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THE HIGH FREQUENCY SPECTRUM OF TUNGSTEN

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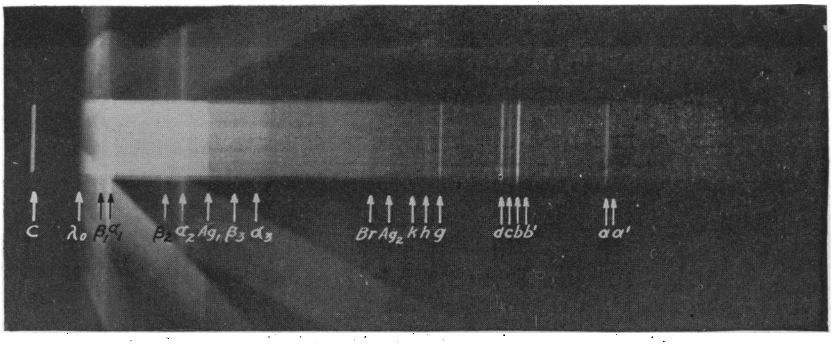
Mosely[1] has shown that the high frequency spectrum given out by the target of an X-ray tube consists of two series of lines superimposed upon a continuous spectrum. The lines, which are known as the *K* and *L* series, respectively, are characteristic of the material of the target. Mosely measured the wave lengths of most of the *K* lines for elements having atomic weights between aluminum and silver, and of the *L* lines for elements from calcium to gold, and showed that for all the lines measured the square roots of the frequencies of corresponding lines are proportional to the atomic numbers of the elements emitting them. Malmer[2] has added to this list the *K* lines of six more elements between silver and lanthanum, and W. H. Bragg[3] and others have studied in great detail the lines of a few of these elements, especially rhodium and platinum.

The continuous or band spectrum was observed qualitatively by Mosely (l.c.), and its short wave-length limit at different voltages measured by Duane and Hunt,[4] who found this limiting frequency, ν_{max}, to be exactly proportional to the voltage on the tube, and given accurately by the quantum relation $h\nu_{max} = eV$, where V is the voltage on the tube, e the charge of an electron, and h Planck's constant.

The spectrum of tungsten is of special interest on account of its use as target material in X-ray tubes, and it has been the subject of several recent investigations. Barnes[5] measured the *L* lines, but was unable to find any *K* lines, although his voltage, 96,000, was sufficiently high for their excitation. Gorton[8] also measured the *L* lines. Rutherford, Barnes and Richardson,[6] using the coefficient of absorption method, measured the effective wave-length of the 'end radiation,' i.e., the short wave-length limit of the continuous spectrum, for different voltages up to 180,000, and found that this minimum wave-length did not decrease continuously with increase of voltage, but approached asymptotically a limiting value of 0.172 angstrom units. As will be shown below, the wave-lengths found by the spectrometer are much shorter, and do not appear to approach any limiting value.

The spectrum shown in figure 1 was taken in the usual manner with a rock salt crystal in continuous slow rotation, photographic plate stationary at 19.13 cm. distance from the crystal, collimating slits 0.2 mm. wide and 20 cm. apart, with a Coolidge tube running at 1 milliampere and 100,000 volts constant potential. The horizontal band gives the reflection from the cubic (100) planes of the crystal, those at 45° from the dodecahedral (110) planes, and those between from the tetrahexahedral (210) planes, etc.



WAVE-LENGTHS OF LINES SHOWN IN FIG. 1

L = Designation of line, D = Distance from C in cm., λ = Wave-length in angstroms

L..............	λ_0	β_1	α_1	β_2	α_2		Ag_1	β_3	α_3		Br
D..............	0.964	1.332	1.496	2.614	2.932	2.994	3.332	3.948	4.386	4.494	6.524
λ..............	0.142	0.196	0.219	0.192	0.215	0.220	0.488	0.192	0.212	0.217	0.463

L..............	Ag_2	k	h		g	d	c	b	b'	a	a'
D..............	6.886	7.368	7.604	7.662	7.884	9.074	9.224	9.382	9.558	11.10	11.19
λ..............	0.487	1.033	1.065	1.073	1.100	1.242	1.260	1.280	1.300	1.468	1.480

The wave-lengths of the lines and bands in the horizontal strip are given in the accompanying table. C, in the photograph, is the undeviated central beam, diminished in intensity by passage through a lead strip 2 mm. thick and 5 mm. wide, placed in front of the plate. The shadow of this strip extends only one fourth of the distance from C to λ_0 and is not visible on the plate.

