[3] C. Kuratowski and C. Zarankiewicz, "A Theorem on Connected Point Sets," *Bull. Amer. Math. Soc.*, **33,** 1927 (571–575).

[4] Menger, K., "Zur allgemeiner Kurventheorie," *Fund. Math.*, **10,** 1927 (96–115).

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[6] A continuous curve is said to be acyclic if it contains no simple closed curve. Cf. Gehman, H. M., "Concerning Acyclic Continuous Curves," *Trans. Amer. Math. Soc.*, **29,** 1927 (553–568).

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[8] Rosenthal, A., "Teilung der Ebene durch irreduzible Kontinua," *Münchener Akademie, Sitzungsber.*, **1919** (91–109).

[9] Mullikin, Anna M., "Certain Theorems Relating to Plane Connected Point Sets," *Trans. Amer. Math. Soc.*, **24,** 1922 (144–162).

[10] Knaster, B., "Un continu dont tout sous-continu est indécomposable," *Fund. Math.*, **3,** 1922 (247–286).

[11] Janiszewski and Kuratowski, "Sur les continus indécomposables," *Fund. Math.*, **1,** 1920 (210–222).

[12] A continuous curve is said to be simple if it is either an arc, an open curve, a simple closed curve or a ray of an open curve. See my paper "Concerning Simple Continuous Curves," *Trans. Amer. Math. Soc.*, **21,** 1920 (333–347).

[13] Whyburn, G. T., "Concerning the Cut Points of Continua." Presented to the American Mathematical Society, Sept. 9, 1927, but not, as yet, published in full.

ON METHODS AND APPLICATIONS IN SPECTROPHOTOMETRY

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1. The interpretation of stellar spectra is not possible without precise photometry. Just as it was necessary that stellar spectra should be photographed before they could be discriminatingly classified, it is essential that the photographed spectra be measured before they can be interpreted in the light of modern theory. The inadequacy of eye estimates for photographic photometry has repeatedly been demonstrated and, accordingly, recent spectroscopic work at Harvard has been largely concerned with the development and standardization of methods of photographic spectrophotometry by means of the Moll microphotometer. The present note is intended to summarize the results of two years' experiments.

The basic problem of the calibration of a photographic plate involves the determination of the blackening that corresponds to a given intensity of incident light. As the photographic plate responds selectively to light of different wave-lengths, and as the nature of this response varies for different emulsions, a complete standardization for any one emulsion would include an intensity-blackening curve for every wave-length.

But stellar spectrophotometry at present cannot usefully be made anything but relative; the selective absorption suffered by starlight in traversing both the atmosphere and the optical train of the telescope makes

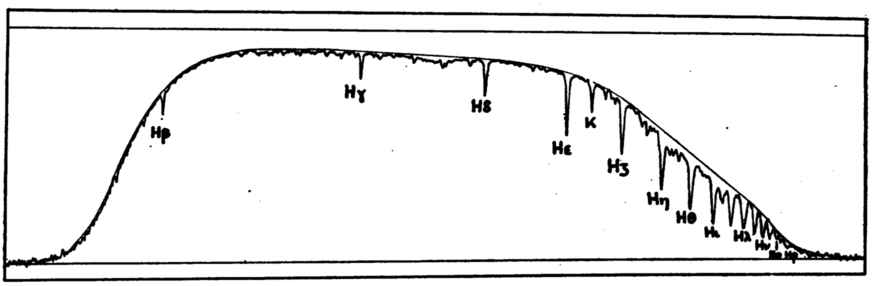

FIGURE 1

Microphotometer tracing from the spectrum of α Cygni. Horizontal lines above and below the tracing represent "darkness" and "clear film." The continuous background is indicated by a light line. The lines of the Balmer series and of Ca^+ are marked.

it impractical to attempt absolute measures. The two fundamental measures that can be made on stellar spectrograms—the absorption at any wave-length and the relative background energy—must both be made differentially; the one is related to the neighboring continuous background, graphically inserted (see Fig. 1), and the other to the background of another star of known energy distribution—both the measures to be compared being made *at the same wave-length.* To compare intensities in this way it is only necessary to know the gradation of the plate in

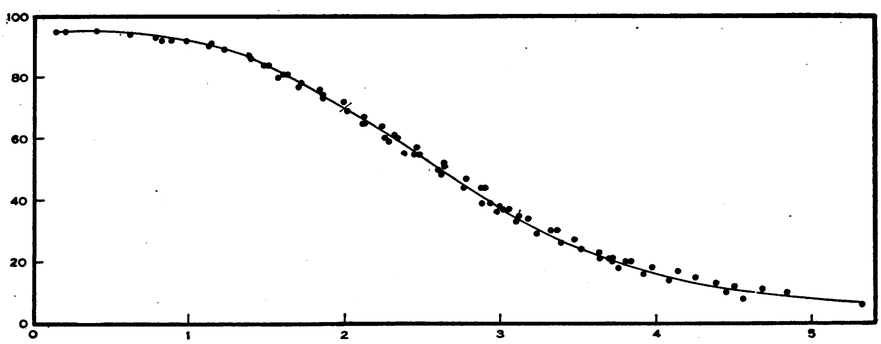

FIGURE 2

Curve for reducing galvanometer deflections (ordinates) to stellar magnitudes (abscissae). Adapted from Harvard Observatory Bulletin 301.

every wave-length; the absolute intensity-blackening relation is irrelevant.

