1922); **9**, 349 (1922).
[11] E. C. Stoner, *Phil. Mag.*, **48**, 719 (1924).
[12] W. H. McCurdy and W. A. MacNair, Unpublished note, *Nature*.