The short wave-length limit of the spectrum, marked λ_0, is the shortest wave-length that is produced by electrons of velocity corresponding to the operating voltage (100,000) and its value, 0.142 angstrom units, agrees very closely with the value given by Duane and Hunt's formula $eV = h\nu_{\max} = hc/\lambda_{\min}$. I have already shown[7] that the proportionality between frequency and voltage holds accurately up to 100,000 volts, and these measurements have since been extended, with less accuracy, up to 150,000 volts. The shortest wave-length so far observed is 8×10^{-10} cm., or 0.08 A.U.

The lines marked $\alpha_1\beta_1$, $\alpha_2\beta_2$, and $\alpha_3\ \beta_3$ are the first, second, and third order reflections respectively of the K_α and K_β lines of tungsten, the α line being a doublet. They are more clearly shown in figure 2, which was taken with narrower slits and greater distance of photographic plate from crystal, so that the doublet is clearly resolved. In this photograph the second order was given an extra exposure, which accounts for the apparent band in the middle. In the photograph of figure 1, all parts were given the same exposure. The wave-lengths of the lines are about 6% less than would be required by Mosely's formula. Slight deviations from the formula have already been noted for the K lines measured by Mosely and Malmer. All these lines, including tungsten, can, however, be correctly represented by the empirical formulae

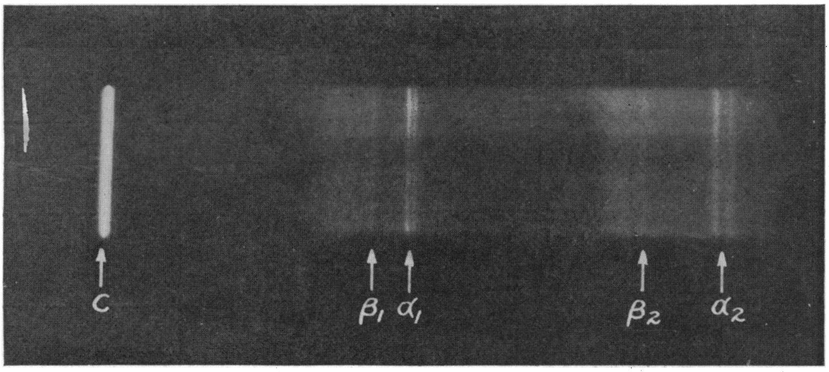


FIG. 2

$$\nu_\alpha = 1.64 \times 10^{15}\ N^{2.10} \text{ for the } \alpha \text{ lines,}$$

and

$$\nu_\beta = 1.56 \times 10^{15}\ N^{2.15} \text{ for the } \beta \text{ lines,}$$

where ν is the frequency and N the atomic number. These formulae, while they have no theoretical significance, may be useful for interpolation.

The bands which terminate on the long wave-length side at Ag_1 and Ag_2 are due to the silver in the photographic plate and are produced by the reflection, in the first and second order respectively, of those wave-lengths which are capable of stimulating the characteristic radiation of silver, that is, those which are shorter than the gamma line of silver. The edge of the band therefore marks the position of the γ line of silver, 0.488 A.U. In the same way the band terminated at Br is due to the bromine in the photographic plate, and marks the position of the γ line of bromine.