Very fortunately it appears, from numerous tests made in the course of the work now described,[1] that the gradation of the plates used, under the standard conditions, does not vary sensibly with wave-length, at least within the region usually photographed. Therefore, one reduction curve, relating increase of blackening to increase of intensity, can be used for the whole length of the spectrum on each plate. It is to this experimental approximation that the possibility of rapid and comprehensive methods of spectrophotometry is due.

2. Various methods have been used for the establishment of reduction curves, all of them involving the illumination of the plate, either successively or simultaneously, by light sources of known intensity ratio. All the satisfactory methods use an exposure of uniform length. According to the several ways of governing the differences of intensity of the various sources, the methods used may be classed as follows:

METHOD	REFERENCE	
(*a*) Various diaphragms on one star	Shapley	*Harv. Obs. Bul.* 805
	Payne and Shapley	*Harv. Repr.* 28
	Payne and Hogg	*Harv. Obs. Circ.* 301
(*b*) Several stars of known brightness, same spectral class	Payne and Hogg	*Harv. Obs. Circ.* 301
(*c*) Several stars of known brightness, different spectral classes	Hogg	*Harv. Obs. Circ.* 309
(*d*) Differences within same spectrum	Payne	Unpublished
(*e*) Laboratory standardization	Dunham	*Harv. Obs. Bul.* 853
(*f*) Exposure ratios on one star	Payne and Hogg	*Harv. Obs. Circ.* 304
(*g*) Prism crossed with grating	Hogg	Unpublished

The methods are further described in the papers quoted and will not be discussed in detail here. It will be noticed that for slit spectra methods (*d*), (*e*) and (*f*) only are available; (*f*) is, however, quite unsatisfactory. The three methods chiefly used at Harvard, (*a*), (*b*) and (*c*), are available only in connection with the objective prism spectrograph.

The prism crossed with grating, which has been used at Greenwich with success,[2] has been the subject of some preliminary experiments, and is probably the best method for very accurate contour work on bright stars. It is proposed to use it in the analysis of long dispersion spectra of the sun.

There is little doubt that method (*c*) will eventually replace the others, because of its speed and accuracy. As the two or more spectra used for standardization are simultaneously photographed, it eliminates changing sky conditions and corrections arising from differential refraction, two factors which may cause serious error for apertured spectra when the stars photographed are faint. Work has been begun on a program of standard pairs of stars, suitably distributed around the sky.

Measurement of the blackening of the plates has been made with the Moll self-registering microphotometer for all the researches discussed. The Schilt visual microphotometer has, however, obvious applications, especially to the measurement of short dispersion spectra, and with it unpublished results have already been derived.

3. When the photographic plate has been calibrated, so that the relation of light intensity to density of image is known, it remains to select significant quantities for measurement. The measurable quantities are evidently the blackness and total absorption of the lines, and the relative energy distribution in the continuous background.

The nature of the quantity measured by the "blackness" or central intensity is discussed by one of the writers in *Harvard Circular* 302, where it is shown that for such wide lines as the H and K lines of ionized calcium in cool stars the central intensity represents the true "residual intensity" of the line, and for the narrow lines it represents the total absorption. The resolving power of the spectrograph affects all measured depths; therefore, only for spectra made with the same instrument can depths be usefully compared. In addition to instrumental effects there are doubtless disturbances arising in the atmosphere of the earth, and possibly also in the atmosphere of the star,[3] which modify the residual intensity, increasing it beyond what would be observed if the theory of the high level chromosphere[4] applied to it rigorously.

The relatively small dispersion of the Harvard spectrograms results in the blending of some important lines. Only lines that are very intense and undoubtedly predominant are included in the discussions, and the possible effects of blending are always kept in mind.

On the other hand, the total absorption of a line is probably unaffected by the resolving power of the spectrograph, and may, therefore, be measured by suitably integrating the line contour. A refined method of doing this is described by Dunham[5] and one suited to more rapid work, by one of the writers.[6] Total energy of lines has an obvious significance, both for stars with absorption spectra, and for emission line objects, such as novae in their later stages, Wolf-Rayet stars, and notably gaseous nebulae. Measures for such objects are now being made at Harvard Observatory.

The determination of the relative energy distribution in the continuous background results from a simple differential comparison with the standard star at various wave-lengths, and may be directly used in the measurement of relative temperature.

4. The results hitherto attained with the aid of the methods enumerated are summarized in the next table. The two main measured parameters, line intensity and relative background energy are the material, respectively, for the spectrum analyses and the measured temperatures.

The researches cover most of the spectral sequence, and are of general as well as special interest. We shall note first a few points brought out by special researches, and then touch lightly on some general implications of the material—implications that must be regarded as preliminary and tentative.

