ADIABATIC EXPANSION IN CASE OF VANISHING INCREMENTS

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1. *Introductory.*—The apparent possibility of reducing measurements in adiabatic expansion of gases to a ratio of successive micrometer shifts of the U-gauge interferometer seems so simple as to be worth investigating. Thus, for instance, the well-known equation of the Clement and Desormes method, in which there are 3 groups of correlative temperatures (absolute) and pressures ($\tau_1 p_1$, $\tau_2 p_2$, $\tau_3 p_3$) may be written (the subscripts denoting the direction of differences), $k/c = \Delta_{12}\log p/(\Delta_{12}\log p - \Delta_{12}\log\tau)$ if $\tau_1 p_1$ change adiabatically to τ_2 and p_2, the latter being atmospheric pressure. In the closed vessel let τ_2, p_2, change to τ_3, p_3 isometrically; then $k/c = \Delta_{12}\log p/(\Delta_{13}\log p - \Delta_{13}\log\tau)$. If now the pressure differences are all very small, so that the interferometer may be used, this equation approaches $k/c = \Delta_{12}p/(\Delta_{13}p - (p/\tau)\Delta_{13}\tau)$. Supposing that the first and final temperatures are appreciably the same so that $(p/\tau)\Delta_{13}\tau$ may be regarded as a correction $k/c = \Delta_{12}p/\Delta_{13}p$ which are the two micrometer shifts stated. The curious difficulty was encountered that the gas would not flow at a rate compatible with the experiments in question.

2. *Apparatus.*—To try this out the apparatus heretofore described was modified as shown in figure 1*a*. m,m' are the (shallow) pools of mercury of the gauge, the side v' being open to the atmosphere. The volume v communicates with the Dewar flask D, containing the sensitive thermometer T and provided with the wide stopcock C, also open to the atmosphere. The experiment begins with an exhaustion (τ_1,p_1) as indicated in the figure. C is then suddenly opened and immediately closed ($\tau_2 p_2$) so that atmospheric pressure p_2 may apparently be assumed. Thereafter with the region Dv closed, τ_3,p_3 is taken. The essential increments are $p_2 - p_1$ and $p_3 - p_1$.

As the total volume Dv was not much over half a liter (555 cm.^{3}), a clear way half-inch stopcock (length 9 cm., diameter 1.3 cm.) seemed sufficient for the transfer of air and was used.

Small increments presupposed, we may deduce from $pv = Rm\tau$ (if as before the total volume $v = Al$, A being the area of the cistern v in figure), since p varies as h, $dh/h + dh/2l = dm/m + d\tau/\tau$. Hence, the virtual pressure increment resulting from the influx of a mass dm of gas is $\Delta h = dh(1 + h/2l)$, if the change of temperature is temporarily disregarded. The corresponding increase of mass will be $dm = mdh(1/h + 1/2l) = m\Delta h/h$. If $m = 0.66$ gram, $h = 76$ cm., $l = 8$ cm., $\Delta h = 5.75\ dh$ and

$dm = 0.0505\ dh$. These constants will presently be used. The micrometer reading $r = 0.71h$.

3. *Data.*—The results obtained with this apparatus were disconcerting. Beginning with the slightly exhausted region (τ_1, p_1) it was found that suddenly opening and closing the stopcock, *C*, produced scarcely any change of pressure p. In fact, to increase this to atmospheric pressure p_2, it was necessary to keep the stopcock fully open more than one second. An example of the early results is given in table 1, where t denotes the time during which the stopcock was open, dh (cm. of mercury) the degree of exhaustion observed thereafter, so that $dh - dh_0$ is the observed rise of pressure due to the influx in t sec. of dm grams of air.

TABLE 1—SHOWING MASS INFLUX (dm) OF GAS IN t SEC.

t	dh	$dh - dh_0$	$\Delta h - \Delta h_0$	Δm
0 *sec.*	−0.00625 *cm.*	0.00000 *cm.*	0.0000 *cm.*	0.000000 *g.*
1	355	270	155	136
2	156	469	270	237
3	021	604	347	305

These data are reproduced in figure 1 with time (t in sec.) as the abscissa. To obtain them the day must be free from wind, since the v' end of the apparatus is open to the atmosphere, while v is closed. This means swaying fringes. Moreover, the results vary in different experiments and the volume v may be filled in 2 seconds, though never in 1 second. Mean initial rates dh/dt during the first second cluster about 0.003 cm. of Hg per sec. The exhaustion being slight the influx during the first second is about 0.00014 gram.