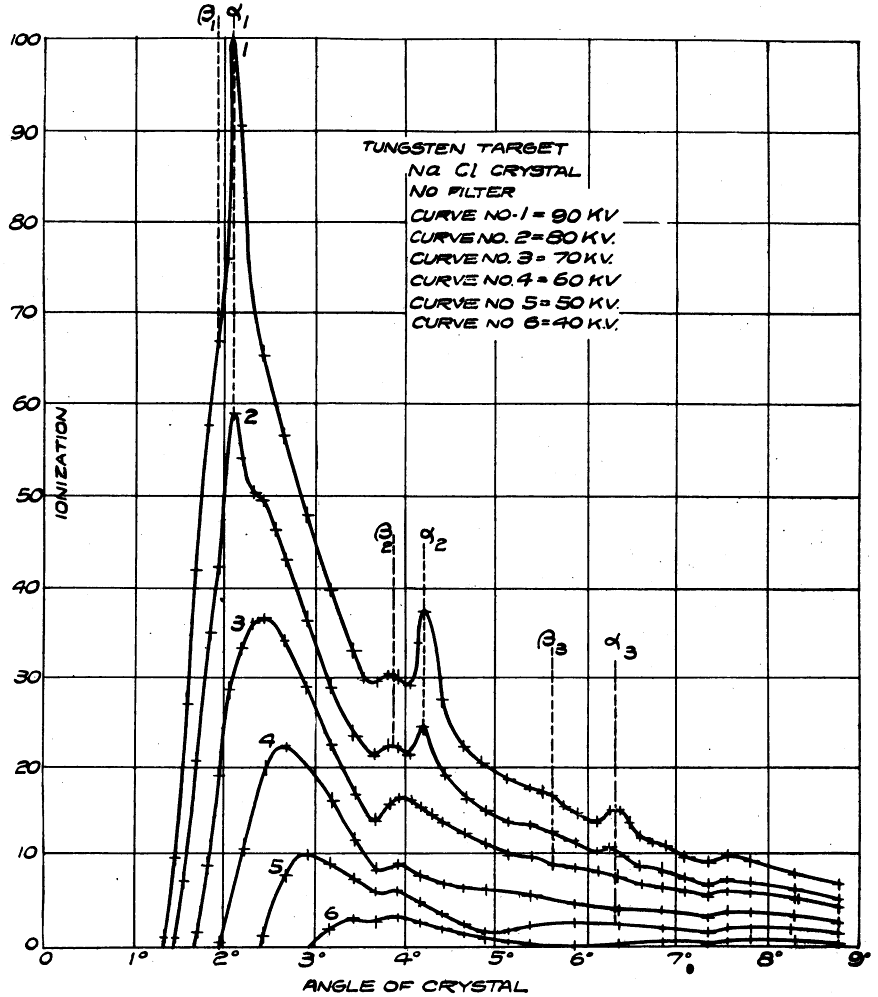


FIG. 3

The remaining nine lines, *a* .... *k*, belong to the *L* spectrum of tungsten, and agree with those found by Barnes,[5] with the inclusion of two faint lines not observed by Barnes. The line marked *h* appears to be a doublet. Gorton's[8] values are all about 2% smaller.

The position of the *K* lines, and their relation to the general radiation at different voltages, has been studied by means of the ionization chamber also. Figure 3 shows the ionization current as a function of the angle of incidence of the rays on the crystal, for five different voltages, and figure 4 a part of the same in the second order. The position of the *K* lines in the first, second and third order respectively, is shown by the dotted lines marked α_1, β_1, α_2, β_2, etc. There is no trace of the lines at 70,000 volts, but at 80,000 they are clearly visible, and increase in intensity as the voltage increases. It is probable that the lowest voltage at which the lines appear is the 'quantum' voltage, i.e., that given by Duane and Hunt's equation, for the γ line, about 70,000 volts for tungsten. This would be in harmony with the mechanism of radiation suggested by Kossel[9] and has already been found by Webster[10] to hold for rhodium. It may be mentioned that the quantum relations