The study of the eight brighter Pleiades shows essential similarity for the six stars of class B5 in the intensities of the Balmer lines. Merope, the most deeply enmeshed in nebulosity, has, however, the shallowest line and the most conspicuous hydrogen emission, and is considerably the reddest of the B5 stars. The emission lines of Alcyone, which are also conspicuous, are probably an accompaniment of its greater luminosity, and not related to the nebula. The relative and actual intensities of the hydrogen lines change with the spectral class, Atlas and Pleione (Class B8) having deeper lines than the other six stars. Definite criteria of luminosity for stars of class B5 are not found.

Mira Ceti was followed from the maximum of 1926 until it fell below visual magnitude 6.5. At maximum the hydrogen lines, which are then at their strongest, contributed not more than a tenth of a magnitude to

	STARS	MEASURES	REFERENCE
(*a*)	Pleiades	Spectrum analysis	*Harv. Obs. Circ.* 303
(*b*)	Pleiades	Temperatures	*Harv. Obs. Circ.* 309
(*c*)	Sirius and Vega	Spectrum analysis	*Harv. Obs. Circ.* 304
(*d*)	Fourteen A stars[5]	Spectrum analysis	*Harv. Obs. Bul.* 853
(*e*)	Nine second type stars	Spectrum analysis	*Harv. Obs. Circ.* 305 and 306
(*f*)	Mira Ceti	Spectrum analysis	*Harv. Obs. Circ.* 308
(*g*)	Wolf-Rayet stars	Temperatures	*Harv. Obs. Bul.* 848
		Spectrum analysis	Unpublished
(*h*)	Mizar	Spectrum analysis	Unpublished
(*i*)	δ Cephei and η Aquilae	Spectrum analysis	Unpublished
(*j*)	Class M stars	Spectrum analysis	Unpublished
(*k*)	High luminosity stars	Spectrum analysis	Unpublished
(*l*)	Standard fields	...	Unpublished

the photographic light of the star—and considerably less to the visual brightness, for Hα is not a strong emission line. The titanium oxide bands, over the interval examined, divert about twenty per cent of the star's photographic light. These results, derived over a part of the period at which the light of the companion is not conspicuous, are probably typical of many long period variable stars.

The estimates of temperature published for the O stars near the cluster N. G. C. 6231 have probably little physical meaning, as it is shown that the energy in the background curves does not correspond to a black body distribution. "Color magnitudes," instead of black body temperatures, are proposed as a better way to express the energy distribution determined spectrophotometrically. The abnormal distribution of energy, and the surprisingly low temperatures found for the northern Wolf-Rayet stars, are borne out by further studies, now in progress, of the spectra of southern stars of the class.

Turning to the general conclusions that can be drawn from the material, it is found that the behavior of integrated line areas along the spectral sequence is roughly parallel to that of the absorption lines, as estimated visually. The observed maxima discussed by ionization theory[7] are no doubt maxima of total absorption. For lines that are wide enough for the measures to be significant, it appears that central intensity for a given line passes through a minimum that coincides approximately with the maximum of total absorption, though there is a gradual change of line contour in passing along the spectral sequence, so that the connection is not quite a simple one. These facts place the Saha-Fowler-Milne theory for the first time on a quantitative basis, and also, as Milne has pointed out, make it possible to develop a theory of atmospheric structure.

The relation of line contour or quality to luminosity within a given spectral class has long been known, and measures such as the present contain the material for placing it too on a quantitative basis. It is found that the total absorption of all lines is greater for intrinsically bright stars, and that contours are more flattened for fainter stars, effects that have a definite empirical use, and possibly a theoretical one, in the discrimination and interpretation of luminosities. The observed intensities of the H and K lines have also an immediate application to the study of the presence and structure of chromospheres.

Some of the matters mentioned in the present section are discussed in *Harvard Circular* 307; the remainder await the publication of the data relating to the supergiant stars, and the discussion of the spectra of the Cepheid variables, which will be used to calibrate the scale by which the other highly luminous stars are to be examined.[8]

Summary.—1. The problem presented by stellar spectrophotometry is briefly surveyed.

2. The methods applied at Harvard to its solution are enumerated and compared.

3. The quantities that can be usefully measured in stellar spectra are discussed.

4. The subjects of recent spectrophotometric studies are tabulated, and the paper concludes with a short account of some special researches, and with a general interpretation of material now available.

[1] Payne and Hogg, *Harv. Obs. Circ.* 301, 1927.

[2] Greaves, Davidson and Martin, *Mon. Not. R. A. S.*, **86**, 1925; *Ibid.*, **87**, 1927.

[3] Eddington, *The Internal Constitution of the Stars*, **1926**, p. 342.

[4] Milne, *Mon. Not. R. A. S.*, **84**, 1924; *Ibid.*, **85**, 1925.

[5] Dunham, *Harv. Obs. Bul.* 853, 1927.

[6] Payne, *Harv. Obs. Circ.* 302, 1927.

[7] Payne, *Harv. Obs. Monograph*, No. 1, **1925**, pp. 116–132.

[8] Shapley, *Harv. Obs. Circ.* 313, 1927.