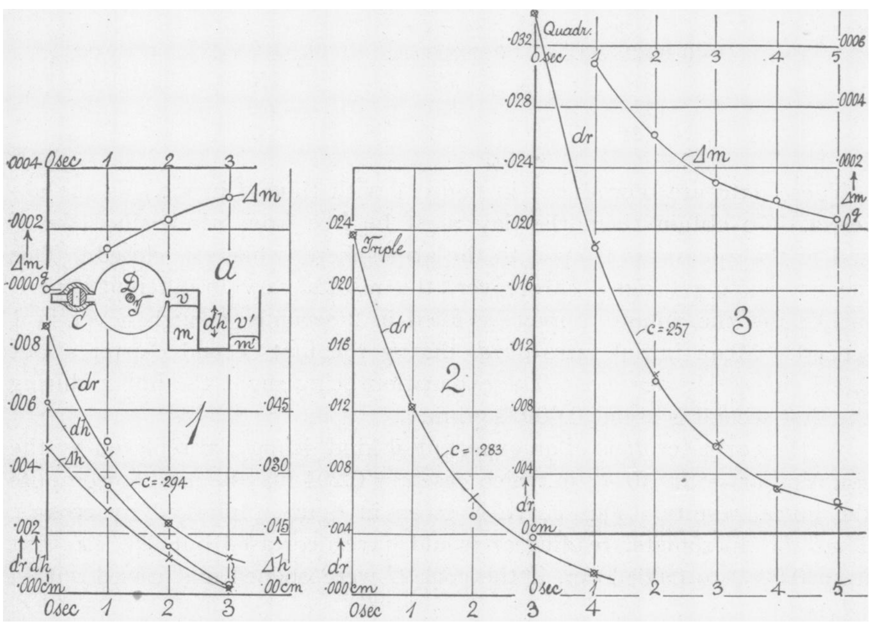
As the curve for dr is of an exponential kind, it may be replaced by the equation $dr = dr_0\ 10^{-ct}$, in which case $c = 0.294$ may be used to compute the initial results. This curve is given in figure 1 marked by crosses. Thus the micrometer reading dr would have decreased but 7%, i.e., but from 0.0089 to 0.0083 cm. if the cock C were opened and closed within $^1/_{10}$ sec.

4. *Viscosity.*—It is therefore necessary to account for this belated influx and the resistance in the stopcock first suggests itself. Since the pressure differences are kept small ($dh < 0.005$ cm. Hg) it seems not unreasonable to use the equation of viscous flow* $vp = Rm\tau = (\pi t r^4/16\eta L)(P^2{}_1 - p^2{}_1)$ neglecting surface slip. Here $r = 0.65$ cm. and $L = 9$ cm. are the radius and length of the influx pipe and P_1 and p_1 the pressures at its ends; p, v, m, τ, referring to the influx gas as above. Since $P^2 - p_1{}^2 = 2pdp$ very nearly, the equation may be written $dm = (\pi r^4/8\eta L)\rho t dp = C\rho t dp$ where $\eta = 0.000181$ is the viscosity, $\rho = 0.00118$ the density of the gas and $dp = dh\rho_m g$, if ρ_m is the density of mercury. Thus $C = 43.0$, $C\rho = 0.0508$ and $dm/dh = 677$ grams per second.

Hence, if such pressure differences *dh* as are given in the table were maintained, the influx of gas would be in 0–1 sec., 3.3 grams, etc., and therefore enormously out of proportion with the data for *dm* there inscribed. In other words, if we compute the observed successive values of the mass influx and compare these with the data obtained from the viscosity of air, the latter are over 22,000 times larger. Hence the viscous resistance of the tube is of no relevant consequence.

The same may be said of the inertia of the gas under the forces available, even if small.

The most likely explanation is to be sought in the U-gauge itself. If the cock *C* is opened and closed rapidly enough, the gas should cool thereafter. Hence the full equation may be taken to be $(dh/h)(1 + h/21) =$



$dm/m - (R/k)(dh/h)$, the last term being the equivalent of adiabatic $d\tau/\tau$. Numerically the equation then becomes $(dh/h)(1 + 4{,}75 + 0.289) = dm/m$. The quantity in parenthesis shows that somewhat less than 17% are to be ascribed to the observed change of head, nearly 80% to the increased volume of the gauge due to the falling head, and nearly 5% to the assumed drop of temperature. Thus the two latter causes, amounting to about 84% of the total, do not appear in the observed change of head *dh*. The apparatus automatically re-exhausts itself. If even more than 84% estimate is not recorded, this may be due to an accelerated fall of the gauge level.

5. *Further Experiments*.—In the preceding work the stopcock was kept open for a specified time t by the aid of a metronome beating seconds. It was found that smoother results were obtained by opening the cock for single second intervals *in succession* and observing the micrometer shift dr after each. To do this effectively, moreover, the above initial exhaustion was increased several fold. Figure 2 gives an example of this kind in which dr, the change of gauge reading ($r = 0.71h$), is taken in the course of seconds and the former exhaustion trebled. Such curves are better adapted to test the equation $dr = dr_0\,10^{-ct}$ and the computed curves (crosses) with their extinction constants c are also given in the diagram. They show that the original exhaustion falls off $^1/_2$ in log $2/c = 1.06$ sec. and this mean rate is pretty well sustained throughout; or that the rate at which the exhaustion vanishes per sec. is proportional to the exhaustion. The rates are slower here where the cock is opened for 1 sec. successively, than when it was kept open for the corresponding number of seconds. The curve, figure 3, was obtained for an expansion four times the original value and is the best of the series when tested by the equation $dh = dh_0\,10^{-ct}$ where $dh = 0.71dr$. The new results thus with increased precision corroborate the remarks already made above.

* O. E. Meyer, *Pogg. Ann.*, **127**, p. 269, 1866.

THE CONDITION OF SELF-OSCILLATION OF A GENERAL TRIODE SYSTEM

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Under the hypotheses of the equivalent plate and grid circuit theorems for voltages and currents of small amplitudes,[1] it can be shown that the general equivalent circuit of a vacuum tube is that shown in figure 1, where Z_c, Z_m and Z_b are the grid-circuit, the mutual-circuit and the plate-circuit complex impedances, respectively, and μ_p, μ_g, r_p and r_g are the usual tube parameters,[2] assumed to be constant. The currents and voltages are as shown. It is proposed in this paper to develop the general relations between the circuit parameters causing the condition of self-oscillation.[3] In order that it may be quite clear what is meant by "the condition of self-oscillation," the following definition is made:

1. If we apply a complex voltage (E) to the vacuum tube circuit, shown in figure 1, at any point such that this voltage causes complex currents to flow in the various branches of the circuit, then the relation