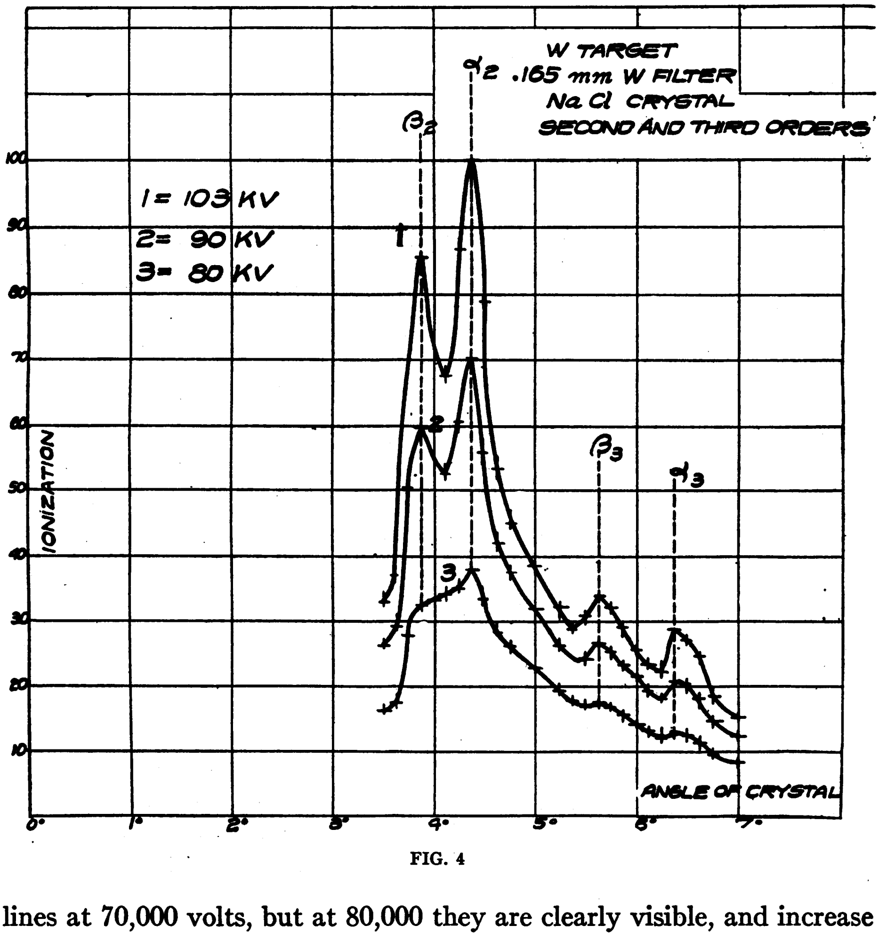


FIG. 4

established by Kossel between the frequencies of the K and L lines hold true for the tungsten lines.

A more detailed account of these and other experiments on the tungsten spectrum, including the distribution of energy in the continuous spectrum, will be published shortly in the *Physical Review.*

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[1] Mosely, *Phil. Mag.*, 26, 210 and 1024 (1913); 27, 710 (1914).

[2] I. Malmer, *Phil. Mag.*, 28, 787 (1914).

[3] W. H. Bragg, *Phil. Mag.*, 29, 407 (1915).

[4] Duane & Hunt, *Physic. Rev.*, 6, 166 (1915).

[5] Barnes, *Phil. Mag.*, 30, 368 (1915).

[6] Rutherford, Barnes and Richardson, *Phil. Mag.*, 30, 339 (1915).

[7] Hull, *Physic. Rev.*, 7, 156 (1916).

[8] Gorton, *Physic. Rev.*, 7, 203 (1916).

[9] Kossel, *Ber. D. Physik. Ges.* 16, 953 (1914).

[10] Webster, these Proceedings, 2, 90 (1916).

ON THE FOUNDATIONS OF PLANE ANALYSIS SITUS

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The notions point, line, plane, order, and congruence are fundamental in Euclidean geometry. Point, line and order (on a line) are fundamental in descriptive geometry. Point, limit-point and regions (of certain types) are fundamental in analysis situs. It seems desirable that each of these doctrines should be founded on (developed from) a set of postulates (axioms) concerning notions that are fundamental for that particular doctrine. Euclidean geometry and descriptive geometry have been so developed.[1] The present paper contains two systems of axioms, Σ_2 and Σ_3, each of which is sufficient for a considerable body of theorems in the domain of plane analysis situs. The axioms of each system are stated in terms of a class, S, of elements called *points* and a class of sub-classes of S called *regions.*

On the basis of Σ_2, *the existence of simple continuous arcs*[2] *is proved as a theorem.*

The system Σ_2 contains an axiom (Axiom 1) which postulates the existence of a countable sequence of regions containing a set of subsequences that close down in a specified way on the points of space. Among other things this axiom implies that the set of all points is separable.[3]

The system Σ_3 is obtained from Σ_2 by replacing Axioms 1, 2, and 4 by three other axioms, Axioms 1′, 2′, and 4′ respectively. Here Axiom 1′ postulates the existence, *for each point P*, of a countable sequence of regions