If B is the interior of a limited analytic curve C and if C_R has the same generic meaning as before, we have (6) *valid not merely for z in B but also for z interior to the largest curve C_R which contains in its interior no singularity of $F_0(z)$—such a curve exists and is exterior to C—and the convergence in* (6) *is uniform on an arbitrary closed point set interior to this C_R.*

These two theorems and generalizations can be proved by methods and results due to Carathéodory and Fejér, Gronwall, Schur, R. Nevanlinna, Carathéodory, and Tonelli. Some of the results of Theorem II have recently been obtained by Julia, for special auxiliary conditions of the form $F(O) = 0$, $F'(O) = 1$. This case is connected with the conformal mapping of the region B onto a circle, as is the case of Theorem I for which $f(z)$ is of the form $1/z$. Detailed references to the literature and detailed proofs of Theorems I and II will appear later in another journal.

[1] The notation here is meant to imply that we allow z in B to approach C in any way whatever, and take the least upper bound of all corresponding limits of $|f(z) - f_0(z)|$.

ON THE SPHERICALLY SYMMETRICAL STATICAL FIELD IN EINSTEIN'S UNIFIED THEORY: A CORRECTION

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In a previous paper[1] the authors of the present note have treated the case of a spherically symmetrical statical field, and stated the conclusions: first, that under Einstein's definition of the electromagnetic potential an electromagnetic *field* is incompatible with the assumptions of static spherical symmetry and of symmetry of past and future; second, that if one uses the Hamiltonian suggested in Einstein's second 1928 paper[2] the electromagnetic potential vanishes and the gravitational field also vanishes. We have recently become aware that the method used in that paper is incorrect because the definition of $\Lambda^{\nu}_{\mu\sigma}$ given on page 353 is valid only if the coördinates in the local axes are taken as Cartesian. In this note we replace the reasoning of the previous paper by an argument employing Cartesian local coördinate axes and Cartesian Gaussian coördinates in the continuum. *None of the conclusions of the previous paper are vitiated by this investigation,* although some of the final formulas are supplemented by an additional term.

Let d^1x, d^2x, d^3x, d^4x be the elements of length along the axes of the local quadruple ("4-Bein"), x, y, z, t, the Gaussian coördinates. We retain the assumptions and notations of the previous paper but introduce the abbreviations $(U-1)/\Lambda^2 = A$, $M/\Lambda = B$, $N/\Lambda = C$. Then

$$\begin{aligned} d^1x &= dx + Ax(x\,dx + y\,dy + z\,dz) + Bxdt \\ d^2x &= dy + Ay(x\,dx + y\,dy + z\,dz) + Bydt \\ d^3x &= dz + Az(x\,dx + y\,dy + z\,dz) + Bzdt \\ d^4x &= Cxdx + Cydy + Czdz + Wdt. \end{aligned}$$

The covariant components ${}^{a}h_{\lambda}$ therefore become

$$\begin{array}{llll} {}^1h_1 = 1 + Ax^2 & {}^1h_2 = Axy & {}^1h_3 = Axz & {}^1h_4 = Bx \\ {}^2h_1 = Axy & {}^2h_2 = 1 + Ay^2 & {}^2h_3 = Ayz & {}^2h_4 = By \\ {}^3h_1 = Axz & {}^3h_2 = Ayz & {}^3h_3 = 1 + Az^2 & {}^3h_4 = Bz \\ {}^4h_1 = Cx & {}^4h_2 = Cy & {}^4h_3 = Cz & {}^4h_4 = W \end{array}$$

The determinant $D = |{}^{a}h_{\lambda}| = UW - MN$ as before. The contravariant components ${}_{a}h^{\lambda}$ are [putting $E = (BC - AW)/D$]

$$\begin{array}{llll} {}_1h^1 = 1 + Ex^2 & {}_1h^2 = Exy & {}_1h^3 = Exz & {}_1h^4 = -Cx/D \\ {}_2h^1 = Exy & {}_2h^2 = 1 + Ey^2 & {}_2h^3 = Eyz & {}_2h^4 = -Cy/D \\ {}_3h^1 = Exz & {}_3h^2 = Eyz & {}_3h^3 = 1 + Ez^2 & {}_3h^4 = -Cz/D \\ {}_4h^1 = -Bx/D & {}_4h^2 = -By/D & {}_4h^3 = -Bz/D & {}_4h^4 = (1 + Ar^2)/D \end{array}$$

The components $\varphi_{\mu} = \Lambda^{\alpha}_{\mu\alpha}$ of the electromagnetic potential are

$$\begin{aligned} \varphi_1 &= \frac{x}{2}\left(\frac{2(U-1)}{r^2} + \frac{M'N - UW'}{Dr}\right) \\ \varphi_2 &= \frac{y}{2}\left(\frac{2(U-1)}{r^2} + \frac{M'N - UW'}{Dr}\right) \\ \varphi_3 &= \frac{z}{2}\left(\frac{2(U-U}{r^2} + \frac{M'N - UW'}{Dr}\right) \\ \varphi_4 &= \frac{M}{r} + \frac{1}{2}\,\frac{M'W - MW'}{D} \end{aligned}$$

Returning to polar coördinates, we obtain

$$\begin{aligned} \varphi_r &= \frac{U-1}{r} + \frac{1}{2}\,\frac{M'N - UW'}{D} \quad (\varphi_1 \text{ of previous paper}) \\ \varphi_{\vartheta} &= \varphi_{\varphi} = 0 \\ \varphi_t &= \frac{M}{r} + \frac{1}{2}\,\frac{WM' - MW'}{D} \quad (\varphi_4 \text{ of previous paper}). \end{aligned}$$

These differ from the expressions of the last paper essentially only in the first terms, although there is also an error of sign in the second term of the previous paper due to a mistake in the signs of ${}_4h^1$ and ${}_1h^4$ on p. 354. Under the assumption of symmetrical time $M = N = 0$, and our potential reduces to

$$\varphi_r = \frac{U-1}{r} - \frac{1}{2}\frac{\partial}{\partial r}\log W$$

$$\varphi_\vartheta = \varphi_\varphi = \varphi_t = 0.$$

Thus the electromagnetic *field* vanishes.

We shall now show that if we use the Hamiltonian $H = g^{\lambda\mu}\Lambda^{\alpha}_{\lambda\beta}\Lambda^{\beta}_{\mu\alpha}$ of Einstein's second 1928 paper, it follows from the field equations that $U = 1$ and $W =$ const. so that the electromagnetic potential vanishes. On the assumptions of symmetry of past and future the Hamiltonian becomes

$$H = \frac{1}{4U^2}\left[\frac{2(U-1)^2}{r^2} + \frac{W'^2}{W^2}\right]$$

and the field equations are obtained from the variational equations

$$\delta\int_0^\infty r^2 DHdr = 0 \qquad (D = UW).$$

Whether W is real or imaginary, W'/W is real, so that the expression within the brackets is the sum of two squares of real quantities. Furthermore, U and W do not pass through zero in any physically significant case. Thus the condition for an absolute minimum is $U = 1$, $W =$ const., and all components of the electromagnetic potential vanish.

In the absence of time symmetry the tensor $\Lambda^{\mu}_{\gamma\delta}$ possesses in general 36 non-vanishing Cartesian components. The condition of time symmetry reduces this number to 18. If now $U = 1$, $W =$ const., all vanish and the space is Euclidean. In this case the gravitational field vanishes as well.

[1] *Proc. Nat. Acad. Sci.*, **15**, 353, 1929.

[2] A. Einstein, *Berliner Berichte*, **1928**, 224–227.